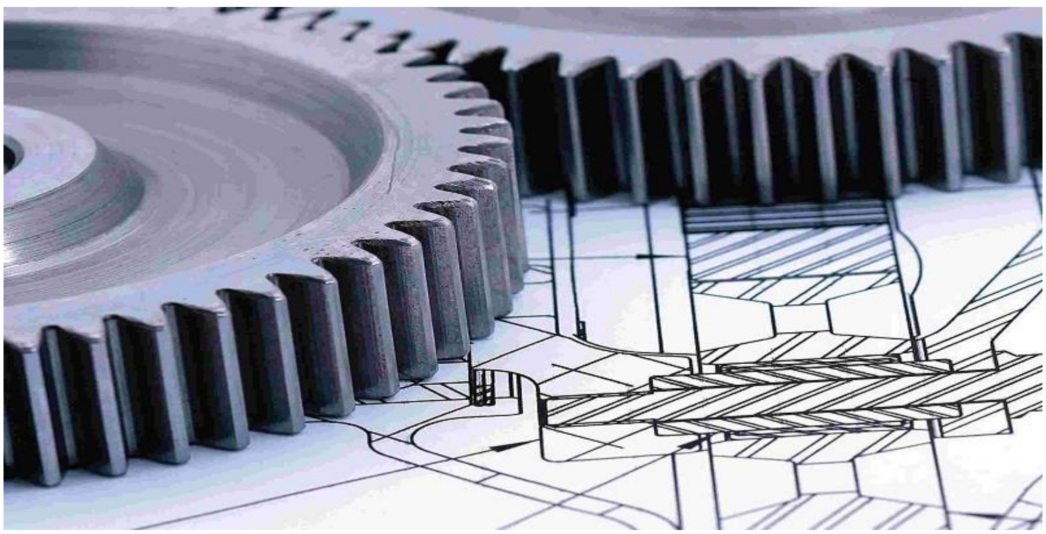
BHADRAK ENGINEERING SCHOOL & TECHNOLOGY
(BEST), ASURALI, BHADRAK

Engineering Mechanics

(Th- 04)

(As per the 2020-21 syllabus of the SCTE&VT, Bhubaneswar, Odisha)



Second Semester

Mechanical Engg.

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ENGINEERING MECHANICS

TOPIC WISE DISTRIBUTION PERIODS

Sl. No.	**Name of the chapter as per the syllabus**	**No of Periods as Per the Syllabus**	**No of Periods Actually Needed**	**Expected marks**
01	Fundamentals of Engineering Mechanics	14	20	24
02	Equilibrium	08	06	17
03	Friction	10	09	19
04	Centroid & moment of Inertia	14	10	19
05	Simple Machines	08	10	19
06	Dynamics	06	08	12
	TOTAL	**60**	**63**	**110**

CHAPTER NO. – 01

FUNDAMENTALS OF ENGINEERING MECHANICS

LEARNING OBJECTIVES:

1.1 Fundamentals: Definitions of Mechanics, Statics, Dynamics, Rigid Bodies,

1.2 Force: Force System. Definition, Classification of force system according to plane & line of action. Characteristics of Force & effect of Force. Principles of Transmissibility & Principles of Superposition. Action & Reaction Forces & concept of Free Body Diagram.

1.3 Resolution of a Force: Definition, Method of Resolution, Types of Component forces, Perpendicular components & non-perpendicular components.

1.4 Composition of Forces: Definition, Resultant Force, Method of composition of forces, such as

1.4.1 Analytical Method such as Law of Parallelogram of forces & method of resolution.

1.4.2. Graphical Method. Introduction, Space diagram, Vector diagram, Polygon law of forces.

1.4.3 Resultant of concurrent, non-concurrent & parallel force system by Analytical & Graphical Method.

1.5 Moment of Force: Definition, Geometrical meaning of moment of a force, measurement of moment of a force & its S.I units. Classification of moments according to direction of rotation, sign convention, Law of moments, Varignon's Theorem, Couple – Definition, S.I. units, measurement of couple, properties of couple.

1.1 Fundamentals: Definitions of Mechanics, Statics, Dynamics, Rigid Bodies:

Engineering Mechanics:

- Mechanics is that branch of physical science which deals with the action of forces on material bodies. Engineering Mechanics, which is very often referred to as Applied Mechanics, deals with the practical applications of mechanics in the field of engineering. Applications of Engineering Mechanics are found in analysis of forces in the components of roof truss, bridge truss, machine parts, parts of heat engines, rocket engineering, aircraft design etc.

Divisions of engineering mechanics:

The subject of Engineering Mechanics may be divided into the following two main groups:

1. Statics and
2. Dynamics.

Statics:

- It is the branch of Engineering Mechanics, which deals with the forces and their effects, while acting upon the bodies at rest.

Dynamics:

- It is the branch of Engineering Mechanics, which deals with the forces and their effects, while acting upon the bodies in motion. Dynamics may be further sub-divided into the following two branches:
    1. Kinematics
    2. Kinetics
- Kinetic deals with the forces acting on moving bodies, whereas kinematics deals with the motion of the bodies without any reference to forces responsible for the motion.

Rigid Body and Elastic Body:

- A body is said to be rigid if it does not undergo deformation whatever force may be applied to the body. In actual practice, there is no body which can be said to be rigid in true sense of terms.
- A body is said to be elastic if it undergoes deformation under the action of force. All bodies are more or less elastic.

1.2 Force: Force System. Definition, Classification of force system according to plane & line of action. Characteristics of Force & effect of Force. Principles of Transmissibility & Principles of Superposition. Action & Reaction Forces & concept of Free Body Diagram

Force:

- Force is that which changes or tends to change the state of rest of uniform motion of a body along a straight line. It may also deform a body changing its dimensions.
- The force may be broadly defined as an agent which produces or tends to produce, destroys or tends to destroy motion. It has a magnitude and direction.
- Mathematically,

  Force = Mass × Acceleration.

  Where, F = force, M = mass and A = acceleration.

Units of force:

- **In C.G.S. System**: In this system, there are two units of force: (1) Dyne and (ii) Gram force (gmf). Dyne is the absolute unit of force in the C.G.S. system. One dyne is that force which acting on a mass of one gram produces in it an acceleration of one centimeter per second^2.
- **In M.K.S. System**: In this system, unit of force is kilogram force (kgf). One kilogram force is that force which acting on a mass of one kilogram produces in it an acceleration of 9.81 m/ sec^2.
- **In S.I. Unit**: In this system, unit of force is Newton (N). One Newton is that force which acting on a mass of one kilogram produces in it an acceleration of one m /sec^2.

  1 Newton = 10^5 Dyne.

Effect of force:

A force may produce the following effects in a body, on which it acts:

- It may change the motion of a body. i.e., if a body is at rest, the force may set it in motion. And if the body is already in motion, the force may accelerate or decelerate it.
- It may retard the forces, already acting on a body, thus bringing it to rest or in equilibrium.
- It may give rise to the internal stresses in the body, on which it acts.
- A force can change the direction of a moving object.
- A force can change the shape and size of an object.

Characteristics of a Force:

In order to determine the effects of a force, acting on a body, we must know the following characteristics of a force:

- Magnitude of the force (i.e., 50 N, 30 N, 20N etc.)
- The direction of the line, along which the force acts (i.e., along West, at 30° North of East etc.). It is also known as line of action of the force
- Natures of the force (push or pull).
- The point at which (or through which) the force acts on the body.

System of forces:

When two or more forces act on a body, they are called to form a system of forces. Force system is basically classified into following types.

- Collinear forces
- Coplanar forces
- Concurrent forces
- Coplanar concurrent forces
- Coplanar non- concurrent forces
- Non-coplanar concurrent forces
- Non- coplanar non- concurrent force

Collinear forces:

- The forces, whose lines of action lie on the same line, are known as collinear forces. They act along the same line. Collinear forces may act in the opposite directions or in the same direction.

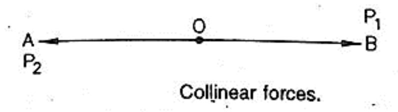


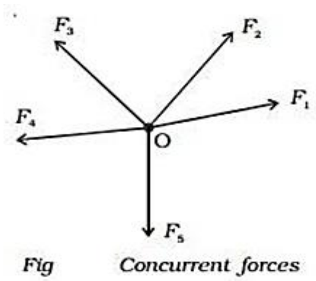
Fig 1.1

Coplanar forces:

- The forces, whose lines of action lie on the same plane, are known as coplanar forces.

Concurrent forces:

- The forces, whose lines of action pass through a common point, are known as concurrent forces. The concurrent forces may or may not be collinear.



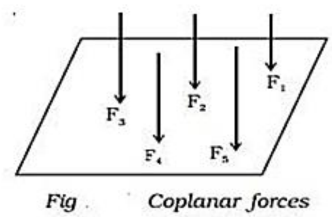


Fig. 1.2 Fig. 1.3

Coplanar concurrent forces:

- The forces, whose lines of action lie in the same plane and at the same time pass through a common point, are known as coplanar concurrent forces.

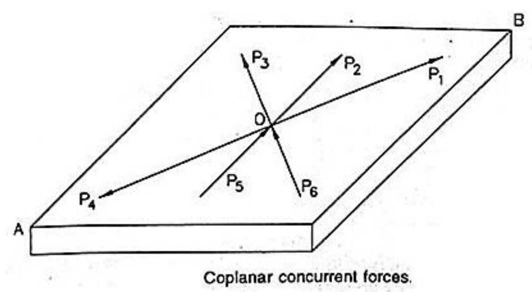


Fig 1.4

Coplanar non-concurrent forces:

- The forces, which do not meet at one point, but their lines of action lie on the same plane, are known as coplanar non-concurrent forces.

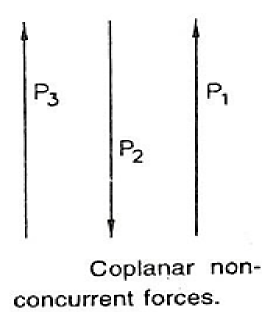


Fig 1.5

Non-coplanar concurrent forces:

- The forces, which meet at one point, but their lines of action do not lie on the same plane, are known as non-coplanar concurrent forces.

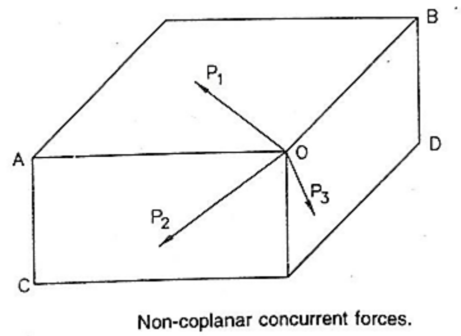


Fig 1.6

Non-coplanar non-concurrent forces:

- The forces, which do not meet at one point and their lines of action do not lie on the same plane, are called non-coplanar non-concurrent forces.

Pull and push:

- Pull is the force applied to a body at its front end to move the body in the direction of the force applied.
- Push is the force applied to a body at its back end in order to move the body in the direction of the force applied.

Fig 1.7 push and pull

Action and reaction:

- When two bodies are in contact with each other, then each force exerts a force on the other. One of these two forces is called Action and the other is called Reaction.
- **Example**: When a body having a weight W (=mg) is placed on a horizontal plane as shown in Fig 1.8, the body exerts a vertically downward force equal to 'W' or 'mg' on the plane. Then 'W' is called action of the body on the plane. According to Newton's 3rd law of motion, every action has an equal and opposite reaction. But action and reaction never act on the same body. So, the horizontal plane will exert equal amount of force 'R' on the body in the vertically upward direction. This vertically upward force acting on the body is called reaction of the plane on the body.

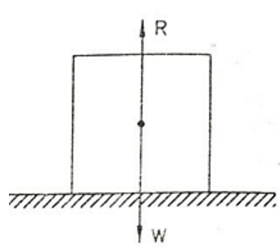
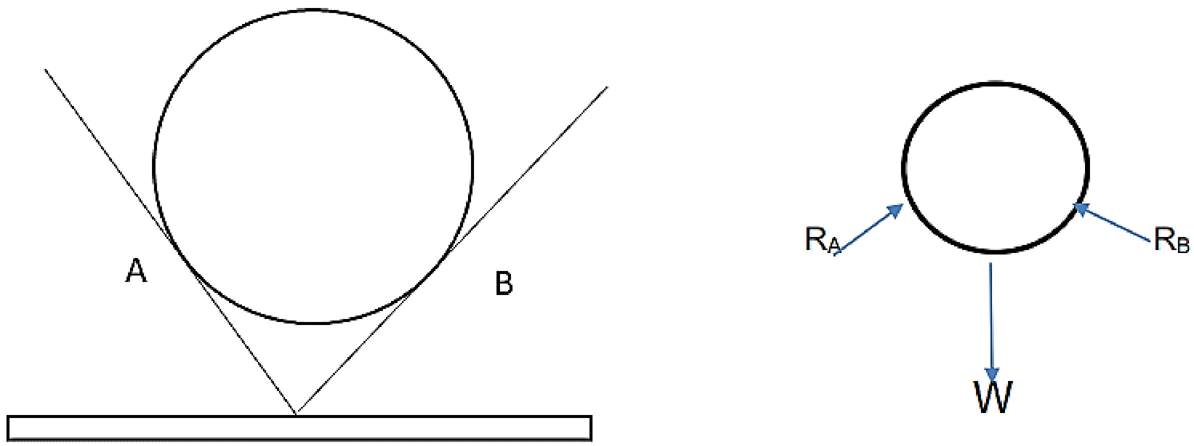


Fig 1.8 Action and reaction

Free body diagram:

- The representation of reaction force on the body by removing all the support or forces act from the body is called free body diagram.



Object with support | Free body diagram

Fig. 1.9

External Force and Internal Force:

- When a force is applied externally to a body; that force is called external force. Internal force is that force which is set up in a body to resist deformation of the body caused by the external force.

Representation of a Force:

Since force is a vector quantity, it can be represented by a straight line. The length of the line represents magnitude of the force, the line itself represents the direction and an arrow put on the head of the straight line indicates the sense in which the force acts.

Denoting A Force by Bow's Notation:

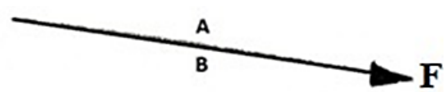


Fig 1.10

- In Bow's notation for denoting a force, two English capital letters are placed, one on each side of the line of action of the force. In figure 1.10 AB denotes the force F.

Principle of Transmissibility of Forces:

- It states, "If a force acts at any point on a rigid body, it may also be considered to act at any other point on its line of action, provided this point is rigidly connected with the body". That means the point of application of a force can be moved anywhere along its line of action without changing the external reaction forces on a rigid body.

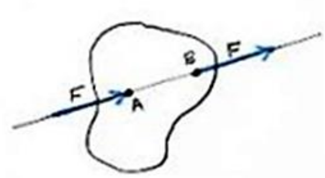


Fig 1.11

- Here, force at point A = force at B (the magnitude of force in the body at any point along the line of action are same)

Principle of Superposition of Forces:

- This principle states that the combined effect of force system acting on a particle or a rigid body is the sum of effects of individual forces.
- Consider two forces P and Q acting at A on a boat as shown in Fig 1.12. Let R be the resultant of these two forces P and Q. According to Newton's second law of motion, the boat will move in the direction of resultant force R with acceleration proportional to R. The same motion can be obtained when P and Q are applied simultaneously.

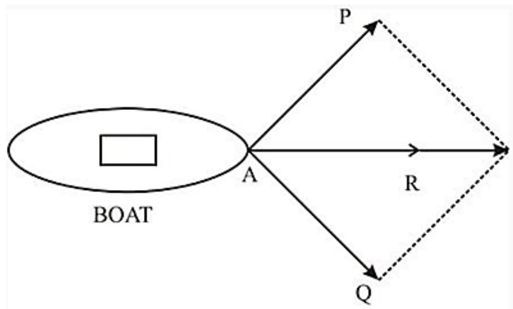


Fig 1.12

1.3 Resolution of a Force: Definition, Method of Resolution, Types of Component forces, Perpendicular components & non-perpendicular components

Resolution of a Force:

- The process of splitting up the given force into a number of components, without changing its effect on the body is called resolution of a force. A force is, generally, resolved along two mutually perpendicular directions.

Method of resolution:

- Resolved all the forces horizontally and find the algebraic sum of all the horizontal components (i.e., $\sum H$).
- Resolved all the forces vertically and find the algebraic sum of all the vertical components (i.e., $\sum V$).
- The resultant R of the given forces will be given by the equation:

$$\boldsymbol{R} = \sqrt{\left(\sum \boldsymbol{H}\right)^{2} + \left(\sum \boldsymbol{V}\right)^{2}}$$

- The resultant force will be inclined at an angle θ, with the horizontal, such that

$$\tan \boldsymbol{\theta} = \frac{\sum V}{\sum H}$$

Types of Component Forces:

Generally, a force is resolved into two types of components

- Mutually perpendicular components
- Non perpendicular components

Mutually Perpendicular Components:

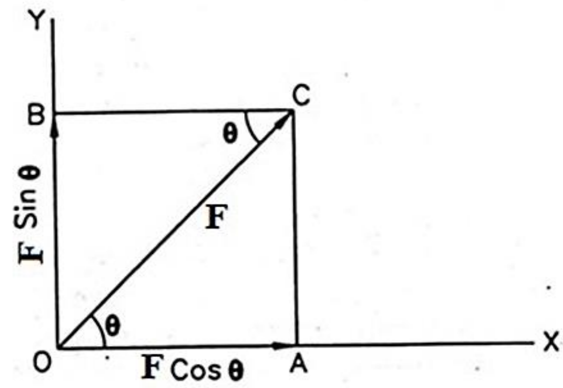


Fig 1.13

- Resolved parts of a force means components of the force along two mutually perpendicular directions.
- Let a force F represented in magnitude and direction by OC make an angle θ with OX. Line OY is drawn through O at right angles to OX as shown in figure 1.13.
- Through C, lines CA and CB are drawn parallel to OY and OX respectively. Then the resolved parts of the force F along OX and OY are represented in magnitude and direction by OA and OB respectively.
- Now in the right angled Δ AOC,

$$\boldsymbol{\cos\theta = \frac{OA}{OC} = \frac{OA}{F}}$$
$$\Rightarrow \boldsymbol{OA = F\cos\theta}$$

$$\boldsymbol{\sin\theta = \frac{AC}{OC} = \frac{AC}{F}}$$
$$\Rightarrow \boldsymbol{AC = OB = F\sin\theta}$$

- Thus, the resolved parts of F along OX and OY are $\boldsymbol{F\cos\theta}$ and $\boldsymbol{F\sin\theta}$ respectively.

Non perpendicular components:

- Let P be the given force represented in magnitude and direction by OB as shown in Fig 1.14. Also let OX and OY be two given directions along which the components of P are to be found out.

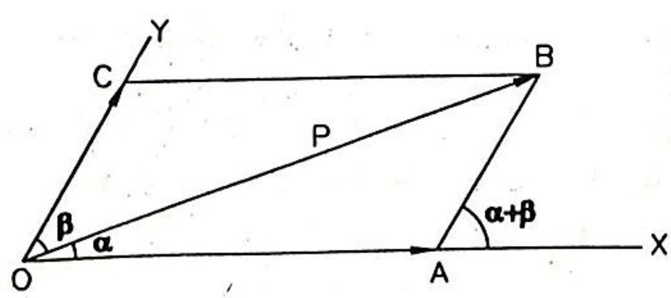


Fig 1.14

- Let ∠BOX = α and ∠BOY = β
- From B, lines BA and BC are drawn parallel to OY and OX respectively. Then the required components of the given force P along OX and OY are represented in magnitude and direction by OA and OC respectively.
- Since AB is parallel to OC, $\boldsymbol{\angle BAX = \angle AOC = \alpha + \beta}$ And $\boldsymbol{\angle OAB = 180^0 - (\alpha + \beta)}$

- Now, in Δ OAB

$$\frac{\boldsymbol{OA}}{\boldsymbol{sin\beta}} = \frac{\boldsymbol{AB}}{\boldsymbol{sin\alpha}} = \frac{\boldsymbol{OB}}{\boldsymbol{sin180^0 - (\alpha + \beta)}}$$

$$\Rightarrow \frac{\boldsymbol{OA}}{\boldsymbol{sin\beta}} = \frac{\boldsymbol{AB}}{\boldsymbol{sin\alpha}} = \frac{\boldsymbol{P}}{\boldsymbol{sin(\alpha + \beta)}}$$

$$\therefore \boldsymbol{OA} = \frac{\boldsymbol{P\ sin\beta}}{\boldsymbol{sin(\alpha + \beta)}} \quad \boldsymbol{And} \quad \boldsymbol{AB = OC} = \frac{\boldsymbol{P\ sin\alpha}}{\boldsymbol{sin(\alpha + \beta)}}$$

1.4 Composition of Forces: Definition, Resultant Force, Method of composition of forces

Composition of Forces:

- The process of finding out the resultant force of a number of given forces is called composition of forces or compounding of forces.

Resultant Force:

- It is a single force replaced by number of forces acting upon a rigid body, whose effect on it is same as the combined effect of all forces.

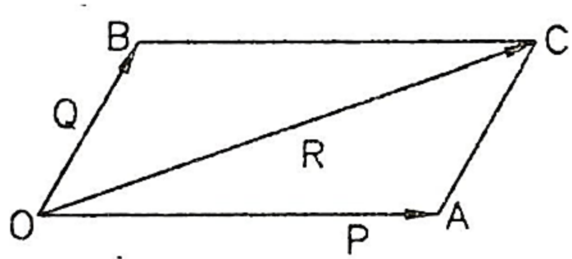


Fig 1.15

- In figure 1.15, ***R*** is the resultant of forces ***P*** and ***Q***. If R is the resultant of two forces P and Q, it means forces P and Q can be replaced by R. Similarly, R can be replaced by two forces P and Q whose joint effect on a body will be the same as R on the body. Then these two forces P and Q are called components of R.

Equilibriant:

- Equilibrant of a system of forces is a single force which will keep the given forces in equilibrium. Evidently, equilibrant is equal and opposite to the resultant of the given forces.

Method of Composition of Forces:

There are two important methods of composition of forces.

- Analytical method
- Graphical method

1.4.1 Analytical Method such as Law of Parallelogram of forces & method of resolution

Analytical Method:

Parallelogram law of forces:

- It states, ***"If two forces, acting simultaneously on a particle, be represented in magnitude and direction by the two adjacent sides of a parallelogram; their resultant may be represented in magnitude and direction by the diagonal of the parallelogram, which passes through their point of intersection."***
- **Explanation:** Let two forces P and Q are acting at a point O be represented in magnitude and direction by OA and OB respectively as shown in Fig 1.16. Then, according to the theorem of parallelogram of forces, the diagonal OC drawn through O represents the resultant of P and Q in magnitude and direction.

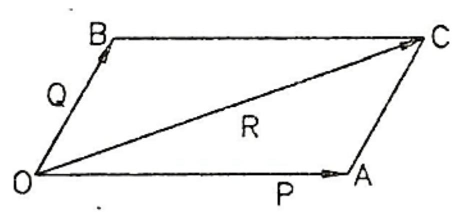


Fig 1.16

- **Proof:**

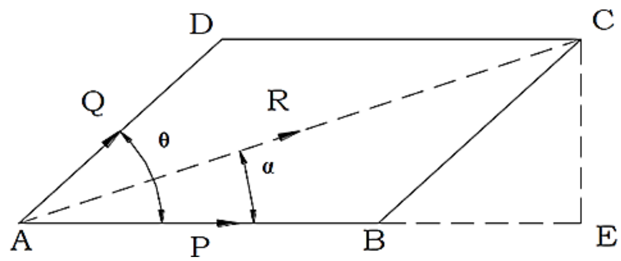


Fig 1.17

- Consider two forces 'P' and 'Q' acting at and away from point 'A' as shown in figure 1.17.
- Let, the forces P and Q are represented by the two adjacent sides of a parallelogram AD and AB respectively as shown in fig. Let, θ be the angle between the force P and Q and α be the angle between R and P. Extend line AB and drop perpendicular from point C on the extended line AB to meet at point E.
- $Consider\ Right\ angle\ triangle\ ACE,$
  $AC^2 = AE^2 + CE^2$
  $= (AB + BE)^2 + CE^2$
  $= AB^2 + BE^2 + 2.AB.BE + CE^2$
  $= AB^2 + BE^2 + CE^2 + 2.AB.BE \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(i)$
- $Consider\ right\ angle\ triangle\ BCE,$
  $BC^2 = BE^2 + CE^2$
  $Cos\ \theta = BE\ /\ BC$
  $\Rightarrow BE = BC.Cos\ \theta$
- $Putting\ BE^2 + CE^2 = BC^2\ \&\ BE = BC.Cos\ \theta\ in\ equation\ (i), we\ get$
  $AC^2 = AB^2 + BC^2 + 2.AB.BC.Cos\ \theta$
  $But, AB = P, BC = Q\ and\ AC = R$

- $So, magnitude\ of\ the\ resultant,$

$$\boldsymbol{R = \sqrt{P^2 + Q^2 + 2PQ\ Cos\ \theta}}$$

- $In\ triangle\ BCE,$
  $Sin\ \theta\ =\ CE\ /\ BC\ \Rightarrow CE\ =\ BC.Sin\ \theta\ =\ Q\ Sin\ \theta$

- $In\ triangle\ ACE,$

$$\boldsymbol{tan\ \alpha = \frac{CE}{AE} = \frac{CE}{AB + BE}}$$

- $So, direction\ of\ the\ resultant,$

$$\boldsymbol{\therefore\ tan\ \alpha = \frac{Q\ Sin\ \theta}{P + Q\ Cos\ \theta}}$$

Now let us consider two forces F_1 and F_2 are represented by the two adjacent sides of a parallelogram

F_1 and F_2 = Forces whose resultant is required to be found out,
θ = Angle between the forces F_1 and F_2, and
α = Angle which the resultant force makes with one of the forces (say F_1).

- Then magnitude of resultant,

$$\boldsymbol{R = \sqrt{{F_1}^2 + {F_2}^2 + 2F_1F_2\ Cos\ \theta}}$$

- And direction of the resultant,

$$\boldsymbol{tan\ \alpha = \frac{F_2\ Sin\ \theta}{F_1 + F_2\ Cos\ \theta}}$$

- If (α) is the angle which the resultant force makes with the other force F_2, then

$$\boldsymbol{tan\ \alpha = \frac{F_1\ Sin\ \theta}{F_2 + F_1\ Cos\ \theta}}$$

<u>CASES</u>:

1. If $\theta = 0$ i.e., when the forces act along the same line, then

$$R_{Max} = F_1 + F_2$$

2. If $\theta = 90°$ i.e., when the forces act at right angle, then

$$R = \sqrt{{F_1}^2 + {F_2}^2}$$

3. If $\theta = 180°$ i.e., when the forces act along the same straight line but in opposite directions, then

$$R_{Min} = F_1 - F_2$$

4. If the two forces are equal i.e., when $F_1 = F_2 = F$ then

$$R = \sqrt{F^2 + F^2 + 2F^2 \cos\theta}$$
$$= \sqrt{2F^2(1 + \cos\theta)}$$
$$= \sqrt{2F^2 \times 2cos^2(\theta/2)}$$
$$= \sqrt{4F^2cos^2(\theta/2)}$$
$$= 2F \cos\theta/2$$

Example – 1: ***Two forces of 100 N and 150 N are acting simultaneously at a point. What is the resultant of these two forces, if the angle between them is 45°?***
Solution. Given: First force (F_1) = 100 N; Second force (F_2) = 150 N and angle between F_1 and F_2 (θ) = 45°.
We know that the resultant force,

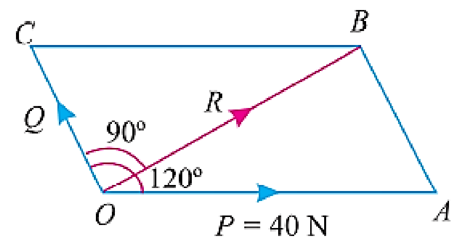
$$R = \sqrt{F_1^{\,2} + F_2^{\,2} + 2F_1F_2\cos\theta}$$

$$= \sqrt{(100)^2 + (150)^2 + 2 \times 100 \times 150 \cos 45^0}$$

$$= \sqrt{10000 + 22500 + (30000 \times 0.707)} = \mathbf{232\ N}$$

Example – 2: ***Two forces act at an angle of 120°. The bigger force is of 40 N and the resultant is perpendicular to the smaller one. Find the smaller force.***

Solution. Given: Angle between the forces ∠AOC = 120^0, Bigger force (F_1) = 40 N and angle between the resultant and F_2 (∠BOC) = 90°;
Let F_2 = Smaller force in N
From the geometry of the figure, we find that ∠AOB,
$\alpha = 120° - 90° = 30°$



We know that,

$$\tan\alpha = \frac{F_2\, sin\,\theta}{F_1 + F_2\, cos\,\theta}$$

$$\Rightarrow \tan 30^0 = \frac{F_2\, sin\, 120^0}{40 + F_2\, cos\, 120^0} = \frac{F_2\, sin\, 60^0}{40 + F_2\,(-cos\, 60^0)}$$

$$\Rightarrow 0.577 = \frac{F_2 \times 0.866}{40 - F_2 \times 0.5} = \frac{0.866\, F_2}{40 - 0.5\, F_2}$$

$$\Rightarrow 40 - 0.5\, F_2 = \frac{0.866\, F_2}{0.577} = 1.5\, F_2$$

$$\Rightarrow 2F_2 = 40$$

$$\Rightarrow \boldsymbol{F_2 = 20\ N}$$

Example – 3: ***Find the magnitude of the two forces, such that if they act at right angles, their resultant is $\sqrt{\mathbf{10}}N$. But if they Act at 60°, their resultant is $\sqrt{\mathbf{13}}N$.***
Solution. Given: Two forces = F_1 and F_2.
First of all, consider the two forces acting at right angles. We know that when the angle between the two given forces is 90°, then the resultant force (R)

$$\sqrt{10} = \sqrt{F_1^{\,2} + F_2^{\,2}}$$

$$\Rightarrow 10 = F_1^{\,2} + F_2^{\,2}$$

$$\ldots (Squaring\ both\ sides)$$

Similarly, when the angle between the two forces is 60°, then the resultant force (R)

$$\sqrt{13} = \sqrt{F_1^{\,2} + F_2^{\,2} + 2\, F_1\, F_2\, \cos 60^0}$$

$$\Rightarrow 13 = F_1^{\,2} + F_2^{\,2} + 2\, F_1\, F_2 \times 0.5 \qquad \ldots (Squaring\ both\ sides)$$

$$F_1 F_2 = 13 - 10 = 3$$

$$\ldots (Substituting\ F_1^{\ 2} + F_2^{\ 2} = 10)$$

We know that,

$$(F_1 + F_2)^2 = F_1^{\ 2} + F_2^{\ 2} + 2\, F_1\, F_2 = 10 + 6 = 16$$

$$\Rightarrow F_1 + F_2 = \sqrt{16} = 4 \ldots \ldots \ldots \ldots \ldots \ldots \ldots (i)$$

Similarly,

$$(F_1 - F_2)^2 = F_1^{\ 2} - F_2^{\ 2} + 2\, F_1\, F_2 = 10 - 6 = 4$$

$$\Rightarrow F_1 - F_2 = \sqrt{4} = 2 \ldots \ldots \ldots \ldots \ldots (ii)$$

Solving equations (i) and (ii),

$$\boldsymbol{F_1 = 3\ N\ and\ F_2 = 1\ N}$$

Method Of Resolution:

- Resolved all the forces horizontally and find the algebraic sum of all the horizontal components (i.e., $\sum H$).
- Resolved all the forces vertically and find the algebraic sum of all the vertical components (i.e., $\sum V$).
- The resultant R of the given forces will be given by the equation:

$$\boldsymbol{R = \sqrt{\left(\sum H\right)^2 + \left(\sum V\right)^2}}$$

- The resultant force will be inclined at an angle θ, with the horizontal, such that

$$\mathbf{tan}\, \boldsymbol{\theta} = \frac{\sum V}{\sum H}$$

Notes: The value of the angle θ will vary depending upon the values of ∑V and ∑H, as discussed below:

1. When ∑V is +ve, the resultant makes an angle between 0° and 180°. But when ∑V is –ve, the resultant makes an angle between 180° and 360°.
2. When ∑H is +ve, the resultant makes an angle between 0° to 90° or 270° to 360°. But when ∑H is –ve, the resultant makes an angle between 90° to 270°.

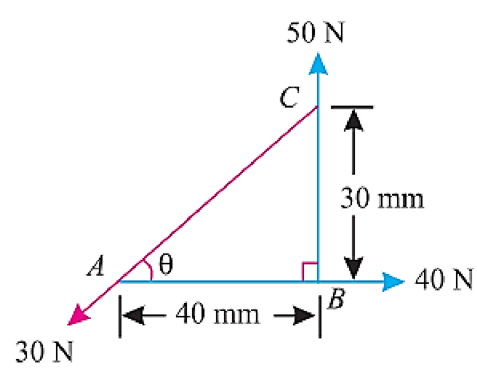
Example – 4: ***A triangle ABC has its side AB = 40 mm along positive x-axis and side BC = 30 mm along positive y-axis. Three forces of 40 N, 50 N and 30 N act along the sides AB, BC and CA respectively. Determine magnitude of the resultant of such a system of forces.***

Solution. The system of given forces is shown in Figure.

From the geometry of the figure, we find that the triangle ABC is a right-angled triangle, in which the side AC = 50 mm. Therefore

$$sin\,\theta = \frac{30}{50} = 0.6$$

$$cos\,\theta = \frac{40}{50} = 0.8$$



Resolving all the forces horizontally (i.e., along AB),

$$\sum H = 40 - 30\, cos\,\theta$$
$$= 40 - (30 \times 0.8) = 16\, N$$

and now resolving all the forces vertically (i.e., along BC)

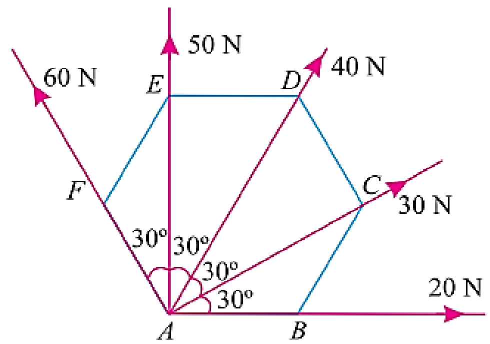
$$\sum V = 50 - 30\, sin\,\theta$$
$$= 50 - (30 \times 0.6) = 32\, N$$

We know that magnitude of the resultant force,

$$R = \sqrt{(\sum H)^2 + (\sum V)^2} = \sqrt{16^2 + 32^2} = 35.8\ N$$

Example – 5: ***The forces 20 N, 30 N, 40 N, 50 N and 60 N are acting at one of the angular points of a regular hexagon, towards the other five angular points, taken in order. Find the magnitude and direction of the resultant force.***

Solution. The system of given forces is shown in Figure.



Magnitude of the resultant force

Resolving all the forces horizontally (i.e., along AB),

$$\begin{aligned}\sum H &= 20\cos 0° + 30\cos 30° + 40\cos 60° + 50\cos 90° + 60\cos 120°\ N \\ &= (20 \times 1) + (30 \times 0.866) + (40 \times 0.5) + (50 \times 0) + 60\,(-0.5)\ N = 36\ N\end{aligned}$$

and now resolving the all forces vertically (i.e., at right angles to AB),

$$\begin{aligned}\sum V &= 20\sin 0° + 30\sin 30° + 40\sin 60° + 50\sin 90° + 60\sin 120°\ N \\ &= (20 \times 0) + (30 \times 0.5) + (40 \times 0.866) + (50 \times 1) + (60 \times 0.866)\ N = 151.6\ N\end{aligned}$$

We know that magnitude of the resultant force,

$$R = \sqrt{(\sum H)^2 + (\sum V)^2} = \sqrt{36^2 + 151.6^2} = 155.8\ N$$

Direction of the resultant force

Let θ = Angle, which the resultant force makes with the horizontal (i.e., AB).

We know that,

$$\tan\theta = \frac{\sum V}{\sum H} = \frac{151.6}{36} = 4.211$$

$$\Rightarrow\ \theta = 76.6°$$

Note. Since both the values of ∑H and ∑V are positive, therefore actual angle of resultant force lies between 0° and 90°.

Example – 6: ***The following forces act at a point:***

(i) 20 N inclined at 30° towards North of East,

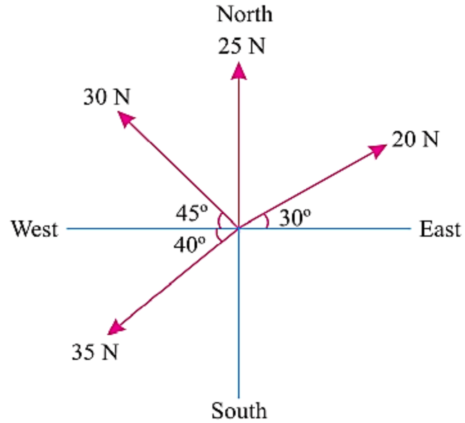
(ii) 25 N towards North,

(iii) 30 N towards North West, and

(iv) 35 N inclined at 40° towards South of West.

Find the magnitude and direction of the resultant force.

Solution. The system of given forces is shown in Figure.



Magnitude of the resultant force

Resolving all the forces horizontally i.e., along East-West line,

$$\sum H = 20\cos 30° + 25\cos 90° + 30\cos 135° + 35\cos 220°\ N$$
$$= (20 \times 0.866) + (25 \times 0) + 30\,(-\,0.707) + 35\,(-\,0.766)N = -30.7\ N$$

and now resolving all the forces vertically i.e., along North-South line,

$$\sum V = 20\sin 30° + 25\sin 90° + 30\sin 135° + 35\sin 220°\ N$$
$$= (20 \times 0.5) + (25 \times 1.0) + (30 \times 0.707) + 35\,(-\,0.6428)N = 33.7\ N$$

We know that magnitude of the resultant force,

$$R = \sqrt{(\sum H)^2 + (\sum V)^2} = \sqrt{(-30.7)^2 + (33.7)^2} = 45.6\ N$$

Direction of the resultant force

Let θ = Angle, which the resultant force makes with the East.

We know that,

$$\tan\theta = \frac{\sum V}{\sum H} = \frac{33.7}{-30.7} = -1.098$$

$$\Rightarrow\ \theta = 47.7°$$

Since ∑H is negative and ∑V is positive, therefore resultant lies between 90° and 180°. Thus, actual angle of the resultant = 180° – 47.7° = 132.3°.

1.4.2. <u>Graphical Method. Introduction, Space diagram, Vector diagram, Polygon law of forces</u>

<u>Graphical Method</u>:

<u>Triangle Law of Forces</u>:

- It states, ***"If two forces acting simultaneously on a particle be represented in magnitude and direction by the two sides of a triangle, taken in order; their resultant may be represented in magnitude and direction by the third side of the triangle, taken in opposite order."***
- **Explanation:** Let two forces P and Q acting at O be such that they can be represented in magnitude and direction by the sides AB and BC of the triangle ABC. Then, according to the theorem of triangle of forces, their resultant R will be represented in magnitude and direction by AC which is the third side of the triangle ABC taken in the reverse order of CA.

- **Proof:**

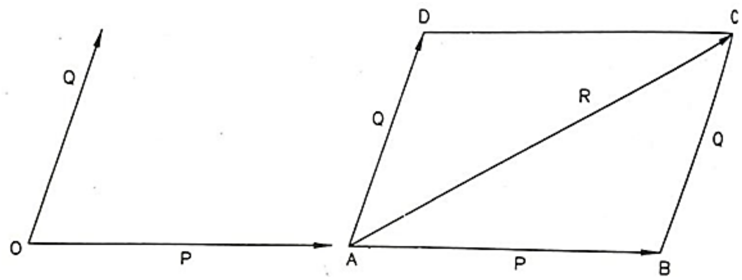


Fig. 1.18

- In Fig.1.18 the parallelogram ABCD is completed with sides AB and BC of the triangle ABC. Side AD is equal and parallel to BC. So, force Q is also represented in magnitude and direction by AD. Now, the resultant of P (represented by AB) and Q (represented by AD) is represented in magnitude and direction by the diagonal AC of the parallelogram ABCD. Thus, the resultant of P and Q is represented in magnitude and direction by the third side AC of the triangle ABC taken in the reverse order.

Polygon Law of Forces:

- It is an extension of Triangle Law of Forces for more than two forces, which states, ***"If a number of forces acting simultaneously on a particle, be represented in magnitude and direction, by the sides of a polygon taken in order then the resultant of all these forces may be represented, in magnitude and direction, by the closing side of the polygon, taken in opposite order."***
- **Proof**:

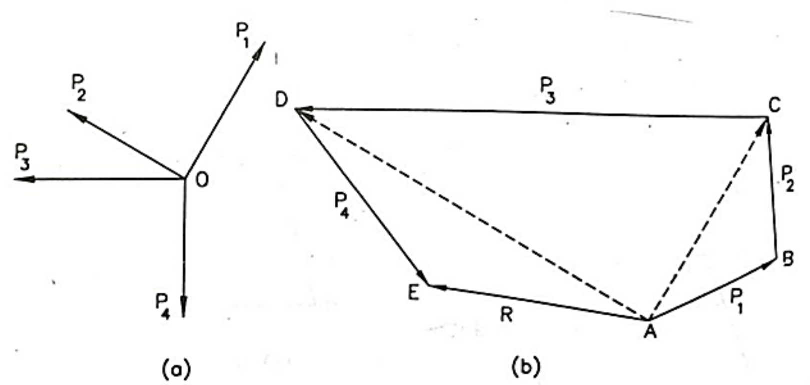


Fig. 1.19

- Let forces P_1, P_2, P_3 and P_4, acting at a point O be such that they can be represented in magnitude and direction by the sides AB, BC, CD and DE of a polygon ABCDE as shown in fig. 1.19.
- We are to prove that the resultant of these forces is represented in magnitude and direction by the side AE in the direction from A towards E.
- According to the triangle law of forces, AC represents the resultant R_1 of P_1 and P_2, AD represents the resultant R_2 of R_1 and P_3. Thus, AD represents the resultant of P_1, P_2 and P_3.
- According to the same law, AE represents the resultant R_3 of R_2 and P_4. Thus, AE represents the resultant of P_1, P_2, P_3 and P_4.

Space Diagram, Vector Diagram and Bow's Notation

Graphical Representation of a Force:

- A force can be represented graphically by drawing a straight line to a suitable scale and parallel to the line of action of the given force and an arrowhead indicates the direction.
- A force in the figure is represented by a vector of length 5 cm (scale 1 cm = 5 N) by drawing a line parallel to the given force and arrowhead indicates the direction of the force.

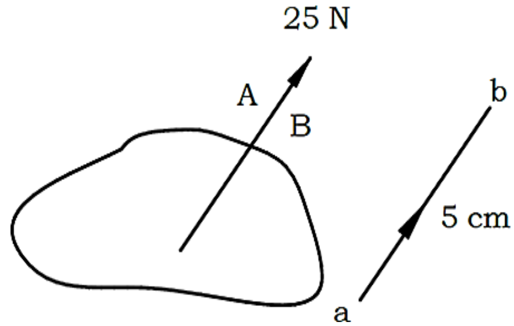


Fig. 1.20

Space Diagram:

- Space diagram is that diagram which shows the forces in space. In a space diagram the actual directions of forces are marked by straight lines with arrow put on their head to indicate the sense in which the forces act. Following Fig. shows the space diagram of forces P_1, P_2, P_3.

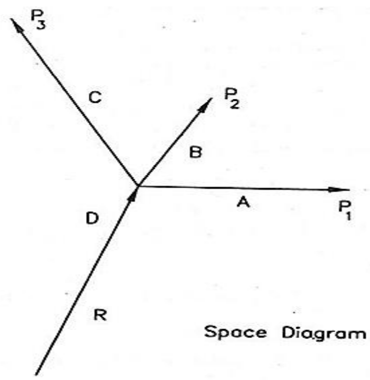


Fig. 1.21

Vector Diagram:

- Vector diagram is a diagram which is drawn according to some suitable scale to represent the given forces in magnitude, direction and sense. The resultant of the given forces is represented by the closing line of the diagram and its sense is from the starting point towards the end point as shown in Fig 1.22.

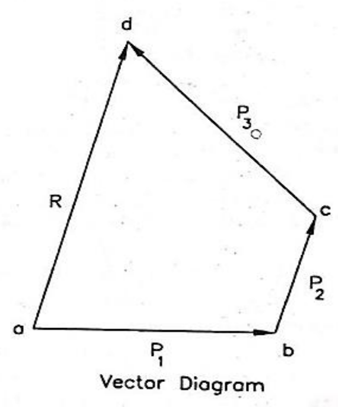


Fig. 1.22

Bow's Notation:

- This is a method of designating forces in space diagram. According to this system of notation, each force in space diagram is denoted by two capital letters, each being placed on two sides of the line of action of the force. In Fig.1.21, forces P_1 P_2 and P_3 are denoted by AB, BC and CD respectively. In the vector diagram, the corresponding forces are represented by ab, bc and cd respectively. Bow's notation is particularly suitable in graphical solution of systems of forces which are in equilibrium.

1.4.3 <u>Resultant of concurrent, non-concurrent & parallel force system by Analytical & Graphical Method</u>

<u>Parallel Forces</u>:

- The forces whose lines of action are parallel to each other, then the forces are known as parallel forces.

<u>Classification Of Parallel Forces</u>:

The parallel forces may be, broadly, classified into the following two categories, depending upon their directions.

- Like parallel forces
- Unlike parallel forces

<u>Like Parallel Forces</u>:

- The forces, whose lines of action are parallel to each other and all of them act in the same direction as shown in Fig. 1.23 (a) are known as like parallel forces.

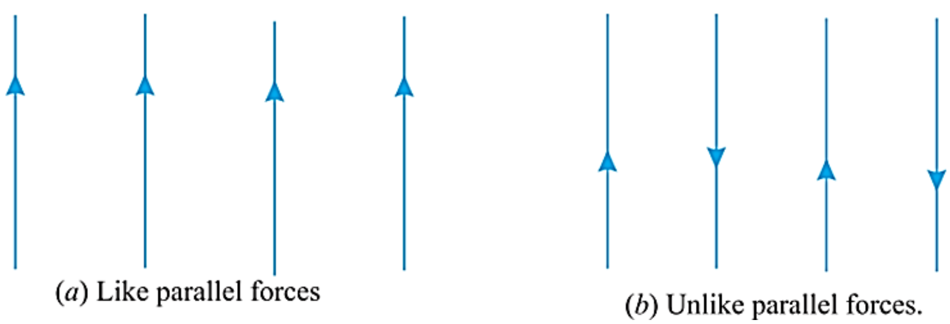


Fig. 1.23

<u>Unlike Parallel Forces</u>:

- The forces, whose lines of action are parallel to each other and all of them do not act in the same direction as shown in Fig. 1.23 (b) are known as like parallel forces.

<u>Methods For Magnitude and Position of The Resultant of Parallel Forces</u>:

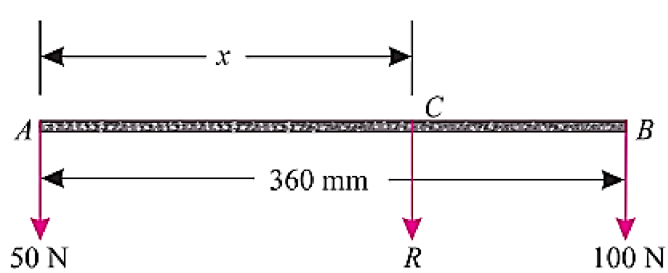
- The magnitude and position of the resultant force, of a given system of parallel forces (like or unlike) may be found out analytically or graphically. Here we shall discuss both the methods one by one.

<u>Analytical Method for The Resultant of Parallel Forces</u>:

- In this method, the sum of clockwise moments is equated with the sum of anticlockwise moments about a point.

Example – 7: ***Two like parallel forces of 50 N and 100 N act at the ends of a rod 360 mm long. Find the magnitude of the resultant force and the point where it acts.***

Solution. Given: The system of given forces is shown in Figure below.



Magnitude of the resultant force

Since the given forces are like and parallel, therefore magnitude of the resultant force,

$$R = 50 + 100 = 150\ N$$

Point where the resultant force acts

Let $\boldsymbol{x}$ = Distance between the line of action of the resultant force (R) and A (i.e., AC) in mm.

Now taking clockwise and anticlockwise moments of the forces about C and equating the same,

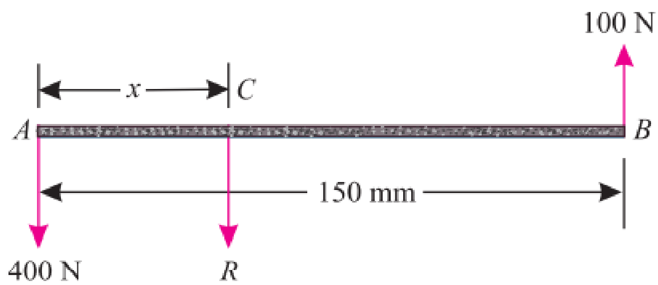
$$50 \times x = 100\,(360 - x) = 36\,000 - 100\,x$$

$$\Rightarrow 150\,x = 36\,000$$

$$\Rightarrow x = \frac{36000}{150} = 240\,mm$$

Example – 8: ***Two unlike parallel forces of magnitude 400 N and 100 N are acting in such a way that their lines of action are 150 mm apart. Determine the magnitude of the resultant force and the point at which it acts.***

Solution. Given: The system of given force is shown in Figure below.



Magnitude of the resultant force

Since the given forces are unlike and parallel, therefore magnitude of the resultant force,

$$R = 400 - 100 = \boldsymbol{300\ N}$$

Point where the resultant force acts

Let $\boldsymbol{x}$ = Distance between the lines of action of the resultant force and A in mm.

Now taking clockwise and anticlockwise moments about A and equating the same,

$$300 \times x = 100 \times 150 = 15\,000$$

$$\Rightarrow x = \frac{15000}{300} = \boldsymbol{50\ mm}$$

<u>Graphical Method for The Resultant of Parallel Forces</u>:

Consider a number of parallel forces (say three like parallel forces) P1, P2 and P3 whose resultant is required to be found out as shown in Fig. 4.6 (a).

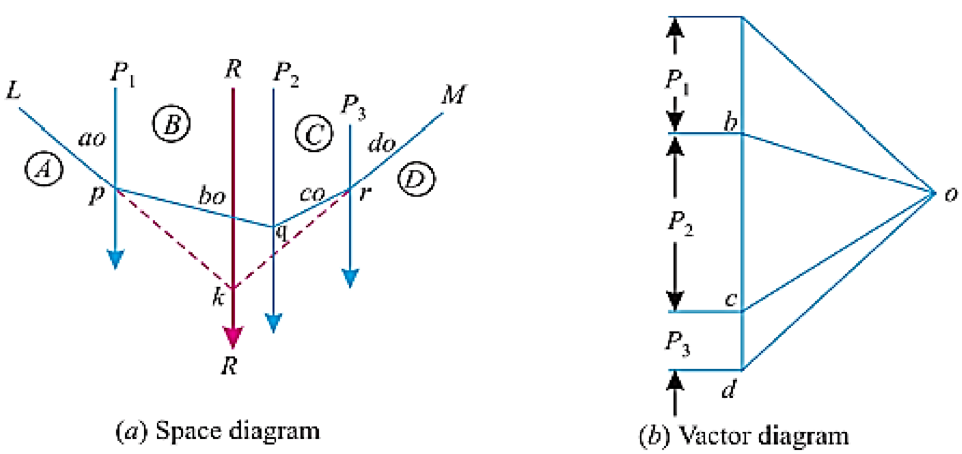


Fig. 1.24

First of all, draw the space diagram of the given system of forces and name them according to Bow's notations as shown in Fig. 1.24 (a). Now draw the vector diagram for the given forces as shown in Fig. 1.24 (b) and as discussed below:

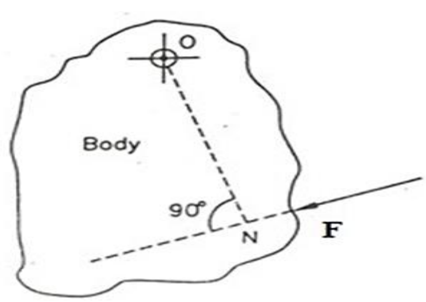
- Select some suitable point *a*, and draw *ab* equal to the force *AB* (P_1) and parallel to it to some suitable scale.
- Similarly draw *bc* and *cd* equal to and parallel to the forces *BC* (P_2) and *CD* (P_3) respectively.
- Now take some convenient point *o* and joint *oa*, *ob*, *oc* and *od*.
- Select some point *p*, on the line of action of the force *AB* of the space diagram and through it draw a line *Lp* parallel to *ao*. Now through *p* draw *pq parallel* to *bo* meeting the line of action of the force *BC* at *q*.
- Similarly draw *qr* and *rM* parallel to *co* and *do* respectively.
- Now extend *Lp* and *Mr to* meet at *k*. Through *k*, draw a line parallel to *ad*, which gives the required position of the resultant force.
- The magnitude of the resultant force is given by *ad* to the scale.

Note. This method for the position of the resultant force may also be used for any system of forces i.e. parallel, like, unlike or even inclined.

1.5 Moment of Force: Definition, Geometrical meaning of moment of a force, measurement of moment of a force & its S.I units. Classification of moments according to direction of rotation, sign convention, Law of moments, Varignon's Theorem, Couple – Definition, S.I. units, measurement of couple, properties of couple

Moment Of Force:

- It is the turning effect produced by a force, on the body, on which it acts.
- The moment of a force is equal to the product of the force and the perpendicular distance of the point, about which the moment is required and the line of action of the force.
- Mathematically, moment,
  $M = P \times l$
  Where, P = Force acting on the body, l = Perpendicular distance between the point, about which the moment is required and the line of action of the force.
- **Example:** Moment of a force about a point is the product of the force and the perpendicular distance of the point from the line of action of the force.



- Let a force F act on a body which is hinged at O. Then, moment of F about the point O in the body is = F × ON, Where: ON = perpendicular distance of O from the line of action of the force F.
- Unit of moment: Newton meter (N-m), kilo Newton meter (kN-m), N-mm.

Types of Moments:

Broadly speaking, the moments are of the following two types:
1. Clockwise moments 2. Anticlockwise moments

Clockwise Moment:

- It is the moment of a force, whose effect is to turn or rotate the body, about the point in the same direction in which hands of a clock move as shown in Fig. 1.25 (a).

Anticlockwise Moment:

- It is the moment of a force, whose effect is to turn or rotate the body, about the point in the opposite direction in which the hands of a clock move as shown in Fig. 1.25 (b).

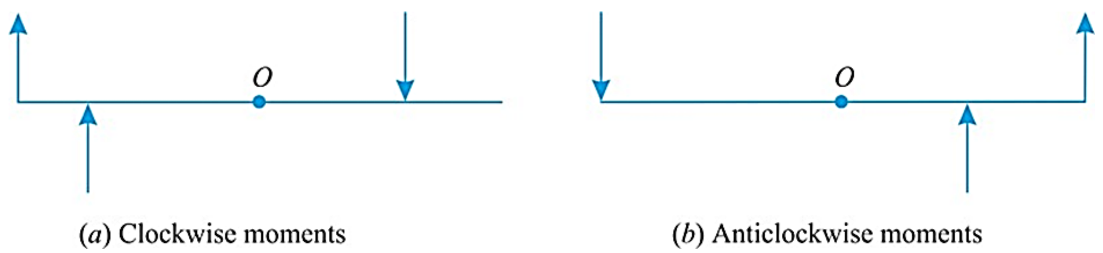


Fig. 1.25

- **Sign convention:** The general convention is to take clockwise moment as positive and anticlockwise moment as negative.

Geometrical representation of moment of a force:

- Consider a force P represented, in magnitude and direction, by the line AB. Let O be a point, about which the moment of this force is required to be found out, as shown in Fig. 1.26 from O, draw OC perpendicular to AB. Join OA and OB.

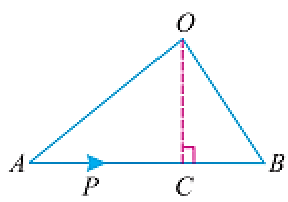


Fig. 1.26

- Now moment of the force P about O $= P \times OC$

$$= AB \times OC$$
$$= 2 \times (1/2 \times AB \times OC)$$
$$= 2 \times Area\ of\ \Delta\ AOB$$

- Thus, the moment of a force, about any point, is equal to twice the area of the triangle, whose base is the line to some scale representing the force and whose vertex is the point about which the moment is taken.

Varignon's theorem:

Varignon's theorem states that the algebraic sum of the moments of two forces about any point in their plane is equal to the moment of their resultant about the same point.

Proof:

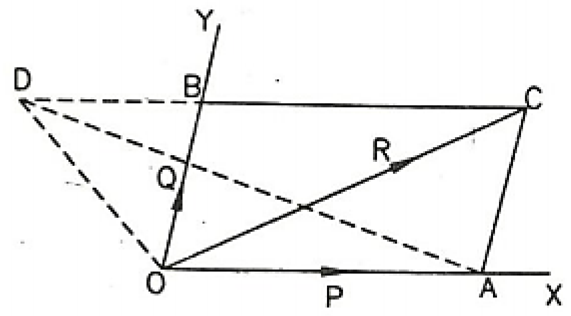


Fig. 1.27

Let P and Q be any two forces acting at a point O along lines OX and OY respectively and let D be any point in their plane as shown in Fig 1.27.

Line DC is drawn parallel to OX to meet OY at B. Let in some suitable scale, line OB represent the force Q in magnitude and direction and let in the same scale, OA represent the force P in magnitude and direction.

With OA and OB as the adjacent sides, parallelogram OACB is completed and OC is joined. Let R be the resultant of forces P and Q. Then, according to the "Theorem of parallelogram of forces", R is represented in magnitude and direction by the diagonal OC of the parallelogram OACB.

The point D is joined with points O and A. The moments of P, Q and R about D are given by 2 × area of Δ AOD, 2 × area of Δ OBD and 2 × area of Δ OCD respectively.

From the geometry of the figure,

$$Area\ of\ \Delta\ OCD = area\ of\ \Delta\ OBC + area\ of\ \Delta\ OBD$$

$$\Rightarrow Area\ of\ \Delta\ OCD = area\ of\ \Delta\ AOC + area\ of\ \Delta\ OBD$$

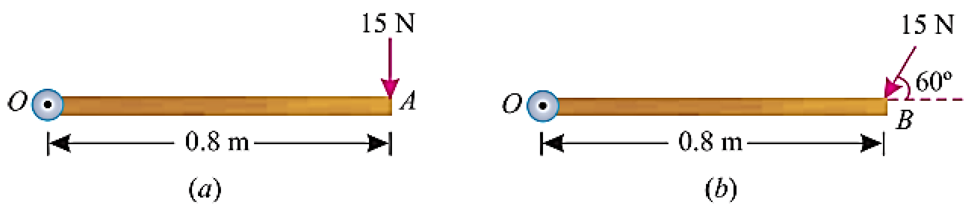
$$\Rightarrow Area\ of\ \Delta\ OCD = area\ of\ \Delta\ AOD + area\ of\ \Delta\ OBD$$

$$\Rightarrow 2 \times Area\ of\ \Delta\ OCD = 2 \times Area\ of\ \Delta\ AOD + 2 \times Area\ of\ \Delta\ OBD$$

$$\Rightarrow Moment\ of\ R\ about\ D = Moment\ of\ P\ about\ D + Moment\ of\ Q\ about\ D$$

[Note: As Δ AOC and Δ AOD are on the same base and have the same altitude. Δ AOC = Δ AOD. Again, As AOC and OBC have equal bases and equal altitudes. ΔAOC = ΔOBC].

Example – 9: ***A force of 15 N is applied perpendicular to the edge of a door 0.8 m wide as shown in Fig. (a). Find the moment of the force about the hinge. If this force is applied at an angle of 60° to the edge of the same door, as shown in Fig. (b), find the moment of this force.***



Solution. Given: Force applied (P) = 15 N and width of the door (l) = 0.8 m

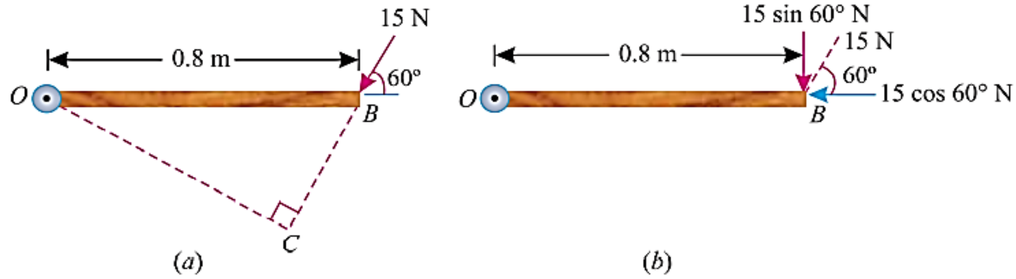
Moment when the force acts perpendicular to the door

We know that the moment of the force about the hinge,

$$Moment = P \times l = 15 \times 0.8 = \mathbf{12.0\ N - m}$$

Moment when the force acts at an angle of 60° to the door

This part of the example may be solved either by finding out the perpendicular distance between the hinge and the line of action of the force as shown in Fig. (a) or by finding out the vertical component of the force as shown in Fig. (b).



From the geometry of Fig. (a), we find that the perpendicular distance between the line of action of the force and hinge,

$$OC = OB \sin 60° = 0.8 \times 0.866 = 0.693\, m$$

$$\therefore\ Moment = 15 \times 0.693 = \mathbf{10.4\ N-m}$$

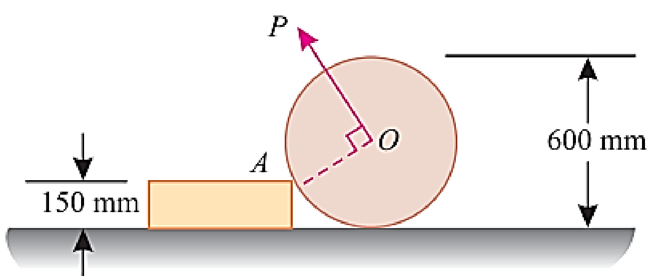
In the second case, we know that the vertical component of the force

$$= 15 \sin 60° = 15 \times 0.866 = 13.0\ N$$

$$\therefore\ Moment = 13 \times 0.8 = \mathbf{10.4\ N-m}$$

Note. Since distance between the horizontal component of force (15 cos 60°) and the hinge is zero, therefore moment of horizontal component of the force about the hinge is also zero.

Example – 10: ***A uniform wheel of 600 mm diameter, weighing 5 kN rests against a rigid rectangular block of 150 mm height as shown in Figure. Find the least pull, through the centre of the wheel, required just to turn the wheel over the corner A of the block. Also find the reaction on the block. Take all the surfaces to be smooth.***



Solution. Given: Diameter of wheel = 600 mm; Weight of wheel = 5 kN and height of the block = 150 mm.

Least pull required just to turn the wheel over the corner.

Let P = Least pull required just to turn the wheel in kN.

A little consideration will show that for the least pull, it must be applied normal to AO. The system of forces is shown in Figure. From the geometry of the figure, we find that

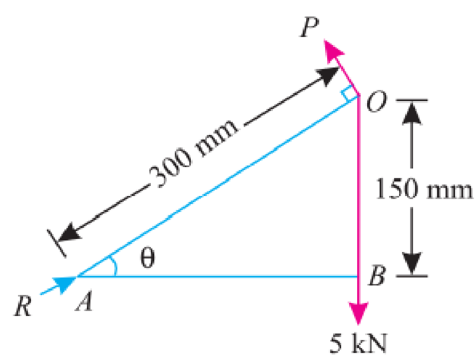
$$\sin\theta = \frac{150}{300} = 0.5 \text{ or } \theta = 30^0$$

$$AB = \sqrt{(300)^2 - (150)^2} = 260\ mm$$

Now taking moments about A and equating the same,

$$P \times 300 = 5 \times 260 = 1300$$

$$\Rightarrow P = \frac{1300}{300} = \mathbf{4.33\ kN}$$



Reaction on the block

Let R = Reaction on the block in kN.

Resolving the forces horizontally and equating the same,

$$R\cos 30° = P\sin 30°$$

$$\Rightarrow R = \frac{P\sin 30°}{\cos 30°} = \frac{4.33 \times 0.5}{0.866} = \mathbf{2.5\ kN}$$

Couple:

- **Definition:** A couple is a pair of two equal and unlike parallel forces acting on a body in such a way that the lines of action of the two forces are not in the same straight line.
- As a matter of fact, a couple is unable to produce any translatory motion (i.e., motion in a straight line). But it produces a motion of rotation in the body, on which it acts.
- The simplest example of a couple is the forces applied to the key of a lock, while locking or unlocking it.

Arm of a Couple:

- The perpendicular distance (a), between the lines of action of the two equal and opposite parallel forces, is known as arm of the couple as shown in Fig. 1.28.

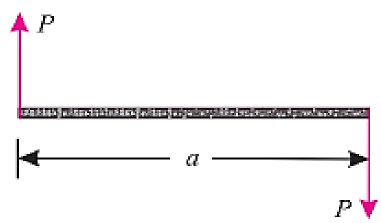


Fig 1.28

Moment of a Couple:

- The moment of a couple is the product of the force (i.e., one of the forces of the two equal and opposite parallel forces) and the arm of the couple.
- Mathematically:

$$\boldsymbol{Moment\ of\ a\ couple = P \times a}$$

$$Where\ P = Magnitude\ of\ the\ force, and\ a = Arm\ of\ the\ couple$$

Units of Couple:

- The SI unit of couple will be Newton-meter (briefly written as N-m). Similarly, the units of couple may also be kN-m (i.e., kN × m), N-mm (i.e., N × mm) etc.

Classification of couples:

The couples may be, broadly, classified into the following two categories, depending upon their direction, in which the couple tends to rotate the body, on which it acts:

1. Clockwise couple, and 2. Anticlockwise couple.

Clockwise couple:

- A couple, whose tendency is to rotate the body, on which it acts, in a clockwise direction, is known as a clockwise couple as shown in Fig. 1.29 (a). Such a couple is also called positive couple.

Anticlockwise couple:

- A couple, whose tendency is to rotate the body, on which it acts, in an anticlockwise direction, is known as an anticlockwise couple as shown in Fig. 1.29 (b). Such a couple is also called a negative couple.

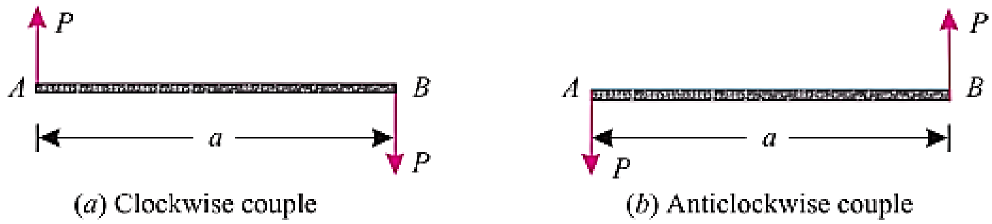


Fig 1.29

<u>Properties of a Couple</u>:

A couple (whether clockwise or anticlockwise) has the following characteristics:

- The algebraic sum of the forces, constituting the couple, is zero.
- The algebraic sum of the moments of the forces, constituting the couple, about any point is the same, and equal to the moment of the couple itself.
- A couple cannot be balanced by a single force. But it can be balanced only by a couple of opposite sense.
- Any no. of coplanar couples can be reduced to a single couple, whose magnitude will be equal to the algebraic sum of the moments of all the couples.

POSSIBLE SHORT TYPE QUESTIONS WITH ANSWER

1. Define force & state its unit in S.I. system. (W – 2016 & 2017)

Ans. The force may be broadly defined as an agent which produces or tends to produce, destroys or tends to destroy motion. S.I. Unit: Newton (N)

2. Define a rigid body. (S – 2019 Old)

Ans. A body is said to be rigid if it does not undergo deformation whatever force may be applied to the body. In actual practice, there is no body which can be said to be rigid in true sense of terms.

3. What is free body diagram? (S – 2019 New)

Ans. The representation of reaction force on the body by removing all the support or forces act from the body is called free body diagram.

4. What do you mean by concurrent forces? (W – 2017, S – 2019 Old)

Ans. The forces, whose lines of action pass through a common point, are known as concurrent forces. The concurrent forces may or may not be collinear.

5. What do you mean by coplanar forces? (S – 2018)

Ans. The forces, whose lines of action lie on the same plane, are known as coplanar forces.

6. State parallelogram law of forces. (W – 2016)

Ans. It states, "If two forces, acting simultaneously on a particle, be represented in magnitude and direction by the two adjacent sides of a parallelogram; their resultant may be represented in magnitude and direction by the diagonal of the parallelogram, which passes through their point of intersection."

7. State polygon law of forces. (S – 2018 & 2019)

Ans. It is an extension of Triangle Law of Forces for more than two forces, which states, "If a number of forces acting simultaneously on a particle, be represented in magnitude and direction, by the sides of a polygon taken in order then the resultant of all these forces may be represented, in magnitude and direction, by the closing side of the polygon, taken in opposite order."

8. Define moment of a force & state its unit in S.I. (W – 2016)

Ans. It is the turning effect produced by a force, on the body, on which it acts. The moment of a force is equal to the product of the force and the perpendicular distance of the point, about which the moment is required and the line of action of the force.

S.I. Unit: Newton meter (N-m), kilo Newton meter (kN-m), N-mm

9. State varignon's theorem. (W – 2017)
Ans. Varignon's theorem states that the algebraic sum of the moments of two forces about any point in their plane is equal to the moment of their resultant about the same point.

10. Define couple. (W – 2017 & S – 2019 Old)
Ans. A couple is a pair of two equal and unlike parallel forces acting on a body in such a way that the lines of action of the two forces are not in the same straight line.

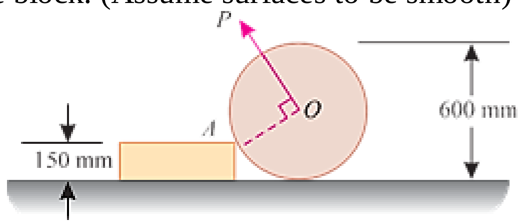
POSSIBLE LONG TYPE QUESTIONS

1. Two forces act at an angle of 120°. The bigger force is of 40 N and the resultant is perpendicular to the smaller one. Find the smaller force. **(W – 2016)**
Hints: refer page - 13, example-2

2. Explain principles of transmissibility & superposition. **(W – 2016)**
Hints: refer page - 07
3. The resultant of two forces ***P*** and 15 N is 20 N inclined at 60^0 to the 15 N force. find the magnitude and direction of ***P***. **(W – 2017)**

4. Find the angle between two equal forces of magnitude P, when their resultant is (*i*) P and (*ii*) P/2. **(S – 2018)**

5. A uniform wheel of 600 mm diameter, weighing 5 kN rests against a rigid rectangular block of 150 mm height as shown in Fig. below. Find the least pull force through the center of the wheel required just to turn the wheel over the corner A of the block. (Assume surfaces to be smooth) **(W – 2016)**



Hints: refer page – 24, example – 10

6. State and explain varignon's theorem. **(W – 2016 & S – 2019 Old)**
Hints: refer page – 22 & 23

7. A uniform rod AB of weight 75 N and 3 m long is simply supported at its ends. Downward forces of 25 N and 60 N are acting at a distance of 0.5 m and 1.2 m from the end A. Find the reactions at A and B. **(S – 2018)**

8. ABCD is a rectangle, in which AB = CD = 150 mm and BC = DA = 75 mm. Forces of 320 N each act along AB and CD and forces of 150 N each act along BC and DA. Find the resultant moment of the two couples. **(S – 2018)**

9. A particle is acted on by three forces 2, $2\sqrt{2}$, and 1 kN. The first force is horizontal and towards the right, the second acts at 45^0 to the horizontal and inclined right upwards and the third is vertical. Determine the resultant of the given forces. **(S – 2019 New)**

10. What do you mean by force? Mention four effects of a force. **(S – 2019 Old)**

11. State and explain triangle law of forces. **(S – 2019 Old)**

CHAPTER NO. – 02

EQUILIBRIUM

LEARNING OBJECTIVES:

2.1 Definition, condition of equilibrium, Analytical & Graphical conditions of equilibrium for concurrent, non-concurrent & Free Body Diagram.
2.2 Lami's Theorem – Statement, Application for solving various engineering problems.

2.1 Definition, condition of equilibrium, Analytical & Graphical conditions of equilibrium for concurrent, non-concurrent & Free Body Diagram:

Definition:

- If the resultant of a number of forces, acting on a particle is zero, the particle will be in equilibrium. Such a set of forces, whose resultant is zero, are called equilibrium forces.
- The force, which brings the set of forces in equilibrium, is called an equilibrant.
- As a matter of fact, the equilibrant is equal to the resultant force in magnitude, but opposite in direction.

Principles Of Equilibrium:

- Though there are many principles of equilibrium, yet the following three are important from the subject point of view:
    1. **Two force principle:** As per this principle, if a body in equilibrium is acted upon by two forces, then they must be equal, opposite and collinear.
    2. **Three force principle:** As per this principle, if a body in equilibrium is acted upon by three forces, then the resultant of any two forces must be equal, opposite and collinear with the third force.
    3. **Four force principle:** As per this principle, if a body in equilibrium is acted upon by four forces, then the resultant of any two forces must be equal, opposite and collinear with the resultant of the other two forces.

Conditions Of Equilibrium:

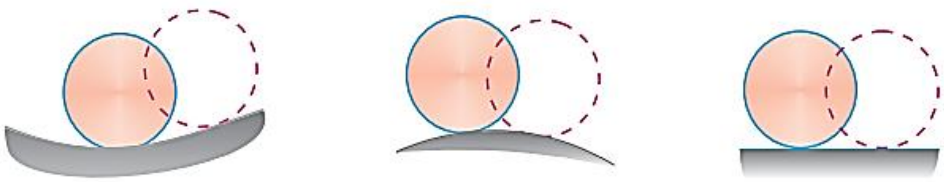
Analytical conditions of equilibrium for concurrent, non-concurrent forces:

- The algebric sum of horizontal components of the forces must be zero. i.e., $\sum H = 0$
- The algebric sum of vertical components of the forces must be zero. i.e., $\sum V = 0$
- The algebric sum of moment of forces about any point in their plane is equal to zero. i.e., $\sum M = 0$

Graphical conditions of equilibrium for concurrent, non-concurrent forces:

1. Stable equilibrium:

- A body is said to be in stable equilibrium, if it returns back to its original position, after it is slightly displaced from its position of rest. This happens when some additional force sets up due to displacement and brings the body back to its original position. A smooth cylinder, lying in a curved surface, is in stable equilibrium. If we slightly displace the cylinder from its position of rest (as shown by dotted lines), it will tend to return back to its original position in order to bring its weight normal to horizontal axis as shown in Fig. (a).



(*a*) Stable (b) Unstable (*c*) Neutral

2. Unstable equilibrium:

- A body is said to be in an unstable equilibrium, if it does not return back to its original position, and heels farther away, after slightly displaced from its position of rest. This happens when the additional force moves the body away from its position of rest. This happens when the additional force moves the body away from its position of rest. A smooth cylinder lying on a convex surface is in unstable equilibrium. If we slightly displace the cylinder from its position of rest (as shown by dotted lines) the body will tend to move away from its original position as shown in Fig. (b).

3. Neutral equilibrium:

- A body is said to be in a neutral equilibrium, if it occupies a new position (and remains at rest in this position) after slightly displaced from its position of rest. This happens when no additional force sets up due to the displacement. A smooth cylinder lying on a horizontal plane is in neutral equilibrium as shown in Fig. (c).

Free Body Diagram:

- Free body diagram of a body is the diagram drawn by showing all the external forces and reactions on the body and by removing the contact surfaces.
- Steps to be followed in drawing a free body diagram.
    1. Isolate the body from all other bodies.
    2. Indicate the external forces on the free body. (The weight of the body should also be included. It should be applied at the Centre of gravity of the body)
    3. The magnitude and direction of the known external forces should be mentioned.
    4. The reactions exerted by the supports on the body should be clearly indicated.
    5. Clearly mark the dimensions in the free body diagram.

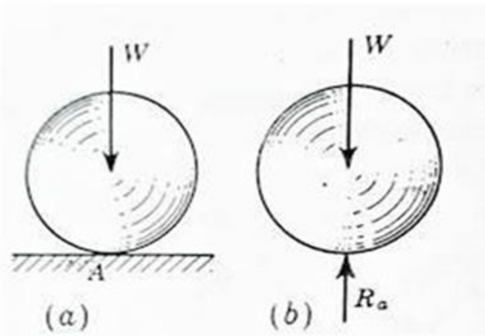


Figure 2.1 (a)

A spherical ball is rested upon a surface as shown in figure 2.1 (a). By following the necessary steps, we can draw the free body diagram for this force system as shown in figure 2.1(b). Similarly, fig 2.2 (b) represents free body diagram for the force system shown in figure 2.2(a).

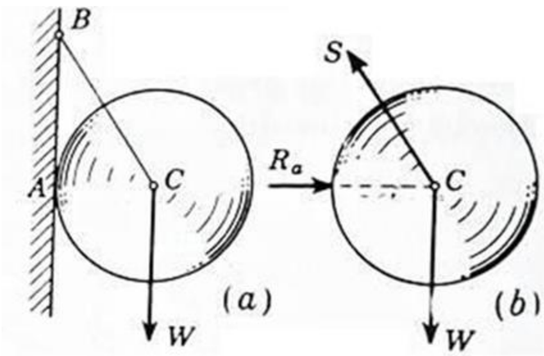
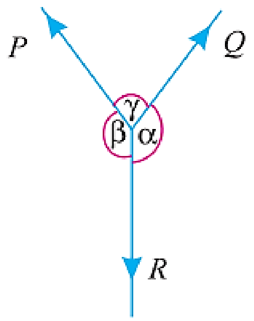


Figure 2.2 (b)

2.2 Lami's Theorem: Statement, Application for solving various engineering problems

Lami's Theorem:

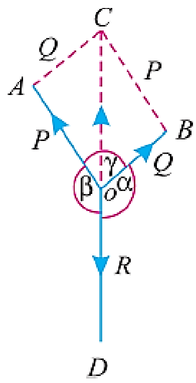


- It states, ***"If three coplanar forces acting at a point be in equilibrium, then each force is proportional to the sine of the angle between the other two."***
- Mathematically,

$$\frac{P}{\sin\alpha}=\frac{Q}{\sin\beta}=\frac{R}{\sin\gamma}$$

Where, P, Q, and R are three forces and α, β, γ are the angles as shown in Figure.

Proof:



- Consider three coplanar forces P, Q, and R acting at a point O. Let the opposite angles to three forces be α, β and γ as shown in Figure. Now let us complete the parallelogram OACB with OA and OB as adjacent sides as shown in the figure. We know that the resultant of two forces P and Q will be given by the diagonal OC both in magnitude and direction of the parallelogram OACB. Since these forces are in equilibrium, therefore the resultant of the forces P and Q must be in line with OD and equal to R, but in opposite direction.

$From\ the\ geometry\ of\ the\ figure, we\ find$

$BC = P\ and\ AC = Q$

$\therefore \angle AOC = (180° - \beta)$

$and \angle ACO = \angle BOC = (180° - \alpha)$

$$\begin{aligned}\therefore \angle CAO &= 180° - (\angle AOC + \angle ACO)\\ &= 180° - [(180° - \beta) + (180° - \alpha)]\\ &= 180° - 180° + \beta - 180° + \alpha\\ &= \alpha + \beta - 180°\end{aligned}$$

$But\ \alpha + \beta + \gamma = 360°$

$Subtracting\ 180°\ from\ both\ sides\ of\ the\ above\ equation,$

$(\alpha + \beta - 180°) + \gamma = 360° - 180° = 180°$

$or \angle CAO = 180° - \gamma$

$We\ know\ that\ in\ triangle\ AOC,$

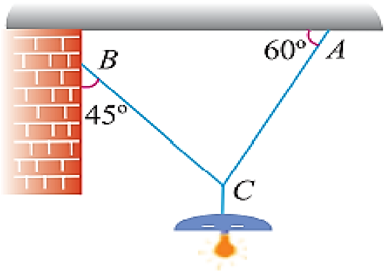
$$\frac{OA}{\sin\angle ACO}=\frac{AC}{\sin\angle AOC}=\frac{OC}{\sin\angle CAO}$$

$$\Rightarrow \frac{OA}{sin(180^0-\alpha)}=\frac{AC}{sin(180^0-\beta)}=\frac{OC}{sin(180^0-\gamma)}$$

$$\Rightarrow \frac{\boldsymbol{P}}{\boldsymbol{sin\,\alpha}}=\frac{\boldsymbol{Q}}{\boldsymbol{sin\,\beta}}=\frac{\boldsymbol{R}}{\boldsymbol{sin\,\gamma}}$$

$\ldots[\because sin\ (180° - \theta) = sin\ \theta]$

Example – 1: ***An electric light fixture weighting 15 N hangs from a point C, by two strings AC and BC. The string AC is inclined at 60° to the horizontal and BC at 45° to the horizontal as shown in Figure. Using Lami's theorem, or otherwise, determine the forces in the strings AC and BC.***



Solution. Given: Weight at C = 15 N

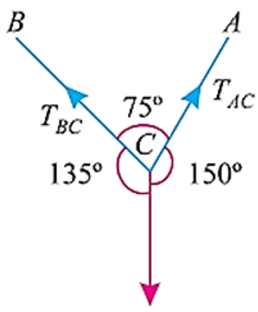
Let T_{AC} = Force in the string AC, and

T_{BC} = Force in the string BC.

The system of forces is shown in Figure. From the geometry of the figure, we find that angle between T_{AC} and 15 N is 150° and angle between T_{BC} and 15 N is 135°.

$$\therefore \angle ACB = 180° - (45° + 60°) = 75°$$

Applying Lami's equation at C,



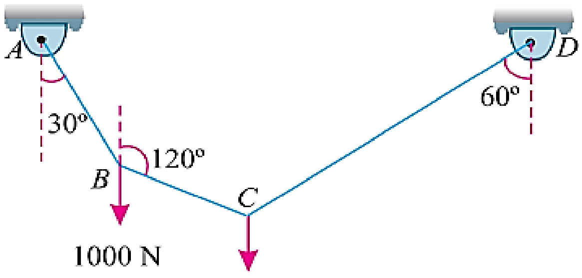
$$\frac{15}{\sin 75^0} = \frac{T_{AC}}{\sin 135^0} = \frac{T_{BC}}{\sin 150^0}$$

$$\Rightarrow \frac{15}{\sin 75^0} = \frac{T_{AC}}{\sin 45^0} = \frac{T_{BC}}{\sin 30^0}$$

$$\Rightarrow T_{AC} = \frac{15 \sin 45^0}{\sin 75^0} = \frac{15 \times 0.707}{0.9659} = \mathbf{10.98\ N}$$

$$\Rightarrow T_{BC} = \frac{15 \sin 30^0}{\sin 75^0} = \frac{15 \times 0.5}{0.9659} = \mathbf{7.76\ N}$$

Example – 2: ***A string ABCD, attached to fixed points A and D has two equal weights of 1000 N attached to it at B and C. The weights rest with the portions AB and CD inclined at angles as shown in Figure. Find the tensions in the portions AB, BC and CD of the string, if the inclination of the portion BC with the vertical is 120°.***

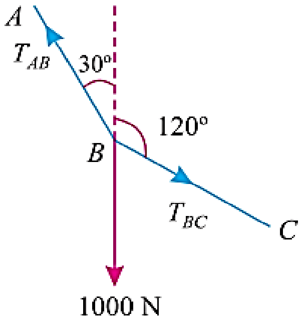


Solution. Given: Load at B = Load at C = 1000 N For the sake of convenience, let us split up the string ABCD into two parts. The system of forces at joints B and is shown in Fig. (a) and (b).

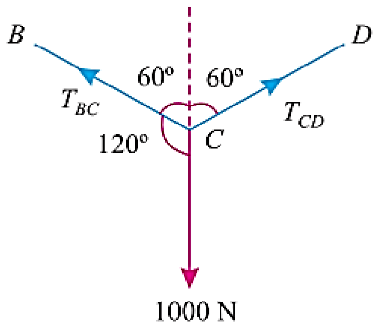
Let T_{AB} = Tension in the portion AB of the string,

T_{BC} = Tension in the portion BC of the string, and

T_{CD} = Tension in the portion CD of the string.



(*a*) Joint *B*



(*b*) Joint *C*

Applying Lami's equation at joint B,

$$\frac{T_{AB}}{sin60^0} = \frac{T_{BC}}{sin150^0} = \frac{1000}{sin150^0}$$

$$\Rightarrow \frac{T_{AB}}{sin60^0} = \frac{T_{BC}}{sin30^0} = \frac{1000}{sin30^0}$$

$$\Rightarrow T_{AB} = \frac{1000sin60^0}{sin30^0} = \frac{1000 \times 0.866}{0.5} = \mathbf{1732}\ \boldsymbol{N}$$

$$\Rightarrow T_{BC} = \frac{1000sin30^0}{sin30^0} = \mathbf{1000}\ \boldsymbol{N}$$

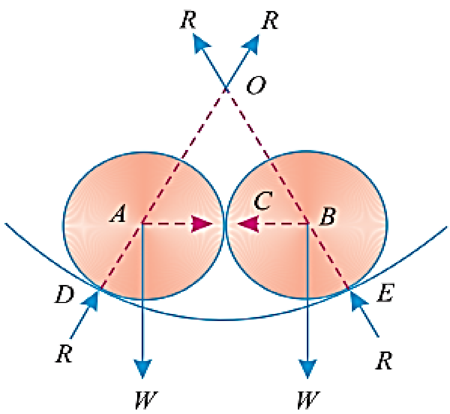
Applying Lami's equation at joint C,

$$\frac{T_{CD}}{sin120^0} = \frac{T_{BC}}{sin120^0} = \frac{1000}{sin120^0}$$

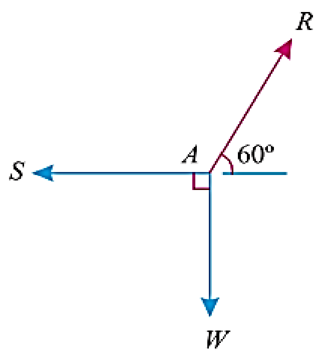
$$\Rightarrow T_{CD} = \mathbf{1000}\ \boldsymbol{N}$$

Example – 3: ***Two equal heavy spheres of 50 mm radius are in equilibrium within a smooth cup of 150 mm radius. Show that the reaction between the cup of one sphere is double than that between the two spheres.***

Solution. Given: Radius of spheres = 50 mm and radius of the cup = 150 mm.



(*a*)



(*b*)

The two spheres with centres A and B, lying in equilibrium, in the cup with O as centre are shown in Fig.(a). Let the two spheres touch each other at C and touch the cup at D and E respectively.

Let R = Reactions between the spheres and cup, and
S = Reaction between the two spheres at C.

From the geometry of the figure, we find that OD = 150 mm and AD = 50 mm. Therefore OA = 100 mm. Similarly, OB = 100 mm. We also find that AB = 100 mm. Therefore, OAB is an equilateral triangle. The system of forces at A is shown in Figure (b).

Applying Lami's equation at A,

$$\frac{R}{sin 90^0} = \frac{W}{sin 120^0} = \frac{S}{sin 150^0}$$

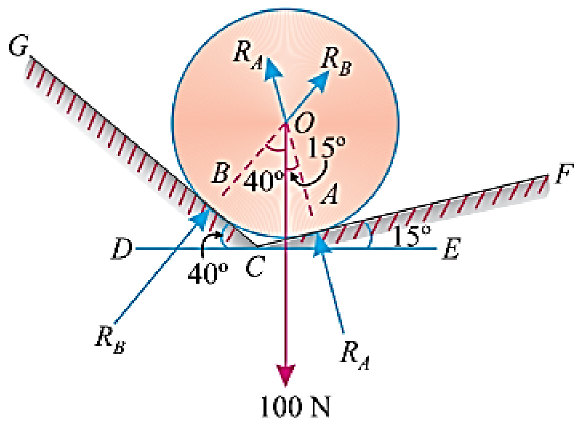
$$\Rightarrow \frac{R}{1} = \frac{W}{sin 60^0} = \frac{S}{sin 30^0}$$

$$\Rightarrow R = \frac{S}{sin 30^0} = \frac{S}{0.5} = 2S$$

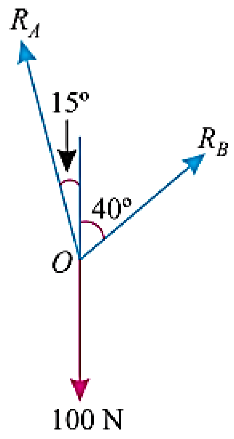
Hence the reaction between the cup and the sphere is double than that between the two spheres.

Example – 4: ***A smooth circular cylinder of radius 1.5 meter is lying in a triangular groove, one side of which makes 15° angle and the other 40° angle with the horizontal. Find the reactions at the surfaces of contact, if there is no friction and the cylinder weights 100 N.***

Solution. Given: Weight of cylinder = 100 N



(*a*)



(*b*)

Let R_A = Reaction at A, and
R_B = Reaction at B.

The smooth cylinder lying in the groove is shown in Fig. (a). In order to keep the system in equilibrium, three forces i.e., R_A, R_B and weight of cylinder (100 N) must pass through the centre of the cylinder. Moreover, as there is no friction, the reactions R_A and R_B must be normal to the surfaces as shown in Fig. (a). The system of forces is shown in Fig. (b).

Applying Lami's equation, at O,

$$\frac{R_A}{sin(180^0 - 40^0)} = \frac{R_B}{sin(180^0 - 15^0)} = \frac{100}{sin(15^0 + 40^0)}$$

$$\Rightarrow \frac{R_A}{sin 40^0} = \frac{R_B}{sin 15^0} = \frac{100}{sin 55^0}$$

$$\Rightarrow R_A = \frac{100 \times sin40^0}{sin55^0} = \frac{100 \times 0.6428}{0.8192} = \mathbf{78.5\ N}$$

$$\Rightarrow R_B = \frac{100 \times sin15^0}{sin55^0} = \frac{100 \times\ 0.2588}{0.8192} = \mathbf{31.6\ N}$$

POSSIBLE SHORT TYPE QUESTIONS WITH ANSWER

1. **State Lami's theorem. (S – 2018)**
   - It states, ***"If three coplanar forces acting at a point be in equilibrium, then each force is proportional to the sine of the angle between the other two."***
   - Mathematically,

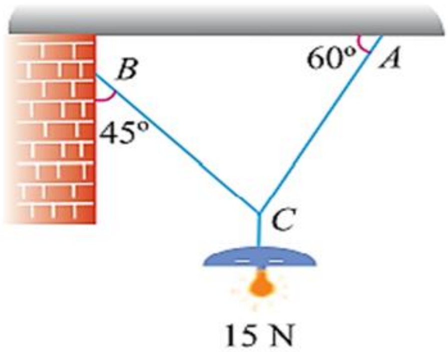
$$\frac{P}{\sin \alpha} = \frac{Q}{\sin \beta} = \frac{R}{\sin \gamma}$$

2. **State the conditions of equilibrium. (S – 2019)**
   - The algebric sum of horizontal components of the forces must be zero. i.e., $\sum H=0$
   - The algebric sum of vertical components of the forces must be zero. i.e., $\sum V=0$
   - The algebric sum of moment of forces about any point in their plane is equal to zero. i.e., $\sum M=0$

POSSIBLE LONG TYPE QUESTIONS

1. State and proof Lami's theorem. **(W – 2016)**

2. An electric light fixture weighting 15 N hangs from a point C, by two strings AC and BC. The string AC is inclined at 60° to the horizontal and BC at 45° to the horizontal as shown in Figure. Using Lami's theorem, or otherwise, determine the forces in the strings AC and BC. **(W – 2016)**



3. Write down the conditions of equilibrium? Also mention it mathematically. **[S – 2019 (O)]**

4. Three forces acting on a particle are in equilibrium, the angles between the first and second is 90^0 and that between the second and third is 120^0. Find the ratio between the forces. **[S – 2019 (O)]**

CHAPTER NO. – 03

FRICTION

LEARNING OBJECTIVE:

3.1 Definition of friction, Frictional forces, Limiting frictional force, Coefficient of Friction. Angle of Friction & Repose, Laws of Friction, Advantages & Disadvantages of Friction.

3.2 Equilibrium of bodies on level plane – Force applied on horizontal & inclined plane (up &down).

3.3 Ladder, Wedge Friction.

3.1 Definition of friction, Frictional forces, Limiting frictional force, Coefficient of Friction. Angle of Friction & Repose, Laws of Friction, Advantages & Disadvantages of Friction

Definition of Friction:

- When a rigid body slides over another rigid body, a resisting force is exerted at the surface of contact, this resisting force is called force of friction or simply friction.
- It always acts in a direction opposite to the direction of motion.

Classification of friction:

- Friction may be classified into two types
    1. Static friction
    2. Dynamic friction

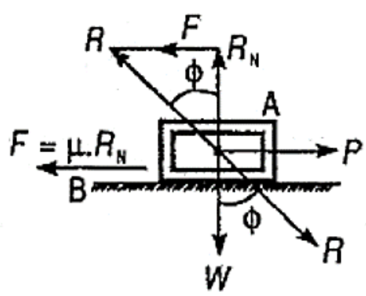
1. **Static friction:** It is the friction experienced by a body when it is at rest. Or in other words, it is the friction when the body tends to move.
2. **Dynamic friction:** It is the friction experienced by a body when it is in motion. It is also called kinetic friction. The dynamic friction is of the following two types :
    - Sliding friction: It is the friction, experienced by a body when it slides over another body.
    - Rolling friction: It is the friction, experienced by a body when it rolls over another body.

Limiting Friction:

- It is the maximum force of friction between two contact surfaces when a body tends to move over another body.

Angle of friction:

- Consider a body of weight ***(W)*** is resting on a horizontal plane. If a force $\boldsymbol{P}$ is applied to the body, no relative motion takes place until the applied force $\boldsymbol{P}$ is equal to the force of friction $\boldsymbol{F}$, acting opposite to the direction of motion.
  Let $\boldsymbol{R_N}$ = Normal reaction acting on the body.
  $\boldsymbol{W}$ = Weight of the body.
  $\boldsymbol{F}$ = Maximum force of friction.
  $\boldsymbol{P}$ = Horizontal force acting on the body.



- From the geometry of the figure, we have

$$\tan\varphi = \frac{F}{R_N} \qquad \Rightarrow \varphi = \tan^{-1}\left(\frac{F}{R_N}\right)$$

- Where, $\varphi = Angle\ of\ friction$

- **Definition of φ:** It is the angle between normal reaction (R_N) and resultant (R) of normal reaction and frictional forces.

Coefficient of friction:

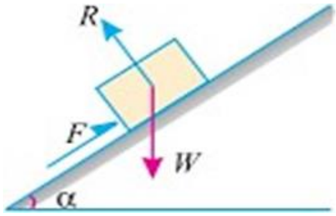
- It is the ratio of limiting friction (F) to the normal reaction (R_N) between two bodies.
- It is generally denoted by $\boldsymbol{\mu}$.
- Mathematically,

$$\boldsymbol{\mu = \frac{F}{R_N} = \tan \varphi}$$

$$\boldsymbol{\Rightarrow \varphi = \tan^{-1} \mu}$$

Angle of repose:

- It is the maximum angle between the inclined plane and horizontal plane at which the body just tends to slide downwards.
- This angle is generally specified by α.
- Angle of friction (φ) = Angle of repose (α)



Laws of Friction:

Prof. Coulomb, after extensive experiments, gave some laws of friction, which may be grouped under the following heads:

- Laws of static friction, and
- Laws of kinetic or dynamic friction.

Laws of Static Friction:

Following are the laws of static friction:

- The force of friction always acts in a direction, opposite to that in which the body tends to move.
- The magnitude of the force of friction is exactly equal to the force, which tends to move the body.
- The magnitude of the limiting friction (F) bears a constant ratio to the normal reaction (R_N) between the two surfaces. Mathematically:

$$\frac{F}{R_N} = constant$$

- The force of friction is independent of the area of contact between the two surfaces.
- The force of friction depends upon the roughness of the surfaces.

Laws of Dynamic Friction:

Following are the laws of kinetic or dynamic friction:

- The force of friction always acts in a direction, opposite to that in which the body is moving.
- The magnitude of kinetic friction bears a constant ratio to the normal reaction between the two surfaces. But this ratio is slightly less than that in case of limiting friction.
- For moderate speeds, the force of friction remains constant. But it decreases slightly with the increase of speed.

Advantages of Friction:

- Friction is responsible for many types of motion.
- It helps us walk on the ground.
- Brakes in a car make use of friction to stop the car.
- Asteroids are burnt in the atmosphere before reaching Earth due to friction.
- It helps in the generation of heat when we rub our hands.

Disadvantages of Friction:

- Friction produces unnecessary heat leading to the wastage of energy.
- The force of friction acts in the opposite direction of motion, so friction slows down the motion of moving objects.
- Forest fires are caused due to the friction between tree branches.
- A lot of money goes into preventing friction and the usual wear and tear caused by it by using techniques like greasing and oiling.

3.2 Equilibrium of bodies on level plane – Force applied on horizontal & inclined plane (up &down)

EQUILIBRIUM OF A BODY ON A ROUGH HORIZONTAL PLANE:

We know that a body, lying on a rough horizontal plane will remain in equilibrium. But when- ever a force is applied on it, the body will tend to move in the direction of the force. In such cases, equilibrium of the body is studied first by resolving the forces horizontally and then vertically.
Now the value of the force of friction is obtained from the relation:

$$\boldsymbol{F = \mu R}$$

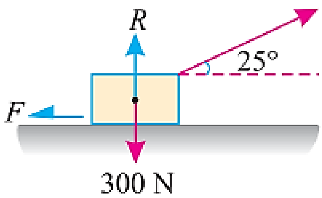
Where, F = Force of friction
μ = coefficient of friction
R = Normal reaction

Example – 1: ***A body of weight 300 N is lying on a rough horizontal plane having a coefficient of friction as 0.3. Find the magnitude of the force, which can move the body, while acting at an angle of 25° with the horizontal.***

Solution. Given: Weight of the body (W) = 300 N; Coefficient of friction (μ) = 0.3 and angle made by the force with the horizontal (α) = 25°

Let P = Magnitude of the force, which can move the body, and
F = Force of friction.

Resolving the forces horizontally,



$$F = P\cos\alpha = P\cos 25° = P \times 0.9063$$

and now resolving the forces vertically,

$$R = W - P\sin\alpha = 300 - P\sin 25° = 300 - P \times 0.4226$$

We know that the force of friction (F),

$$0.9063\,P = \mu R = 0.3 \times (300 - 0.4226\,P) = 90 - 0.1268\,P$$

$$\Rightarrow 90 = 0.9063\,P + 0.1268\,P = 1.0331\,P$$

$$\Rightarrow P = \frac{90}{1.0331} = \boldsymbol{87.1\ N}$$

Example – 2: ***A body, resting on a rough horizontal plane, required a pull of 180 N inclined at 30° to the plane just to move it. It was found that a push of 220 N inclined at 30° to the plane just moved the body. Determine the weight of the body and the coefficient of friction.***

Solution. Given: Pull = 180 N; Push = 220 N and angle at which force is inclined with horizontal plane (α) = 30°
Let W = Weight of the body
R = Normal reaction, and
μ = Coefficient of friction.

First of all, consider a pull of 180 N acting on the body. We know that in this case, the force of friction (F_1) will act towards left as shown in Fig. (a).

Resolving the forces horizontally,

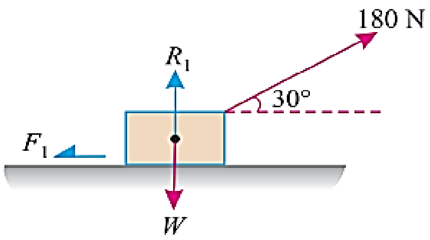
$$F_1 = 180 \cos 30° = 180 \times 0.866 = 155.9\, N$$

and now resolving the forces vertically,

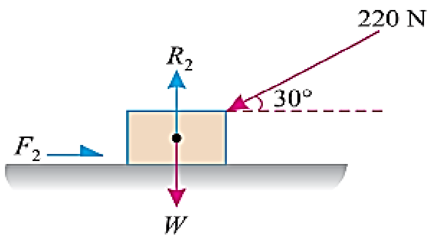
$$R_1 = W - 180 \sin 30° = W - 180 \times 0.5 = W - 90\, N$$

We know that the force of friction (F_1),

$$155.9 = \mu R_1 = \mu\,(W - 90) \ldots\ldots\ldots\ldots (i)$$



(a) Pull of 180 N



(b) Pull of 220 N

Now consider a push of 220 N acting on the body. We know that in this case, the force of friction (F_2) will act towards right as shown in Fig. (b).

Resolving the forces horizontally,

$$F_2 = 220 \cos 30° = 220 \times 0.866 = 190.5\, N$$

and now resolving the forces horizontally,

$$R_2 = W + 220 \sin 30° = W + 220 \times 0.5 = W + 110\, N$$

We know that the force of friction (F_2),

$$190.5 = \mu . R_2 = \mu\,(W + 110) \ldots\ldots\ldots\ldots (ii)$$

Dividing equation (*i*) by (*ii*)

$$\frac{155.9}{190.5} = \frac{\mu\,(W - 90)}{\mu\,(W + 110)} = \frac{(W - 90)}{(W + 110)}$$

$$\Rightarrow 155.9\, W + 17\,149 = 190.5\, W - 17\,145$$

$$\Rightarrow 34.6\, W = 34\,294$$

$$\Rightarrow W = \frac{34\,294}{34.6} = \mathbf{991.2\, N}$$

Now substituting the value of W in equation (*i*),

$$155.9 = \mu\,(991.2 - 90) = 901.2\, \mu$$

$$\Rightarrow \mu = \frac{155.9}{901.2} = \mathbf{0.173}$$

<u>EQUILIBRIUM OF A BODY ON A ROUGH INCLINED PLANE SUBJECTED TO A FORCE ACTING ALONG THE INCLINED PLANE</u>:

Consider a body lying on a rough inclined plane subjected force acting along the inclined plane, which keeps it in equilibrium as shown in Fig. (a) and (b).

Let W = Weight of the body,

α = Angle, which the inclined plane makes with the horizontal,

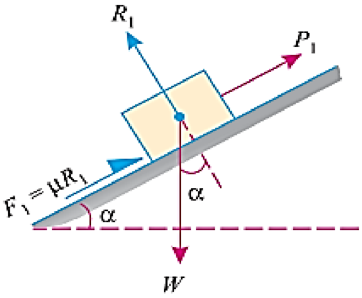
R = Normal reaction,

μ = Coefficient of friction between the body and the inclined plane, and

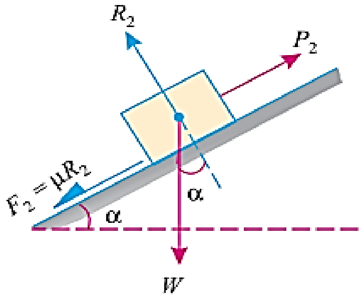
ϕ = Angle of friction, such that $\mu = \tan \phi$.

A little consideration will show that if the force is not there, the body will slide down the plane. Now we shall discuss the following two cases:

1. Minimum force (P_1) which will keep the body in equilibrium, when it is at the point of sliding downwards



(*a*) Body at the point of sliding downwards



(*b*) Body at the point of sliding upwards

In this case, the force of friction ($F_1 = \mu.R_1$) will act upwards, as the body is at the point of sliding downwards as shown in Fig. (*a*). Now resolving the forces along the plane,

$$P_1 = W \sin \alpha - \mu.\ R_1 \text{ ……………… } (i)$$

And now resolving the forces perpendicular to the plane.

$$R_1 = W \cos \alpha \text{ .... } (ii)$$

Substituting the value of R_1 in equation (i),

$$P_1 = W \sin \alpha - \mu\ W \cos \alpha = W (\sin \alpha - \mu \cos \alpha)$$

And now substituting the value of $\mu = \tan \phi$ in the above equation,

$$P_1 = W (\sin \alpha - \tan \phi \cos \alpha)$$

Multiplying both sides of this equation by $\cos \phi$,

$$P_1 \cos \phi = W (\sin \alpha \cos \phi - \sin \phi \cos \alpha) = W \sin (\alpha - \phi)$$

$$\therefore\ \boldsymbol{P_1 = W \times \frac{\sin(\alpha - \emptyset)}{\cos \emptyset}}$$

2. Maximum force (P_2) which will keep the body in equilibrium, when it is at the point of sliding upwards

In this case, the force of friction ($F_2 = \mu.\ R_2$) will act downwards as the body is at the point ofsliding upwards as shown in Fig. (b). Now resolving the forces along the plane,

$$P_2 = W \sin \alpha + \mu.\ R_2 \text{ …………... } (i)$$

And now resolving the forces perpendicular to the plane,

$$R_2 = W \cos \alpha \text{ .... } (ii)$$

Substituting the value of R_2 in equation (i),

$$P_2 = W \sin \alpha + \mu\ W \cos \alpha = W (\sin \alpha + \mu \cos \alpha)$$

And now substituting the value of $\mu = \tan \phi$ in the above equation,

$$P_2 = W (\sin \alpha + \tan \phi \cos \alpha)$$

Multiplying both sides of this equation by cos ϕ,

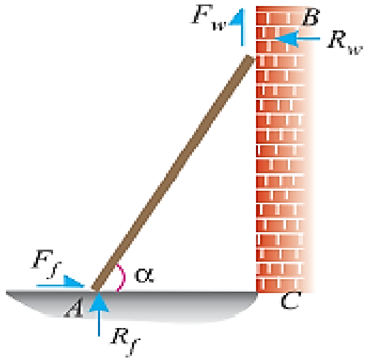
$$P_2 \cos \phi = W (\sin \alpha \cos \phi + \sin \phi \cos \alpha) = W \sin (\alpha + \phi)$$

$$\therefore\ \boldsymbol{P_2 = W \times \frac{\sin(\alpha + \emptyset)}{\cos \emptyset}}$$

3.3 Ladder, Wedge Friction

Ladder Friction:

- The ladder is a device for climbing or scaling on the roofs or walls. It consists of two long uprights of wood, iron or rope connected by a number of cross pieces called rungs. These runing serve as steps.
- Consider a ladder *AB* resting on the rough ground and leaning against a wall, as shown in the figure



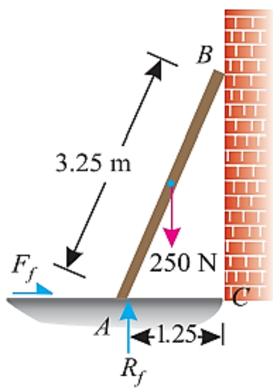
Here, F_f = Force of friction between the ladder and the floor will be towards the wall.
F_w = Force of friction between the ladder and the wall will be upwards.
R_f = Normal reaction of the floor will act perpendicular to the ladder.
R_w = Normal reaction of the wall will also act perpendicular to the ladder.
α = Angle between the ladder and the floor.

- Since the system is in equilibrium, therefore the algebraic sum of the horizontal and vertical components of the forces must also be equal to zero.

Example – 3: ***A uniform ladder of length 3.25 m and weighing 250 N is placed against a smooth vertical wall with its lower end 1.25 m from the wall. The coefficient of friction between the ladder and floor is 0.3. What is the frictional force acting on the ladder at the point of contact between the ladder and the floor? Show that the ladder will remain in equilibrium in this position.***

Solution. Given: Length of the ladder (*l*) = 3.25 m; Weight of the ladder (w) = 250 N; Distance between the lower end of ladder and wall = 1.25 m and coefficient of friction between the ladder and floor (μ_f) = 0.3.

Frictional force acting on the ladder.
The forces acting on the ladder are shown in Figure.



Let F_f = Frictional force acting on the ladder at the Point of contact between the ladder and floor, and
R_f = Normal reaction at the floor.

Since the ladder is placed against a smooth vertical wall, therefore there will be no friction at the point of contact between the ladder and wall. Resolving the forces vertically,

$$R_f = 250\ N$$

From the geometry of the figure, we find that

$$BC = \sqrt{(3.25)^2 - (1.25)^2} = 3.0\ m$$

Taking moments about B and equating the same,

$$F_f \times 3 = (R_f \times 1.25) - (250 \times 0.625) = (250 \times 1.25) - 156.3 = 156.2\ N$$

$$\Rightarrow F_f = \frac{156.2}{3} = 52.1\ N$$

Equilibrium of the ladder

We know that the maximum force of friction available at the point of contact between the ladder and the floor

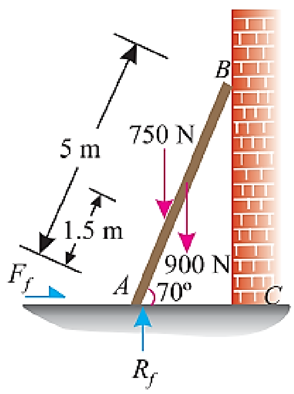
$$= \mu R_f = 0.3 \times 250 = 75\ N$$

Thus, we see that the amount of the force of friction available at the point of contact (75 N) is more than the force of friction required for equilibrium (52.1 N). Therefore, the ladder will remain in an equilibrium position.

Example – 4: ***A ladder 5 meters long rests on a horizontal ground and leans against a smooth vertical wall at an angle 70° with the horizontal. The weight of the ladder is 900 N and acts at its middle. The ladder is at the point of sliding, when a man weighing 750N stands on a rung 1.5 metre from the bottom of the ladder. Calculate the coefficient of friction between the ladder and the floor.***

Solution. Given: Length of the ladder (l) = 5 m; Angle which the ladder makes with the horizontal (α) = 70°; Weight of the ladder (w_1) = 900 N; Weight of man (w_2) = 750 N and distance between the man and bottom of ladder = 1.5 m.

Forces acting on the ladder are shown in Figure.



Let μ_f = Coefficient of friction between ladder and floor and

R_f = Normal reaction at the floor.

Resolving the forces vertically,

$$R_f = 900 + 750 = 1650\ N \ldots\ldots\ldots\ldots (i)$$

∴ Force of friction at A

$$F_f = \mu_f \times R_f = \mu_f \times 1650 \ldots\ldots\ldots\ldots (ii)$$

Now taking moments about B, and equating the same,

$$R_f \times 5 \sin 20° = (F_f \times 5 \cos 20°) + (900 \times 2.5 \sin 20°) + (750 \times 3.5 \sin 20°)$$

$$= (F_f \times 5 \cos 20°) + (4875 \sin 20°)$$

$$= (\mu_f \times 1650 \times 5 \cos 20°) + 4875 \sin 20°$$

and now substituting the values of R_f and F_f from equations (i) and (ii)

$$1650 \times 5 \sin 20° = (\mu_f \times 1650 \times 5 \cos 20°) + (4875 \sin 20°)$$

Dividing both sides by 5 sin 20°,

$$1650 = (\mu_f \times 1650 \cot 20°) + 975$$

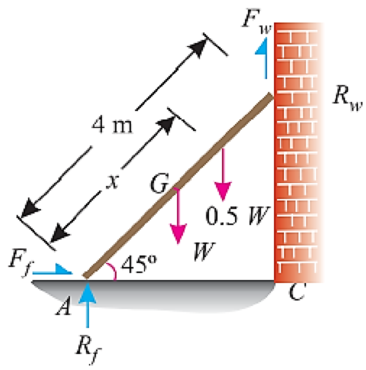
$$= (\mu_f \times 1650 \times 2.7475) + 975 = 4533\,\mu_f + 975$$

$$\Rightarrow \mu_f = \frac{1650 - 975}{4533} = \mathbf{0.15}$$

Example – 5: ***A uniform ladder of 4 m length rests against a vertical wall with which it makes an angle of 45°. The coefficient of friction between the ladder and the wall is 0.4 and that between ladder and the floor is 0.5. If a man, whose weight is one-half of that of the ladder ascends it, how high will it be when the ladder slips?***

Solution. Given: Length of the ladder (l) = 4 μ; Angle which the ladder makes with the horizontal (α) = 45°; Coefficient of friction between the ladder and the wall (μ_w) = 0.4 and coefficient of friction between the ladder and the floor (μ_f) = 0.5.

The forces acting on the ladder are shown in Figure.



Let x = Distance between A and the man, when the ladder is at the point of slipping.

W = Weight of the ladder, and

R_f = Normal reaction at floor.

$$\therefore \text{ Weight of the man} = \frac{W}{2} = 0.5\,W$$

We know that frictional force at the floor,

$$F_f = \mu_f R_f = 0.5\,R_f \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$$

and frictional force at the wall,

$$F_w = \mu_w R_w = 0.4\,R_w \ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

Resolving the forces vertically,

$$R_f + F_w = W + 0.5W = 1.5\,W \ldots\ldots\ldots\ldots\ldots\ldots\ldots (iii)$$

and now resolving the forces horizontally,

$$R_w = F_f = 0.5\,R_f \text{ or } R_f = 2R_w$$

Now substituting the values of R_f and F_w in equation (iii),

$$2R_w + 0.4\,R_w = 1.5\,W$$

$$\therefore R_w = \frac{1.5\,W}{2.4} = 0.625\,W$$

$$F_w = 0.4\,R_w = 0.4 \times 0.625\,W = 0.25\,W \ldots\ldots\ldots\ldots\ldots\ldots (iv)$$

Taking moments about A and equating the same,

$$(W \times 2\cos 45°) + (0.5\,W \times x\cos 45°) = (R_w \times 4\sin 45°) + (F_w \times 4\cos 45°)$$

Substituting values of R_w and F_w from equations (iii) and (iv) in the above equation,

$$(W \times 2\cos 45°) + (0.5\,W \times x\cos 45°) = (0.625\,W \times 4\sin 45°) + (0.25\,W \times 4\cos 45°)$$

Dividing both sides by (W sin 45°),

$$2 + 0.5\,x = 2.5 + 1 = 3.5$$

$$\Rightarrow x = \frac{3.5 - 2}{0.5} = \mathbf{3\ m}$$

Wedge Friction:

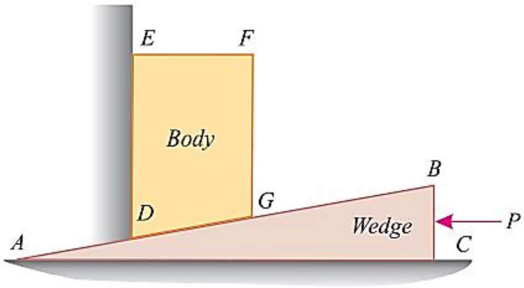
A wedge is, usually, of a triangular or trapezoidal in cross-section. It is, generally, used for slight adjustements in the position of a body i.e., for tightening fits or keys for shafts. Sometimes, a wedge is also used for lifting heavy weights as shown in Figure.

It will be interesting to know that the problems on wedges are basically the problems of equilibrium on inclined planes. Thus, these problems may be solved either by the equilibrium method or by applying Lami's theorem. Now consider a wedge ABC, which is used to lift the body DEFG.

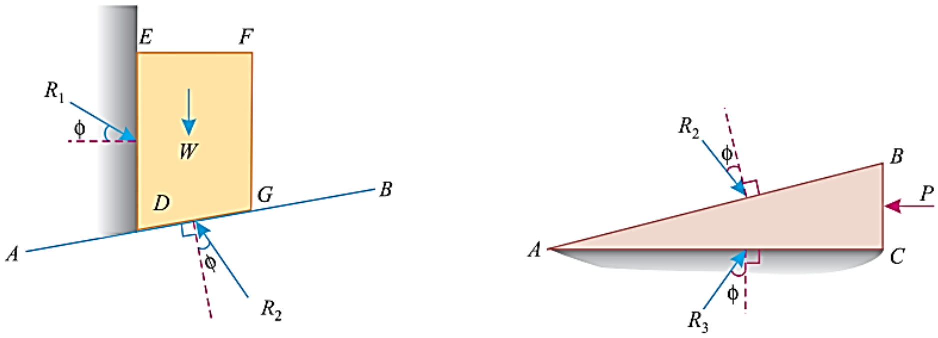
Let W = Weight of the body DEFG,

P = Force required to lift the body, and

μ = Coefficient of friction on the planes AB, AC and DE such that tan φ = μ.



A little consideration will show that when the force is sufficient to lift the body, the sliding will take place along three planes AB, AC and DE will also occur as shown in Figure.



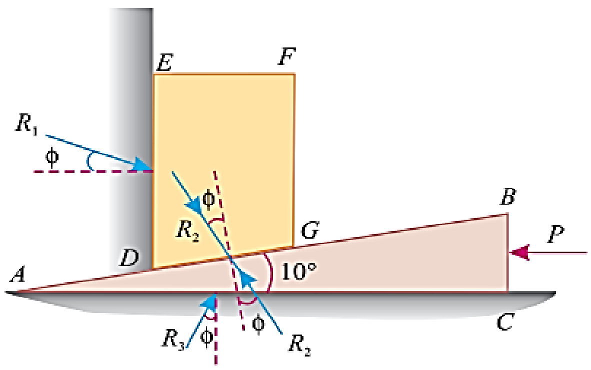
The three reactions and the horizontal force (P) may now be found out either by graphical method or analytical method as discussed below:

Graphical method

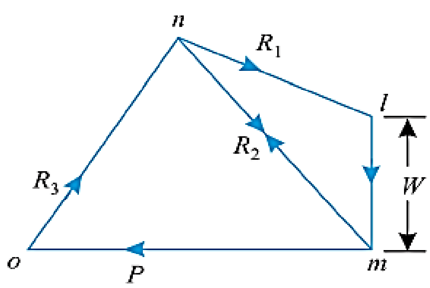
1. First of all, draw the space diagram for the body DEFG and the wedge ABC as shown in Figure (a). Now draw the reactions R_1, R_2 and R_3 at angle φ with normal to the faces DE, AB and AC respectively (such that tan φ = μ).

2. Now consider the equilibrium of the body DEFG. We know that the body is in equilibrium under the action of
   (a) Its own weight (W) acting downwards
   (b) Reaction R_1 on the face DE, and
   (c) Reaction R_2 on the face AB.

   Now, in order to draw the vector diagram for the above mentioned three forces, take some suitable point *l* and draw a vertical line *lm* parallel to the line of action of the weight (W) and cut off *lm* equal to the weight of the body to some suitable scale. Through *l* draw a line parallel to the reaction R_1. Similarly, through *m* draw a line parallel to the reaction R_2, meeting the first line at n as shown in Figure (b).



(*a*) Space diagram



(*b*) Vector diagram

3. Now consider the equilibrium of the wedge ABC. We know that it is equilibrium under the action of
   (a) Force acting on the wedge (P),
   (b) Reaction R_2 on the face AB, and
   (c) Reaction R_3 on the face AC.

   Now, in order to draw the vector diagram for the above mentioned three forces, through *m* draw a horizontal line parallel to the force (P) acting on the wedge. Similarly, through *n* draw a line parallel to the reaction R_3 meeting the first line at O as shown in Figure (b).

4. Now the force (P) required on the wedge to raise the load will be given by *mo* to the scale.

Analytical method

1. First of all, consider the equilibrium of the body DEFG. And resolve the forces W, R_1 and R_2 horizontally as well as vertically.

2. Now consider the equilibrium of the wedge ABC. And resolve the forces P, R_2 and R_3 horizontally as well as vertically.

POSSIBLE SHORT TYPE QUESTIONS WITH ANSWER

1. Define friction. **(W – 2016)**
Ans: When a rigid body slides over another rigid body, a resisting force is exerted at the surface of contact, this resisting force is called force of friction or simply friction. It always acts in a direction opposite to the direction of motion.

2. Define limiting friction. **(Possible)**
Ans: It is the maximum force of friction between two contact surfaces when a body tends to move over another body.

3. Define angle of friction. **(S – 2019)**
Ans: It is the angle between normal reaction (R_N) and resultant (R) of normal reaction and frictional forces. It is denoted by $\boldsymbol{\varphi}$.

$$\Rightarrow\ \boldsymbol{\varphi = \tan^{-1}\left(\frac{F}{R_N}\right)}$$

4. Define coefficient of friction. **(W – 2016, 2017 & S – 2018)**
Ans:

- It is the ratio of limiting friction (F) to the normal reaction (R_N) between two bodies.
- It is generally denoted by $\boldsymbol{\mu}$.
- Mathematically,

$$\boldsymbol{\mu = \frac{F}{R_N} = \tan\varphi}$$
$$\boldsymbol{\Rightarrow \varphi = \tan^{-1}\mu}$$

5. Define angle of repose. **(S – 2019 Old)**
Ans: It is the maximum angle between the inclined plane and horizontal plane at which the body just tends to slide downwards. This angle is generally specified by α.

6. Write any two disadvantages of friction. **(S – 2019)**
Ans:

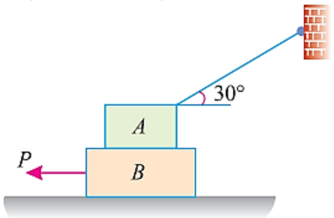
- Friction produces unnecessary heat leading to the wastage of energy.
- The force of friction acts in the opposite direction of motion, so friction slows down the motion of moving objects.

POSSIBLE LONG TYPE QUESTIONS

1. Explain laws of friction? **(W – 2016)**

2. What are the advantages and disadvantages of friction? **(Possible)**

3. Two blocks A and B of weights 1 kN and 2 kN respectively are in equilibrium position as shown in Fig. below. If the coefficient of friction between the two blocks as well as the block B and the floor is 0.3, find the force (P) required to move the block B. **(W – 2016)**



4. A body, resting on a rough horizontal plane, required a pull of 18 N inclined at 30° to the plane just to move it. It was found that a push of 22 N inclined at 30° to the plane just moved the body. Determine the weight of the body and the coefficient of friction. **(W – 2017)**

5. A force of 250 N pulls a body of weight 500 N up an inclined plane, the force being applied parallel to the plane. If the inclination of the plane to the horizontal is 15^0, find the coefficient of friction. **(S – 2018)**

4. A uniform ladder 3 m long weighs 200 N. It is placed against a wall making an angle of 60° with the floor. The coefficient of friction between the wall and the ladder is 0.25 and that between the floor and ladder is 0.35. The ladder, in addition to its own weight, has to support a man of 1000 N at its top at B. Calculate the horizontal force P to be applied to ladder at the floor level to prevent slipping. **(S – 2018)**

5. A body of weight 50 N is pulled along a rough horizontal plane by a force of 18 N acting at an angle of 14^0 with the horizontal. Find the coefficient of friction. **(S – 2019)**

6. A ladder 5 meters long rests on a horizontal ground and leans against a smooth vertical wall at an angle 70° with the horizontal. The weight of the ladder is 900 N and acts at its middle. The ladder is at the point of sliding, when a man weighing 750N stands on a rung 1.5 metre from the bottom of the ladder. Calculate the coefficient of friction between the ladder and the floor. **(S – 2019)**

CHAPTER NO. – 04

CENTROID & MOMENT OF INERTIA

LEARNING OBJECTIVES

4.1 Centroid – Definition, Moment of an area about an axis, centroid of geometrical figures such as squares, rectangles, triangles, circles, semicircles & quarter circles, centroid of composite figures.

4.2 Moment of Inertia – Definition, Parallel axis & Perpendicular axis Theorems. M.I. of plane lamina & different engineering sections.

4.1 Centroid – Definition, Moment of an area about an axis, centroid of geometrical figures such as squares, rectangles, triangles, circles, semicircles & quarter circles, centroid of composite figures

Centre of gravity:

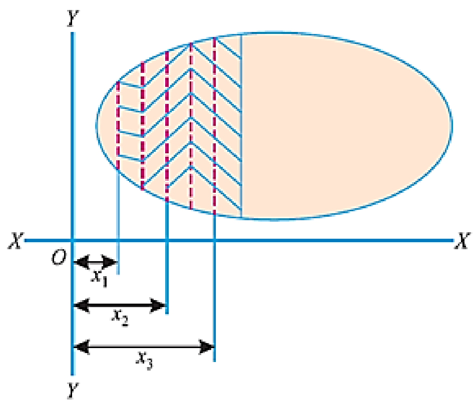
- Centre of gravity of a body may be defined as the point through which the whole weight of a body may be assumed to act.

Centroid:

- Centroid or Centre of area is defined as the point where the whole area of the figure is assumed to be concentrated.

Moment of an area about an axis:

Consider a body of area *A* whose centroid is required to be found out. Divide the body into small masses, Let a_1, a_2, a_3 ……. etc. be the masses of the particles and (x_1, y_1), (x_2, y_2), (x_3, y_3) ……. be the co-ordinates of the centroid from a fixed-point *O* as shown in Fig. below.



Let $\overline{x}$ and $\overline{y}$ be the co-ordinates of the centroid of the body. From the principle of moments, we know that

$$A\overline{x} = a_1x_1 + a_2x_2 + a_3x_3 + \ldots\ldots\ldots.$$

$$\Rightarrow \overline{x} = \frac{\sum ax}{A}$$

Similarly,

$$\overline{y} = \frac{\sum ay}{A}$$

Where,

$$A = a_1 + a_2 + a_3 + \ldots\ldots\ldots.$$

Centroid by geometrical consideration:

The centroid of simple figures may be found out from the geometry of the figure as given below.

1. The centroid of uniform rod is at its middle point.

2. The centroid of a rectangle (or a parallelogram) is at the point, where its diagonals meet each other. It is also a middle point of the length as well as the breadth of the rectangle as shown in Fig. 6.1

3. The centroid of a triangle is at the point, where the three medians (a median is a line connecting the vertex and middle point of the opposite side) of the triangle meet as shown in Fig. 6.2.

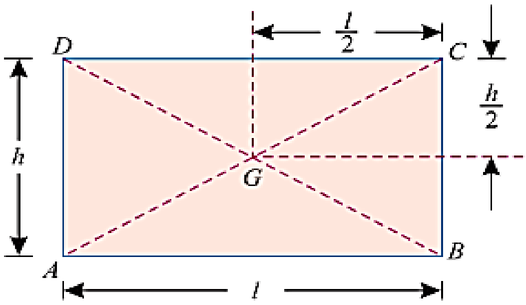


Fig. 6.1. Rectangle

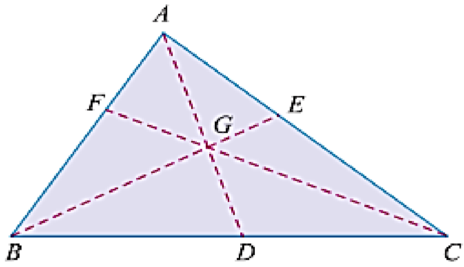


Fig. 6.2. Triangle

4. The centroid of a trapezium with parallel sides a and b is at a distance of $\frac{h}{3} \times \left(\frac{b+2a}{b+a}\right)$ measured form the side b as shown in Fig. 6.3.

5. The centroid of a semicircle is at a distance of $\frac{4r}{3\pi}$ from its base measured along the vertical radius as shown in Fig. 6.4.

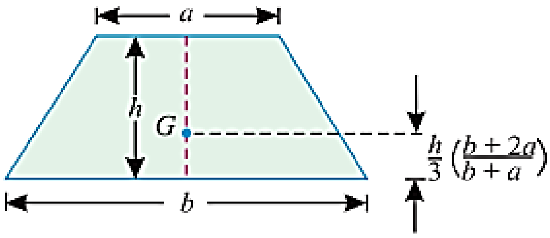


Fig. 6.3. Trapezium

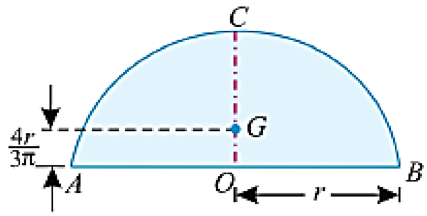


Fig. 6.4. Semicircle

6. The centroid of a circular sector making semi-vertical angle α is at a distance of $\left(\frac{2r}{3}\frac{\sin\alpha}{\alpha}\right)$ from the centre of the sector measured along the central axis as shown in Fig. 6.5.

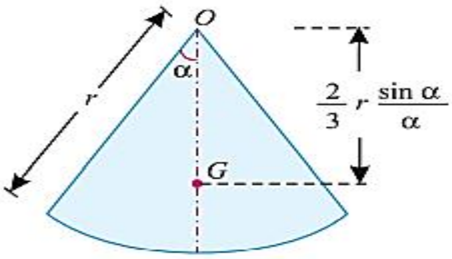


Fig. 6.5. Circular sector

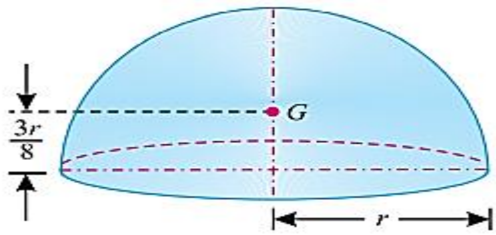


Fig. 6.6. Hemisphere

7. The centroid of a cube is at a distance of $l/2$ from every face (where l is the length of each side).

8. The centroid of a sphere is at a distance of d/2 from every point (where d is the diameter of the sphere).

9. The centroid of a hemisphere is at a distance of 3r/8 from its base, measured along the vertical radius as shown in Fig. 6.6.

10. The centroid of right circular solid cone is at a distance of h/4 from its base, measured along the vertical axis as shown in Fig. 6.7.

11. The centroid of a segment of sphere of a height h is at a distance of $\frac{3}{4}\frac{(2r-h)^2}{(3r-h)}$ from the centre of the sphere measured along the height. as shown in Fig. 6.8.

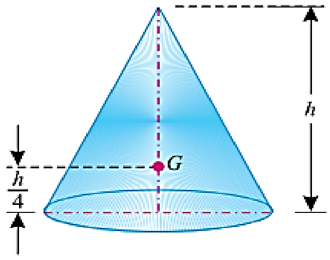


Fig. 6.7. Right circular solid cone

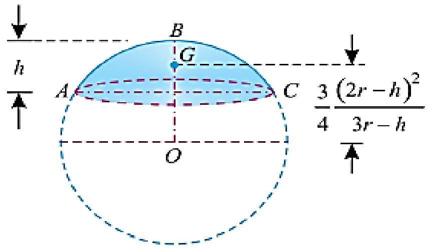
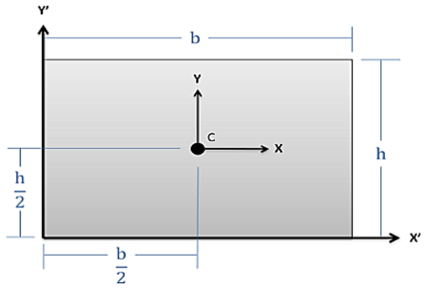


Fig.6.8. Segment of a sphere

Centroid of geometrical figures:

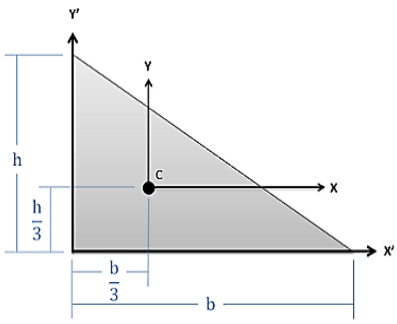
1. Rectangle:



$$Area = bh$$

$$\bar{x} = \frac{b}{2} \text{ and } \bar{y} = \frac{h}{2}$$

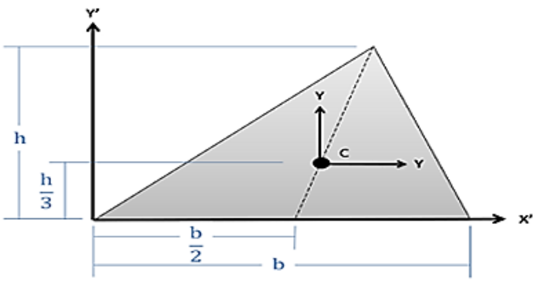
2. Right triangle:



$$Area = \frac{1}{2}bh$$

$$\bar{x} = \frac{b}{3} \text{ and } \bar{y} = \frac{h}{3}$$

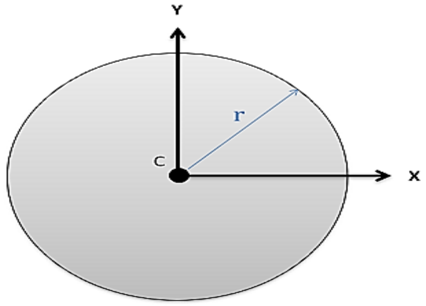
3. Triangle:



$$Area = \frac{1}{2}bh$$

$$\bar{x} = \frac{b}{2} \text{ and } \bar{y} = \frac{h}{3}$$

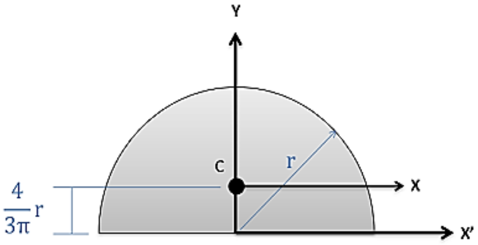
4. Circle:



$$Area = \pi r^2$$

$$\overline{\boldsymbol{x}} = \boldsymbol{r} \textbf{ and } \overline{\boldsymbol{y}} = \boldsymbol{r}$$

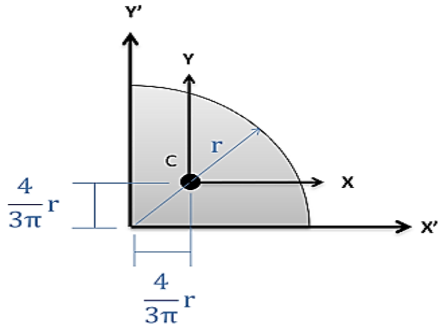
5. Semicircle:



$$Area = \frac{\pi}{2}r^2$$

$$\overline{\boldsymbol{x}} = \boldsymbol{r} \textbf{ and } \overline{\boldsymbol{y}} = \frac{\boldsymbol{4r}}{\boldsymbol{3\pi}}$$

6. Quadrant:



$$Area = \frac{\pi}{4}r^2$$

$$\overline{\boldsymbol{x}} = \frac{\boldsymbol{4r}}{\boldsymbol{3\pi}} \textbf{ and } \overline{\boldsymbol{y}} = \frac{\boldsymbol{4r}}{\boldsymbol{3\pi}}$$

Note: $\overline{\boldsymbol{x}}$ **=** Horizontal distance between the centroid and the vertical axis.
$\overline{y}$ = Vertical distance between the centroid and the horizontal axis.

<u>Centroid of plane figures</u>:

- The plane geometrical figures (such as T-section, I-section, L-section etc.) have only areas but no mass. The centre of gravity of such figures is found out in the same way as that of solid bodies. The centre of area of such figures is known as centroid, and coincides with the centre of gravity of the figure.
- Let $\overline{x}$ and $\overline{y}$ be the co-ordinates of the centre of gravity with respect to some axis of reference, then

$$\overline{\boldsymbol{x}} = \frac{\boldsymbol{a_1 x_1 + a_2 x_2 + a_3 x_3 + \cdots}}{\boldsymbol{a_1 + a_2 + a_3 + \cdots}}$$

$$\overline{\boldsymbol{y}} = \frac{\boldsymbol{a_1 y_1 + a_2 y_2 + a_3 y_3 + \cdots}}{\boldsymbol{a_1 + a_2 + a_3 + \cdots}}$$

Centre of gravity of symmetrical sections:

Sometimes, the given section, whose centre of gravity is required to be found out, is symmetrical about X-X axis or Y-Y axis. In such cases, the procedure for calculating the centre of gravity of the body is very much simplified; as we have only to calculate either $\overline{\boldsymbol{x}}$ or $\overline{\boldsymbol{y}}$. This is due to the reason that the centre of gravity of the body will lie on the axis of symmetry.

Example – 1: ***Find the centre of gravity of a 100 mm × 150 mm × 30 mm T-section.***

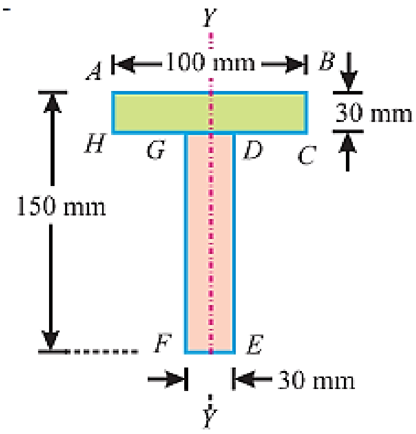
Solution: As the section is symmetrical about Y-Y axis, bisecting the web, therefore its Centre of gravity will lie on this axis. Split up the section into two rectangles ABCH and DEFG as shown in figure.

Let bottom of the web FE be the axis of reference.

(*i*) Rectangle ABCH

$$a_1 = 100 \times 30 = 300mm^2$$

$$y_1 = \left(150 - \frac{30}{2}\right) = 135mm$$



(*ii*) Rectangle DEFG

$$a_2 = 120 \times 30 = 3600mm^2$$

$$y_2 = \frac{120}{2} = 60mm$$

We know that the distance between the centre of gravity of the section and bottom of the flange FE

$$\overline{y} = \frac{a_1 + y_1}{a_1 + a_2} = \frac{(3000 \times 135) + (3600 \times 60)}{3000 + 3600} = 94.1mm\ (\boldsymbol{Ans})$$

Example – 2: ***Find the centre of gravity of a channel section 100 mm × 50 mm × 15 mm.***

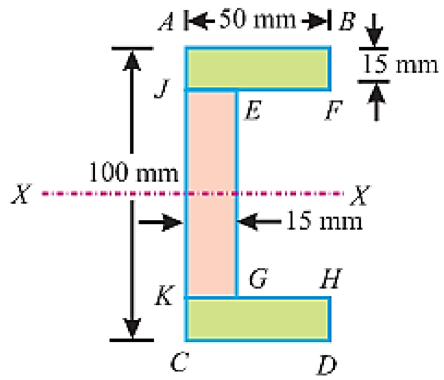
Solution: As the section is symmetrical about X-X axis, therefore its centre of gravity will lie on this axis. Now split up the whole section into three rectangles ABFJ, EGKJ and CDHK as shown in figure.

Let the face AC be the axis of reference.

(*i*) Rectangle ABFJ

$$a_1 = 50 \times 15 = 750mm^2$$

$$x_1 = \frac{50}{2} = 25mm$$



(*ii*) Rectangle EGKJ

$$a_2 = (100 - 30) \times 15 = 1050mm^2$$

$$x_2 = \frac{15}{2} = 7.5mm$$

(*iii*) Rectangle CDHK

$$a_3 = 15 \times 50 = 750mm^2$$

$$x_3 = \frac{50}{2} = 25mm$$

We know that the distance between the centres of gravity of the section and left face of the section AC

$$\overline{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3} = \frac{(720 \times 25) + (1050 \times 7.5) + (750 \times 20)}{750 + 1050 + 750} = 17.8mm\ (\boldsymbol{Ans})$$

Example – 3: ***An I-section has the following dimensions in mm units:***
Bottom flange = 300 × 100, Top flange = 150 × 50, Web = 300 × 50.
Determine mathematically the position of centre of gravity of the section.

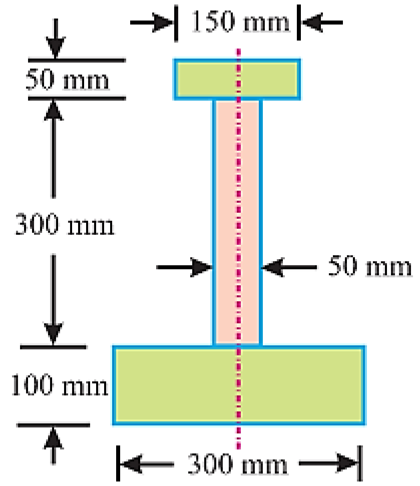
Solution: As the section is symmetrical about Y-Y axis, bisecting the web, therefore its centre of gravity will lie on this axis. Now split up the section into three rectangles as shown in figure.

Let bottom of the bottom flange be the axis of reference.

(*i*) Bottom flange

$$a_1 = 300 \times 100 = 30000mm^2$$

$$y_1 = \frac{100}{2} = 50mm$$



(*ii*) Web

$$a_2 = 300 \times 50 = 15000mm^2$$

$$y_2 = 100 + \frac{300}{2} = 250mm$$

(*iii*) Top flange

$$a_3 = 150 \times 50 = 7500mm^2$$

$$y_3 = 100 + 300 + \frac{50}{2} = 425mm$$

We know that the distance between centre of gravity of the section and bottom of the flange,

$$\overline{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3} = \frac{(30000 \times 50) + (15000 \times 250) + (7500 \times 425)}{30000 + 15000 + 7500} = 160.7mm\ (\boldsymbol{Ans})$$

<u>Centre of gravity of unsymmetrical sections</u>:

Sometimes, the given section, whose centre of gravity is required to be found out, is not symmetrical either about X-X axis or Y-Y axis. In such cases, we have to find out both the values of $\boldsymbol{\overline{x}}$ and $\boldsymbol{\overline{y}}$.

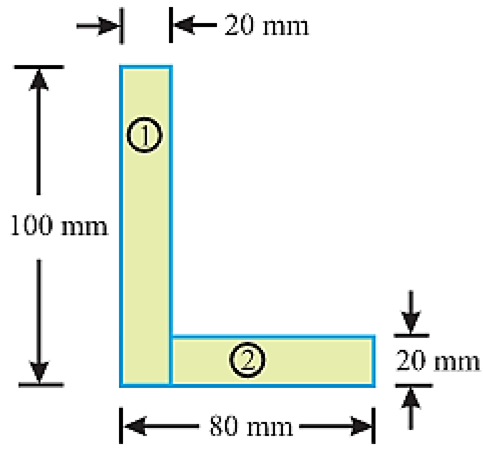
Example – 4: ***Find the centroid of an unequal angle section 100 mm × 80 mm × 20 mm.***

Solution: As the section is not symmetrical about any axis, therefore we have to find out the values of $\overline{x}$ and $\overline{y}$ for the angle section. Split up the section into two rectangles as shown in figure.
Let left face of the vertical section and bottom face of the horizontal section be axes of reference.

(*i*) Rectangle – 1

$$a_1 = 100 \times 20 = 2000mm^2$$

$$x_1 = \frac{20}{2} = 10mm$$



(*ii*) Rectangle – 2

$$a_2 = (80 - 20) \times 20 = 1200mm^2$$

$$x_2 = 20 + \frac{60}{2} = 50mm$$

$$y_2 = \frac{20}{2} = 10mm$$

And we know that the distance between the centre of gravity of the section and left face

$$\overline{x} = \frac{a_1x_1 + a_2x_2}{a_1 + a_2} = \frac{(2000 \times 10) + (1200 \times 50)}{(2000 + 1200)} = 25mm\ (\boldsymbol{Ans})$$

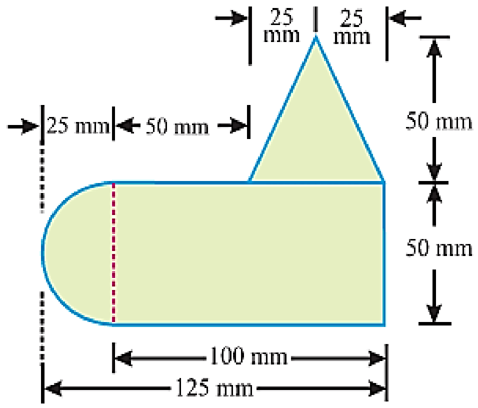
Similarly, the distance between centres of gravity of the section and bottom face

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(2000 \times 50) + (1200 \times 10)}{2000 + 1200} = 35mm\ (\boldsymbol{Ans})$$

Example – 5: ***A uniform lamina shown in Figure consists of a rectangle, a circle and a triangle. Determine the centre of gravity of the lamina. All dimensions are in mm.***

Solution: As the section is not symmetrical about any axis, therefore we have to find out the values of both $\bar{x}$ and $\bar{y}$ for the lamina.

Let left edge of circular portion and bottom face rectangular portion be the axes of reference.



(*i*) Rectangular portion

$$a_1 = 100 \times 50 = 5000mm^2$$

$$x_1 = 25 + \frac{100}{2} = 75mm$$

$$y_1 = \frac{50}{2} = 25mm$$

(*ii*) Semicircular portion

$$a_2 = \frac{\pi}{2} \times r^2 = \frac{\pi}{2}(25)^2 = 982mm^2$$

$$x_2 = 25 - \frac{4r}{3\pi} = 25 - \frac{4 \times 25}{3\pi} = 14.4mm$$

$$y_2 = \frac{50}{2} = 25mm$$

(*iii*) Triangular portion

$$a_3 = \frac{50 \times 50}{2} = 1250mm^2$$

$$x_3 = 25 + 50 + 25 = 100mm$$

$$y_3 = 50 + \frac{50}{3} = 66.7mm$$

We know that the distance between the center of gravity of the section and the left edge of the circular portion

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3} = \frac{(5000 \times 75) + (982 \times 14.4) + (1250 \times 100)}{5000 + 982 + 1250} = 71.1\ mm\ (\boldsymbol{Ans})$$

Similarly, the distance between the centre of gravity of the section and the bottom face of the rectangular portion.

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3} = \frac{(5000 \times 25) + (982 \times 25) + (1250 \times 66.7)}{5000 + 982 + 1250} = 32.2mm\ (\boldsymbol{Ans})$$

<u>Centre of gravity of sections with cut out holes</u>:

The centre of gravity of such a section is found out by considering the main section, first as a complete one, and then deducting the area of the cut-out hole i.e., by taking the area of the cut-out hole as negative. Now substituting a_2 (i.e., the area of the cut-out hole) as negative, in the general equation for the centre of gravity, we get

$$\boldsymbol{\bar{x} = \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2}} \quad \boldsymbol{and} \quad \boldsymbol{\bar{y} = \frac{a_1 y_1 - a_2 y_2}{a_1 - a_2}}$$

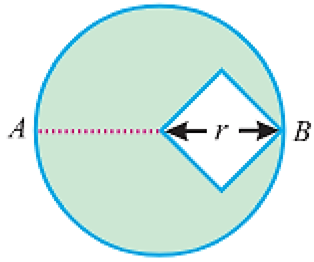
Example – 6: ***A square hole is punched out of circular lamina, the diagonal of the square being the radius of the circle as shown in Figure. Find the centre of gravity of the remainder, if r is the radius of the circle.***

Solution: As the section is symmetrical about X-X axis, therefore its centre of gravity will lie on this axis. Let A be the point of reference

(*i*) Main circle

$$a_1 = \pi r^2$$

$$x_1 = r$$



(*ii*) Cut out square

$$a_2 = \frac{r \times r}{2} = 0.5r^2$$

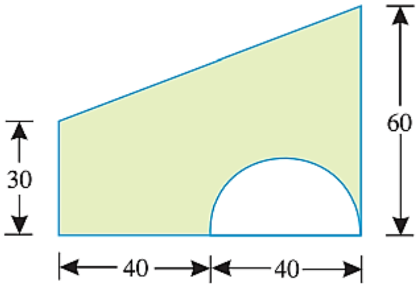
$$x_2 = r + \frac{r}{2} = 1.5r$$

We know that distance between centres of gravity of the section and A,

$$\bar{x} = \frac{a_1x_1 - a_2x_2}{a_1 - a_2} = \frac{(\pi r^2 \times r) - (0.5\, r^2 \times 1.5\, r)}{\pi r^2 - 0.5\, r^2}$$

$$= \frac{r^3(\pi - 0.75)}{r^2(\pi - 0.5)} = \frac{r(\pi - 0.75)}{\pi - 0.5} \; (\boldsymbol{Ans})$$

Example – 7: ***A semi-circular area is removed from a trapezium as shown in Figure (dimensions in mm) Determine the centroid of the remaining area (shown hatched).***



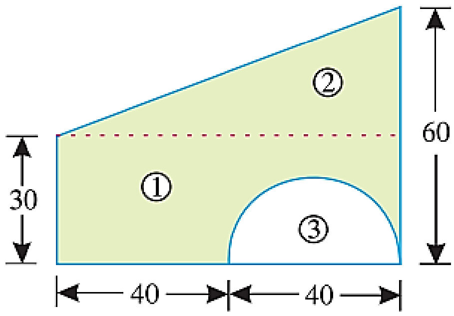
Solution: As the section in not symmetrical about any axis, therefore we have to find out the values of x and y for the area. Split up the area into three parts as shown in Fig. 6.25. Let left face and base of the trapezium be the axes of reference.

(*i*) Rectangle

$$a_1 = 80 \times 30 = 2400mm^2$$

$$x_1 = \frac{80}{2} = 40mm$$

$$y_1 = \frac{30}{2} = 15mm$$



(*ii*) Triangle

$$a_2 = \frac{80 \times 30}{2} = 1200mm^2$$

$$x_2 = \frac{80 \times 2}{3} = 53.3mm$$

$$y_2 = 30 + \frac{30}{3} = 40mm$$

(iii) Semi circle

$$a_3 = \frac{\pi}{2} \times r^2 = \frac{\pi}{2} 20^2 = 628.3mm^2$$

$$x_3 = 40 + \frac{40}{2} = 60mm$$

$$y_3 = \frac{4 \times 20}{3\pi} = \frac{4r}{3\pi} = 8.5mm$$

We know that the distance between center of gravity of the area and left face of trapezium

$$\overline{x} = \frac{a_1 x_1 + a_2 x_2 - a_3 x_3}{a_1 + a_2 - a_3} = \frac{(2400 \times 40) + (1200 \times 53.3) - (628.3 \times 60)}{2400 + 1200 - 628.3} = 41.1mm\ (\boldsymbol{Ans})$$

Similarly, the distance between center of gravity of the area and the base of the trapezium

$$\overline{y} = \frac{a_1 y_1 + a_2 y_2 - a_3 y_3}{a_1 + a_2 - a_3} = \frac{(2400 \times 15) + (1200 \times 40) - (628.3 \times 8.5)}{2400 + 1200 - 628.3} = 26.5mm\ (\boldsymbol{Ans})$$

4.2 Moment of Inertia – Definition, Parallel axis & Perpendicular axis Theorems. M.I. of plane lamina & different engineering sections

Moment of inertia

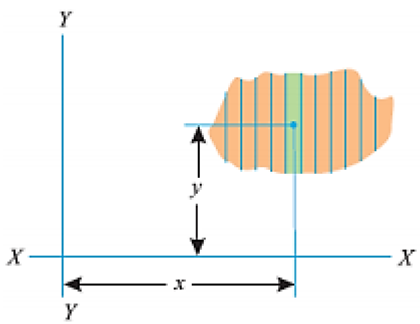
- Moment of a force (P) about a point is the product of the force and perpendicular distance (x) between the point and the line of action of the force (i.e., P. x). If this moment is again multiplied by the perpendicular distance (x) between the point and the line of action of the force i.e., P. x (x) = Px^2, then this quantity is called moment of inertia.
- It is defined as the sum of second moment of area of individual sections of a body about an axis.

Calculation of moment of inertia by integration method

The moment of inertia of an area may be found out by the method of integration:
Consider a plane figure, whose moment of inertia is required to be found out about X-X axis and Y-Y axis as shown in Figure below. Let us divide the whole area into a no. of strips. Consider one of these strips.

Let dA = Area of the strip
x = Distance of the centre of gravity of the strip on X-X axis and
y = Distance of the centre of gravity of the strip on Y-Y axis.

We know that the moment of inertia of the strip about Y-Y axis = $dA.x^2$
And the moment of inertia of the strip about X-X axis = $dA.y^2$
Now the moment of inertia of the whole area may be found out by integrating above equations.

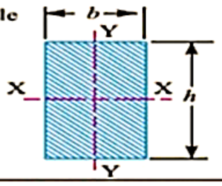
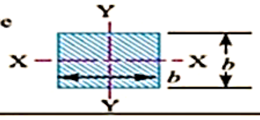
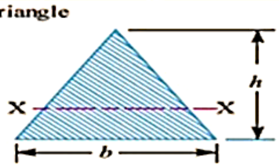
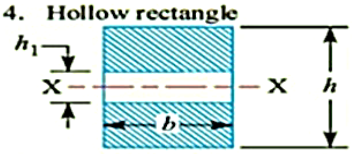
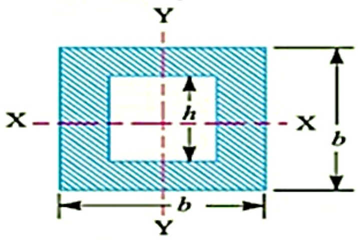
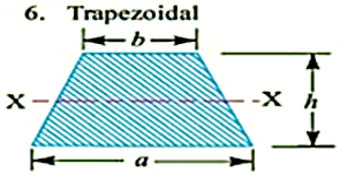
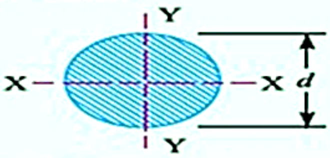
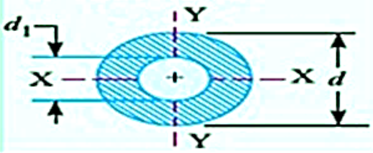
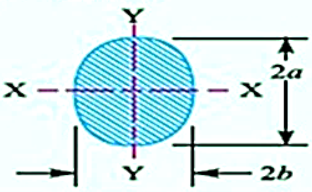


$$\boldsymbol{I_{YY} = \sum dA\, x^2 \quad Or \quad \int dA\, x^2}$$

$$\boldsymbol{I_{XX} = \sum dA\, y^2 \quad Or \quad \int dA\, y^2}$$

Unit: It depends on units of area and length
If area=m^2, length = m then, M.I. = m^4
If area = mm^2, length = mm then, M.I. = mm^4

Moment of inertia of some geometric shapes:

Section	Area (A)	Moment of inertia (I)	*Distance from the neutral axis to the extreme fibre (y)
1. Rectangle 	bh	$I_{xx} = \frac{bh^3}{12}$ $I_{yy} = \frac{hb^3}{12}$	$\frac{h}{2}$ $\frac{b}{2}$
2. Square 	b^2	$I_{xx} = I_{yy} = \frac{b^4}{12}$	$\frac{b}{2}$
3. Triangle 	$\frac{bh}{2}$	$I_{xx} = \frac{bh^3}{36}$	$\frac{h}{3}$
4. Hollow rectangle 	$b\,(h - h_1)$	$I_{xx} = \frac{b}{12}(h^3 - h_1^3)$	$\frac{h}{2}$
5. Hollow square 	$b^2 - h^2$	$I_{xx} = I_{yy} = \frac{b^4 - h^4}{12}$	$\frac{b}{2}$
6. Trapezoidal 	$\frac{a+b}{2} \times h$	$I_{xx} = \frac{h^2\,(a^2 + 4ab + b^2)}{36\,(a+b)}$	$\frac{a + 2b}{3\,(a+b)} \times h$
Section	**(A)**	**(I)**	**(y)**
7. Circle 	$\frac{\pi}{4} \times d^2$	$I_{xx} = I_{yy} = \frac{\pi d^4}{64}$	$\frac{d}{2}$
8. Hollow circle 	$\frac{\pi}{4}(d^2 - d_1^2)$	$I_{xx} = I_{yy} = \frac{\pi}{64}(d^4 - d_1^4)$	$\frac{d}{2}$
9. Elliptical 	πab	$I_{xx} = \frac{\pi}{4} \times a^3 b$ $I_{yy} = \frac{\pi}{4} \times ab^3$	a b

Theorem of perpendicular axis:

- It states, If I_{XX} and I_{YY} be the moments of inertia of a plane section about two perpendicular axis meeting at O, the moment of inertia I_{ZZ} about the axis Z-Z, perpendicular to the plane and passing through the intersection of X-X and Y-Y is given by:

$$\boldsymbol{I_{ZZ} = I_{XX} + I_{YY}}$$

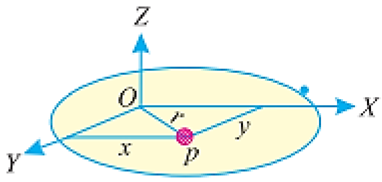
Proof:

- Consider a small lamina (P) of area ***da*** having co-ordinates as x and y along OX and OY two mutually perpendicular axes on a plane section as shown in the figure.

- Now consider a plane OZ perpendicular to OX and OY. Let (r) be the distance of the lamina (P) from Z-Z axis such that OP = r.

- From the geometry of the figure, we find that

$$r^2 = x^2 + y^2$$



- We know that the moment of inertia of the lamina P about X-X axis,

$$I_{XX} = da.y^2 \quad \ldots\ldots\ldots (I = Area \times Distance^2)$$

- Similarly,

$$I_{YY} = da.x^2$$

$$I_{ZZ} = da.r^2 = da(x^2 + y^2)$$

$$\Rightarrow I_{ZZ} = da.x^2 + da.y^2$$

$$\Rightarrow \boldsymbol{I_{ZZ} = I_{XX} + I_{YY}}$$

Theorem of parallel axis

- It states, If the moment of inertia of a plane area about an axis through its centre of gravity is denoted by IG, then moment of inertia of the area about any other axis AB, parallel to the first, and at a distance h from the centre of gravity is given by:

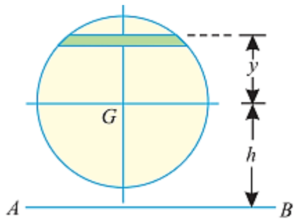
$$\boldsymbol{I_{AB} = I_G + ah^2}$$

- Where, $\boldsymbol{I_{AB}}$ = Moment of inertia of the area about an axis AB,
  $\boldsymbol{I_G}$ = Moment of Inertia of the area about its centre of gravity
  a = Area of the section, and
  h = Distance between centre of gravity of the section and axis AB.

Proof:

- Consider a strip of a circle, whose moment of inertia is required to be found out about a line AB as shown in the figure.

- Let, δa = Area of the strip
  y = Distance of the strip from the centre of gravity the section and
  h = Distance between centre of gravity of the section and the axis AB.



- We know that moment of inertia of the strip about an axis passing through the centre of gravity of the section = $\delta a.y^2$

- And moment of inertia of the whole section about an axis passing through its centre of gravity,

$$I_G = \sum \delta a.y^2$$

- ∴ Moment of inertia of the section about the axis AB,

$$I_{AB} = \sum \delta a\,(h + y)^2$$
$$= \sum \delta a\,(h^2 + y^2 + 2\,h\,y)$$
$$= (\sum h^2.\delta a) + (\sum y^2.\delta a) + (\sum 2\,h\,y.\delta a)$$
$$= a\,h^2 + I_G + 0$$
$$= I_G + ah^2$$

It may be noted that $\sum h^2.\ \delta a = a\ h^2$ and $\sum y^2.\ \delta a = I_G$ [as per equation above] and $\sum \delta a.\ y$ is the algebraic sum of moments of all the areas, about an axis through centre of gravity of the section and is equal to $a\,\overline{y}$., where $\overline{y}$ is the distance between the section and the axis passing through the centre of gravity, which obviously is zero.

Example – 8: ***Find the moment of inertia of a T-section with flange as 150 mm × 50 mm and web as 150 mm × 50 mm about X-X and Y-Y axes through the centre of gravity of the section.***

Solution: The given T-section is shown in Fig. First of all, let us find out centre of gravity of the section. As the section is symmetrical about Y-Y axis, therefore its centre of gravity will lie on this axis. Split up the whole section into two rectangles viz., 1 and 2 as shown in figure. Let bottom of the web be the axis of reference

(*i*) Rectangle – 1

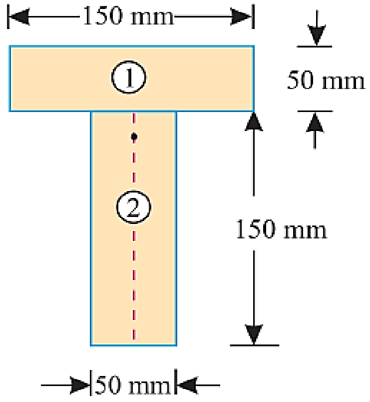
$$a_1 = 150 \times 50 = 7500mm^2$$

$$y_1 = 150 + \frac{50}{2} = 75mm$$

(*ii*) Rectangle – 2

$$a_2 = 150 \times 50 = 7500mm^2$$

$$y_2 = \frac{150}{2} = 75mm$$



We know that the distance between the centres of gravity of the section and bottom of the web,

$$\overline{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(7500 \times 175) + (7500 \times 75)}{7500 + 7500} = 125mm$$

Moment of inertia about X-X axis

We also know that M.I of the rectangle – 1 about an axis through its center of gravity and parallel to X-X axis.

$$I_{G_1} = \frac{150 \times (50)^3}{12} = 1.5625 \times 10^6 mm^4$$

And distance between the center of gravity of rectangle – 1 and X-X axis

$$h_1 = 175 - 125 = 50mm$$

Moment of inertia of rectangle – 1 about X-X axis

$$I_{G_1} + a_1 h^2 = (1.5625 \times 10^6) + [7500 \times (50)^2] = 20.3125 \times 10^6 mm^4$$

Similarly, moment of inertia of rectangle – 2 about an axis through its center of gravity and parallel to X-X axis

$$I_{G_2} = \frac{50 \times (150)^3}{12} = 14.0625 \times 10^6 mm^4$$

And distance between center of gravity of the rectangle – 2 and X-X axis

$$h_2 = 125 - 75 = 50mm$$

Moment of inertia of the rectangle – 2 about X-X axis

$$= I_{G_2} + a_2 {h_2}^2 = (14.0625 \times 10^6) + (7500 + 50^2) = 32.8125 \times 10^6 mm^4$$

Now moment of inertia of the whole section about X-X axis

$$I_{XX} = (20.3125) \times 10^6 + (32.8125 \times 10^6) = 53.125 \times 10^6 mm^4 \ (\boldsymbol{Ans})$$

Moment of inertia about Y-Y axis

We know that M.I of rectangle – 1 about Y-Y axis

$$= \frac{50 \times (50)^3}{12} = 14.0625 \times 10^6 mm^4$$

Moment of inertia of rectangle – 2 about Y-Y axis

$$= \frac{150 \times (50)^3}{12} = 1.5625 \times 10^6 mm^4$$

Moment of inertia of the whole section about Y-Y axis

$$I_{YY} = (14.0625 \times 10^6) + (1.5625 \times 10^6) = 15.625 \times 10^6 mm^4 \ (\boldsymbol{Ans})$$

Example – 9: ***An I-section is made up of three rectangles as shown in Figure. Find the moment of inertia of the section about the horizontal axis passing through the centre of gravity of the section.***

Solution: First of all, let us find out centre of gravity of the section. As the section is symmetrical about Y-Y axis, therefore its centre of gravity will lie on this axis. Split up the whole section into three rectangles 1, 2 & 3 as shown in figure.
Let bottom face of the bottom flange be the axis of reference.

(i) Rectangle – 1

$$a_1 = 60 \times 20 = 1200mm$$

$$y_1 = 20 + 100 + \frac{20}{2} = 130mm$$

(ii) Rectangle – 2

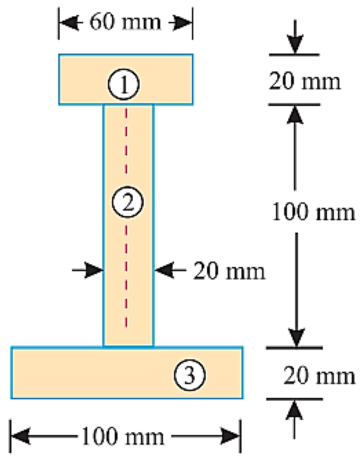
$$a_2 = 100 \times 20 = 2000mm^2$$

$$y_2 = 20 + \frac{100}{2} = 70mm$$

(iii) Rectangle – 3

$$a_3 = 100 \times 20 = 2000mm^2$$

$$y_3 = \frac{20}{2} = 10mm$$



We know that the distance between centre of gravity of the section and bottom face

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3} = \frac{(1200 \times 130) + (2000 \times 70) + (2000 \times 10)}{1200 + 2000 + 2000} = 60.8 \ mm$$

We know that the moment of inertia of rectangle – 1 about an axis through its centre of gravity and parallel to X-X axis.

$$I_{G_1} = \frac{(60 \times 20^3)}{12} = 40 \times 10^3 mm^4$$

And distance between centres of gravity of rectangle – 1 and X-X axis

$$h_1 = 130 - 60.8 = 69.2mm$$

Moment of inertia of rectangle1 about x-x axis

$$= I_{G_1} + a_1{h_1}^2 = (40 \times 10^3) + [1200 \times 69.2^2] = 5786 \times 10^3 mm^4$$

Similarly, the moment of inertia of rectangle – 2 about an axis through its centre of gravity and parallel to X-X axis.

$$I_{G_2} = \frac{20 \times 100^3}{12} = 1666.7 \times 10^3 mm^4$$

And distance between the centre of gravity of rectangle – 2 and x-x axis

$$h_2 = 70 - 60.8 = 9.2mm$$

Moment of inertia of rectangle – 2 about X-X axis

$$= I_{G_2} + a_2{h_2}^2 = (1666.7 \times 10^3) + [2000 \times 9.2^2] = 1836 \times 10^3 mm^4$$

Moment of inertia of rectangle – 3 about an axis through its centre of gravity and parallel to X-X axis

$$I_{G_3} = \frac{20^3 \times 100}{12} = 66.7 \times 10^3 mm^4$$

And distance between centre of gravity of rectangle – 3 and X-X axis

$$h_3 = 60.8 - 10 = 50.8mm$$

Moment of inertia of rectangle – 3 about X-X axis

$$= I_{G_3} + a_3{h_3}^2 = (66.7 \times 10^3) + [1200 \times 50.8^2] = 5228 \times 10^3 mm^4$$

Now moment of inertia of the whole action about an X-X axis

$$I_{XX} = (5786 \times 10^3) + (1836 \times 10^3) + (5228 \times 10^3) = 12850 \times 10^3 mm^4 \; (\boldsymbol{Ans})$$

POSSIBLE SHORT TYPE QUESTIONS WITH ANSWER

1. Define centre of gravity. (Possible)
Ans: Centre of gravity of a body may be defined as the point through which the whole weight of a body may be assumed to act.

2. Define centroid. (W – 2016 & S – 2019)
Ans: Centroid or Centre of area is defined as the point where the whole area of the figure is assumed to be concentrated.

3. State perpendicular axis theorem. **(S – 2019)**
Ans: It states, If I_{XX} and I_{YY} be the moments of inertia of a plane section about two perpendicular axis meeting at O, the moment of inertia I_{ZZ} about the axis Z-Z, perpendicular to the plane and passing through the intersection of X-X and Y-Y is given by:

$$I_{ZZ} = I_{XX} + I_{YY}$$

4. State parallel axis theorem. (Possible)
Ans: It states, If the moment of inertia of a plane area about an axis through its centre of gravity is denoted by IG, then moment of inertia of the area about any other axis AB, parallel to the first, and at a distance h from the centre of gravity is given by:

$$I_{AB} = I_G + ah^2$$

5. What is the distance of centroid of a semi-circular area from the base? (W – 2017 & S – 2018)
Ans:

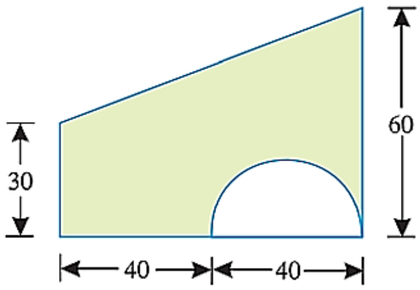
$$\bar{y} = \frac{4r}{3\pi} = 0.424\, r$$

POSSIBLE LONG TYPE QUESTIONS

1. State and prove perpendicular axis theorem. **(Possible)**

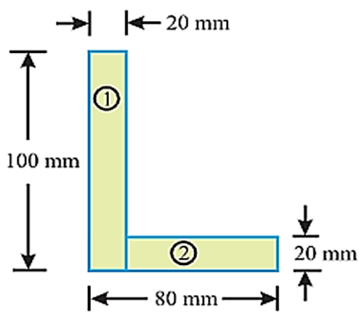
2. State and prove parallel axis theorem. **(S – 2019 Old)**

3. A semi-circular area is removed from a trapezium as shown in Figure (dimensions in mm) Determine the centroid of the remaining area (shown hatched). **(W – 2016)**



4. Find the position of the centroid of an angle section (L – section) having dimension of 150 mm × 200 mm × 20 mm. **(S – 2018)**

5. Find the position of centroid of a L section as shown in the figure below. **(S – 2019)**

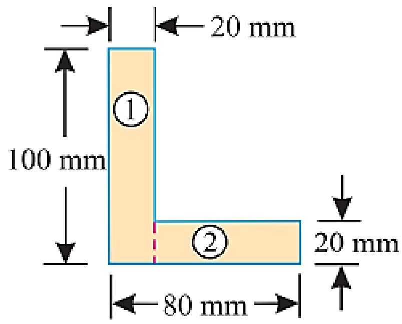


6. Find the moment of inertia of an I – section having following dimensions about centroidal X-X axis and Y-Y axis.

Top flange: 150 mm × 20 mm
Web: 150 mm × 20 mm
Bottom flange: 300 mm × 30 mm

(S – 2018)

7. Find the moment of inertia of the given section about the centroidal X-X and Y-Y axes. **(S – 2019 old)**



CHAPTER NO. – 05

SIMPLE MACHINES

LEARNING OBJECTIVES

5.1 Definition of simple machine, velocity ratio of simple and compound gear train, explain simple & compound lifting machine, define M.A, V.R. & Efficiency & State the relation between them, State Law of Machine, Reversibility of Machine, Self-Locking Machine.
5.2 Study of simple machines – simple axle & wheel, single purchase crab winch & double purchase crab winch, Worm & Worm Wheel, Screw Jack.
5.3 Types of hoisting machine like derricks etc, their use and working principle. No problems.

5.1 Definition of simple machine, velocity ratio of simple and compound gear train, explain simple & compound lifting machine, define M.A, V.R. & Efficiency & State the relation between them, State Law of Machine, Reversibility of Machine, Self-Locking Machine

Introduction:

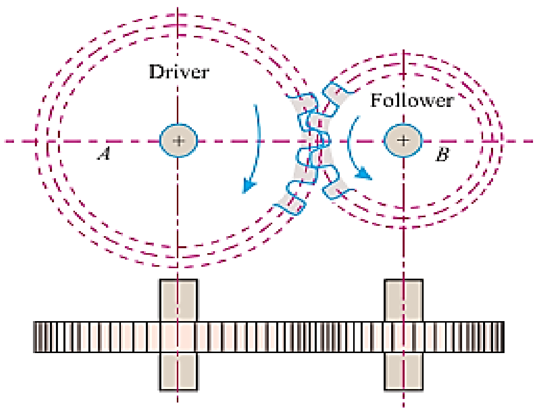
- Man invented various types of machines for his easy work. Sometimes, one person cannot do heavy work, but with the help of machine, the same work can be easily done.
- To change the tyre of a car, number of persons will be required. But with the help of a "Jack", the same work can be done by a single man. Therefore, jack acts as a machine by which the load of a car can be lifted by applying very small force as compared to the load of car.

Definition of simple machine:

- A simple machine may be defined as a device, which enables us to do some useful work at some point or to overcome some resistance, when an effort or force is applied to it, at some other convenient point.

Gear:

- Gear is a rotating machine element used for transmission of power from one shaft to another when it meshes with other one.
- On the periphery (circumference), numbers of teeth are provided for proper meshing between two mating gears.
- Between two mating gears, one is called as driver (pinion) and other one is called as follower (driven or spur wheel).



Velocity Ratio of a Simple Gear Drive:

- It is the ratio between the velocities of the driver and the follower or driven. Now consider a simple gear drive as shown in the above Figure.

- Let, N_1 = Speed of the driver,
  T_1 = No. of teeth on the driver,

d_1 = Diameter of the pitch circle of the driver,
N_2, T_2, d_2 = Corresponding values for the follower, and
p = Pitch of the two wheels.

- We know that the pitch of the driver

$$p = \frac{\pi d_1}{T_1} \ldots\ldots\ldots\ldots (i)$$

- Similarly, pitch of the follower

$$p = \frac{\pi d_2}{T_2} \ldots\ldots\ldots\ldots (ii)$$

- Since the pitch of both the wheels is the same, therefore equating (i) and (ii),

$$\frac{\pi d_1}{T_1} = \frac{\pi d_2}{T_2}$$
$$\Rightarrow \frac{d_1}{d_2} = \frac{T_1}{T_2}$$

- ∴ Velocity ratio,

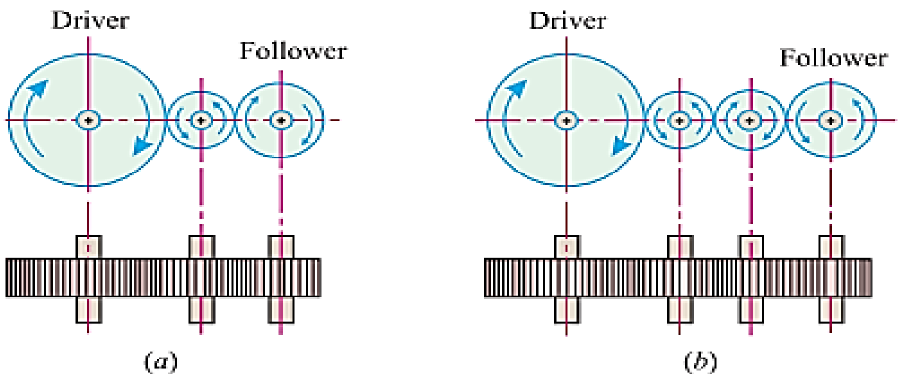
$$\boldsymbol{\frac{N_2}{N_1} = \frac{d_1}{d_2} = \frac{T_1}{T_2}}$$

Gear train:

- When two or more gears are made to mesh with each other, so as to operate as a single system, to transmit power from one shaft to another, then the combination is called gear train or train of wheels.
- Following are the two types of train of wheels depending upon the arrangement of wheels:
    1. Simple gear train
    2. Compound gear train

Simple gear train:

- When there is only one gear on each shaft, the train of wheels is called simple gear train.
- The gears in between driver and follower are called idle gears or intermediate gears.
- It may be noted that when the number of intermediate wheels is odd, then the driver and follower rotate in the same direction. But, if the number of intermediate wheels is even, then the driver and follower rotate in the opposite direction.



Velocity ratio of simple gear train:

- Now consider a simple train of wheels with one intermediate wheel as shown in Fig. (a).

- Let, N_1 = Speed of the driver
  T_1 = No. of teeth on the driver,
  N_2, T_2 = Corresponding values for the intermediate wheel, and
  N_3, T_3 = Corresponding values for the follower.

- Since the driver gears with the intermediate wheel, therefore

$$\frac{N_2}{N_1} = \frac{T_1}{T_2} \dots \dots \dots \dots \dots \dots \dots (i)$$

- Similarly, as the intermediate wheel gears with the follower, therefore

$$\frac{N_3}{N_2} = \frac{T_2}{T_3} \dots \dots \dots \dots \dots \dots \dots (ii)$$

- Multiplying equation (*ii*) by (*i*),

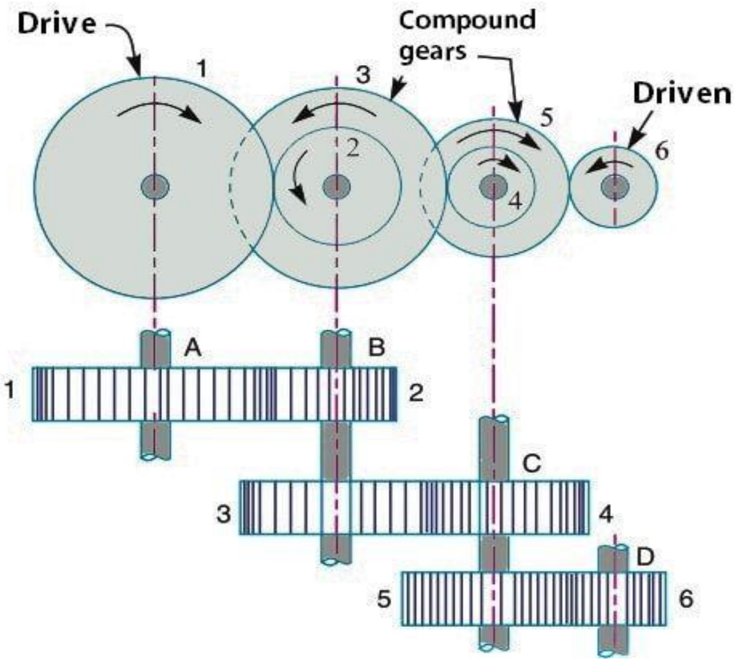
$$\frac{N_3}{N_2} \times \frac{N_2}{N_1} = \frac{T_2}{T_3} \times \frac{T_1}{T_2}$$

$$\Rightarrow \boldsymbol{\frac{N_3}{N_1} = \frac{T_1}{T_3}}$$

$$\therefore \quad \boldsymbol{\frac{Speed\ of\ the\ follower}{Speed\ of\ the\ driver} = \frac{No\ of\ teeth\ on\ the\ driver}{No\ of\ teeth\ on\ the\ follower}}$$

<u>Compound gear train</u>:

- When there are more than one gear on each shaft, the train of such wheels is called compound gear train.



<u>Velocity Ratio of Compound Gear Train</u>:

- Let, N_1 = Speed of the driver 1
  T_1 = No. of teeth on the driver 1,
  $N_2, N_3 \dots N_6$ = Speed of the respective wheels
  $T_2, T_3 \dots T_6$ = No. of teeth on the respective wheels.

- Since the wheel 1 gears with the wheel 2, therefore

$$\frac{N_2}{N_1} = \frac{T_1}{T_2} \dots \dots \dots \dots \dots (i)$$

- Similarly,

$$\frac{N_4}{N_3} = \frac{T_3}{T_4} \dots \dots \dots \dots .. (ii)$$

$$\frac{N_6}{N_5} = \frac{T_5}{T_6} \dots \dots \dots \dots . (iii)$$

- Multiplying equations (i), (ii) and (iii),

$$\frac{N_2}{N_1} \times \frac{N_4}{N_3} \times \frac{N_6}{N_5} = \frac{T_1}{T_2} \times \frac{T_3}{T_4} \times \frac{T_5}{T_6}$$

$$\Rightarrow \frac{N_6}{N_1} = \frac{T_1 \times T_3 \times T_5}{T_2 \times T_4 \times T_6} \qquad (\because N_2 = N_3 \; and \; N_4 = N_5)$$

$$\therefore \boldsymbol{\frac{N_6}{N_1} = \frac{Product\ of\ the\ teeth\ on\ the\ drivers}{Product\ of\ the\ teeth\ on\ the\ followers}}$$

Simple lifting machine:

- It is a device, which enables us to lift a heavy load ($\boldsymbol{W}$) by applying a comparatively smaller effort ($\boldsymbol{P}$).

Compound lifting machine:

- A compound lifting machine may be defined as a device, consisting of a number of simple machines, which enables us to do some useful work at a faster speed or with a much less effort as compared to a simple machine.

Effort:

- It may be defined as, the force which is applied so as to overcome the resistance or to lift the load. It is denoted by '$\boldsymbol{P}$'.

Load:

- The weight to be lifted or the resistive force to be overcome with the help of a machine is called as load ($\boldsymbol{W}$).

Mechanical Advantage:

- The mechanical advantage (briefly written as M.A.) is the ratio of weight lifted ($\boldsymbol{W}$) to the effort applied ($\boldsymbol{P}$) and is always expressed in pure number.
- Mathematically, mechanical advantage,

$$\boldsymbol{M.A. = \frac{W}{P}}$$

Velocity Ratio:

- The velocity ratio (briefly written as V.R.) is the ratio of distance moved by the effort ($\boldsymbol{y}$) to the distance moved by the load ($\boldsymbol{x}$) and is always expressed in pure number.
- Mathematically, velocity ratio,

$$\boldsymbol{V.R. = \frac{y}{x}}$$

Input of a Machine:

- The input of a machine is the work done on the machine. In a lifting machine, it is measured by the product of effort ($\boldsymbol{P}$) and the distance ($\boldsymbol{y}$) through which it has moved.
- Mathematically,

$$\boldsymbol{Input\ of\ a\ machine = P \times y}$$

Output of a Machine:

- The output of a machine is the actual work done by the machine. In a lifting machine, it is measured by the product of the weight lifted ($\boldsymbol{W}$) and the distance ($\boldsymbol{x}$) through which it has been lifted.
- Mathematically,

$$\boldsymbol{Output\ of\ a\ machine = W \times x}$$

Efficiency of a Machine:

- It is the ratio of output to the input of a machine and is generally expressed as a percentage.
- Mathematically, efficiency,

$$\boldsymbol{\eta = \frac{Output}{Input} \times 100}$$

Ideal Machine:

- If the efficiency of a machine is 100% i.e., if the output is equal to the input, the machine is called as a perfect or an ideal machine.
- In practical cases no machine is 100% efficient. All are real machines whose efficiencies are less than unity.

Relation between efficiency, mechanical advantage and velocity ratio of a lifting machine:

- It is an important relation of a lifting machine, which throws light on its mechanism. Now consider a lifting machine, whose efficiency is required to be found out.

- Let, W = Load lifted by the machine,
  P = Effort required to lift the load,
  y = Distance moved by the effort, in lifting the load, and
  x = Distance moved by the load.

- We know that,

$$M.A. = \frac{W}{P} \text{ and } V.R. = \frac{y}{x}$$

- We also know that input of a machine = Effort applied × Distance through which the effort has moved = $P \times y$.............. (*i*)
- output of a machine = Load lifted × Distance through which the load has been lifted = $W \times x$ ....(*ii*)
- ∴ Efficiency,

$$\boldsymbol{\eta = \frac{Output}{Input} = \frac{W \times x}{P \times y} = \frac{W/P}{y/x} = \frac{M.A.}{V.R.}}$$

Example – 1: ***In a certain weight lifting machine, a weight of 1 kN is lifted by an effort of 25 N. While the weight moves up by 100 mm, the point of application of effort moves by 8 m. Find mechanical advantage, velocity ratio and efficiency of the machine.***

Solution. Given: Weight (W) = 1 kN = 1000 N; Effort (P) = 25 N; Distance through which the weight is moved (x) = 100 mm = 0.1 m and distance through which effort is moved (y) = 8 m.

Mechanical advantage of the machine.

We know that mechanical advantage of the machine

$$M.A. = \frac{W}{P} = \frac{1000}{25} = \mathbf{40}$$

Velocity ratio of the machine

We know that velocity ratio of the machine

$$V.R. = \frac{y}{x} = \frac{8}{0.1} = \mathbf{80}$$

Efficiency of the machine

We also know that efficiency of the machine

$$\eta = \frac{M.A.}{V.R.} = \frac{40}{80} = 0.5 = \mathbf{50}\%$$

Reversibility of a Machine:

- When a machine is capable of doing some work in the reverse direction even on removal of effort, it is called as reversible machine and its action is known as reversibility of the machine.

Condition For the Reversibility of a Machine:

- Consider a reversible machine, whose condition for the reversibility is required to be found out.

- Let, W = Load lifted by the machine,
  P = Effort required to lift the load,
  y = Distance moved by the effort, and
  x = Distance moved by the load.

- We know that input of the machine = $P \times y$ and Output of the machine = $W \times x$
- We also know that machine friction = Input – Output = $(P \times y) - (W \times x)$
- A little consideration will show that in a reversible machine, the output of the machine should be more than the machine friction, when the effort (P) is zero. i.e.

$$(W \times x) > (P \times y) - (W \times x)$$

$$\Rightarrow 2(W \times x) > (P \times y)$$

$$\Rightarrow \frac{W \times x}{P \times y} > \frac{1}{2}$$

$$\Rightarrow \frac{W/P}{y/x} > \frac{1}{2}$$

$$\Rightarrow \frac{M.A.}{V.R.} > \frac{1}{2}$$

$$\Rightarrow \boldsymbol{\eta} > \frac{\mathbf{1}}{\mathbf{2}} = \mathbf{0.5} = \mathbf{50}\%$$

- Hence the condition for a machine, to be reversible, is that its **efficiency should be more than 50%.**

Self-Locking Machine:

- When a machine is not capable of doing some work in the reverse direction even on removal of effort, it is called as irreversible machine or non-reversible machine or self-locking machine.
- The condition for a machine to be non – reversible or self – locking is that its efficiency should not be more than 50% i.e., **η < 50%.**

Example – 2: ***A certain weight lifting machine of velocity ratio 30 can lift a load of 1500 N with the help of 125 N effort. Determine if the machine is reversible.***
Solution. Given: Velocity ratio (V.R.) = 30; Load (W) = 1500 N and effort (P) = 125 N.

We know that

$$M.A. = \frac{W}{P} = \frac{1500}{125} = 12$$

and efficiency

$$\eta = \frac{M.A.}{V.R.} = \frac{12}{30} = 0.4 = \mathbf{40}\%$$

Since efficiency of the machine is less than 50%, therefore the machine is non-reversible.

Example – 3: ***In a lifting machine, whose velocity ratio is 50, an effort of 100 N is required to lift a load of 4 kN. Is the machine reversible? If so, what effort should be applied, so that the machine is at the point of reversing?***
Solution. Given: Velocity ratio (V.R.) = 50; Effort (P) = 100 N and load (W) = 4 kN = 4000 N.
Reversibility of the machine
We know that

$$M.A. = \frac{W}{P} = \frac{4000}{100} = 40$$

and efficiency

$$\eta = \frac{M.A.}{V.R.} = \frac{40}{50} = 0.8 = \mathbf{80}\%$$

Since efficiency of the machine is **more than 50%,** therefore the machine is reversible.

Effort to be applied
A little consideration will show that the machine will be at the point of reversing, when its efficiency is 50% or 0.5.

Let P_1 = Effort required to lift a load of 4000 N when the machine is at the point of reversing.

We know that

$$M.A. = \frac{W}{P_1} = \frac{4000}{P_1}$$

and efficiency

$$0.5 = \frac{M.A.}{V.R.} = \frac{4000/P_1}{50} = \frac{80}{P_1}$$

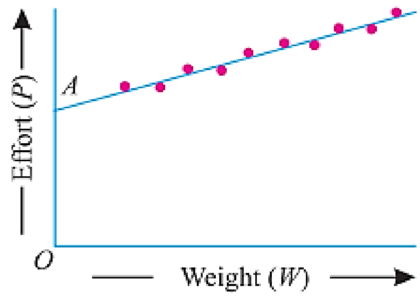
$$\Rightarrow P_1 = \frac{80}{0.5} = \mathbf{160\ N}$$

Law of Machine:

- The equation which gives the relation between load lifted and effort applied in the form of a slope and intercept of a straight line is called as Law of a machine.
- Mathematically, the law of a lifting machine is given by the relation:

$$\boldsymbol{P = mW + C}$$

- Where, P = Effort applied to lift the load,
  m = A constant (called coefficient of friction) which is equal to the slope of the line AB,
  W = Load lifted, and
  C = another constant, which represents the machine friction, (i.e. OA)
-



Maximum Mechanical Advantage of a Lifting Machine:

- We know that mechanical advantage of a lifting machine,

$$M.A. = \frac{W}{P}$$

- For maximum mechanical advantage, substituting the value of $P = mW + C$ in the above equation,

$$\boldsymbol{Max.M.A. = \frac{W}{mW + C} = \frac{1}{m + \frac{C}{W}} = \frac{1}{m}}$$

$$\ldots \ldots \left(Neglecting\ \frac{C}{W}\right)$$

Maximum Efficiency of a Lifting Machine:

- We know that efficiency of a lifting machine,

$$\eta = \frac{M.A.}{V.R.}$$

- A little consideration will show that the efficiency will be maximum, when the mechanical advantage will be maximum.

$$\boldsymbol{Max.\eta = \frac{Max.\ M.A.}{V.R.} = \frac{1}{m \times V.R.}}$$

Example – 4: ***What load can be lifted by an effort of 120 N, if the velocity ratio is 18 and efficiency of the machine at this load is 60%? Determine the law of the machine, if it is observed that an effort of 200 N is required to lift a load of 2600 N and find the effort required to run the machine at a load of 3.5 kN.***
Solution. Given: Effort (P) = 120 N; Velocity ratio (V.R.) = 18 and efficiency (η) = 60% = 0.6.
Load lifted by the machine.
Let W = Load lifted by the machine.
We know that

$$M.A. = \frac{W}{P} = \frac{W}{120}$$

and efficiency,

$$0.6 = \frac{M.A.}{V.R.} = \frac{W/120}{18} = \frac{W}{2160}$$

$$\therefore\ W = 0.6 \times 2160 = 1296\,N$$

Law of the machine
In the second case, P = 200 N and W = 2600 N

Substituting the two values of P and W in the law of the machine, i.e., $P = mW + C$,

$$120 = m \times 1296 + C \ldots\ldots\ldots\ldots (i)$$

$$200 = m \times 2600 + C \ldots\ldots\ldots\ldots (ii)$$

Subtracting equation (*i*) from (*ii*),

$$80 = 1304\,m$$

$$\Rightarrow m = \frac{80}{1304} = 0.06$$

and now substituting the value of *m* in equation (*ii*)

$$200 = (0.06 \times 2600) + C = 156 + C$$

$$\Rightarrow C = 200 - 156 = 44$$

Now substituting the value of m = 0.06 and C = 44 in the law of the machine,

$$P = 0.06\,W + 44$$

Effort required to run the machine at a load of 3.5 kN.
Substituting the value of W = 3.5 kN or 3500 N in the law of machine,

$$P = (0.06 \times 3500) + 44 = 254\,N$$

Example – 5: ***In a lifting machine, an effort of 40 N raised a load of 1 kN. If efficiency of the machine is 0.5, what is its velocity ratio? If on this machine, an effort of 74 N raised a load of 2 kN, what is now the efficiency? What will be the effort required to raise a load of 5 kN?***
Solution. Given: When Effort (P) = 40 N; Load (W) = 1 kN = 1000 N; Efficiency (η) = 0.5; When effort (P) = 74 N and load (W) = 2 kN = 2000 N.
Velocity ratio when efficiency is 0.5.
We know that

$$M.A. = \frac{W}{P} = \frac{1000}{40} = 25$$

and efficiency

$$0.5 = \frac{M.A.}{V.R.} = \frac{25}{V.R.}$$

$$\Rightarrow V.R. = \frac{25}{0.5} = 50$$

Efficiency when P is 74 N and W is 2000 N

We know that

$$M.A. = \frac{W}{P} = \frac{2000}{74} = 27$$

and efficiency

$$\eta = \frac{M.A.}{V.R.} = \frac{27}{50} = 0.54 = 54\%$$

Effort required to raise a load of 5 kN or 5000 N

Substituting the two values of P and W in the law of the machine, i.e., $P = mW + C$

$$40 = m \times 1000 + C \ldots\ldots\ldots\ldots (i)$$

$$74 = m \times 2000 + C \ldots\ldots\ldots\ldots (ii)$$

Subtracting equation (i) from (ii),

$$34 = 1000\,m$$

$$\Rightarrow m = \frac{34}{1000} = 0.034$$

and now substituting this value of m in equation (i),

$$40 = (0.034 \times 1000) + C = 34 + C$$

$$\Rightarrow C = 40 - 34 = 6$$

Substituting these values of m = 0.034 and C = 6 in the law of machine,

$$P = 0.034\,W + 6 \ldots\ldots\ldots\ldots (iii)$$

Effort required to raise a load of 5000 N,

$$P = (0.034 \times 5000) + 6 = 176\,N$$

N.B: **Friction in a machine**

$$* \boldsymbol{F_{effort} = P - \frac{W}{V.R.}}$$

$$* \boldsymbol{F_{load} = (P \times V.R.) - W}$$

5.2 Study of simple machines – simple axle & wheel, single purchase crab winch & double purchase crab winch, Worm & Worm Wheel, Screw Jack

Study Of Simple Machines:

Simple Wheel and Axle:

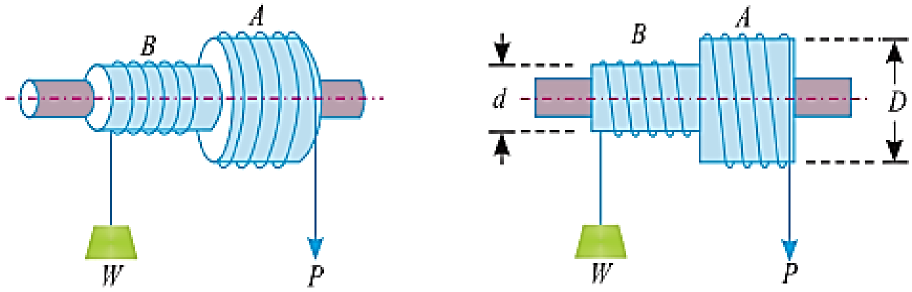


Fig. Simple wheel and axle

- The above figure shows a simple wheel and axle, in which the wheel A and axle B are keyed to the same shaft. The shaft is mounted on ball bearings, order to reduce the frictional resistance to a minimum. A string is wound round the axle B, which carries the load to be lifted. A second string is wound round the wheel A in the opposite direction to that of the string on B.

- Let, D = Diameter of effort wheel,
  d = Diameter of the load axle,
  W = Load lifted, and
  P = Effort applied to lift the load.
- One end of the string is fixed to the wheel, while the other is free and the effort is applied to this end. Since the two strings are wound in opposite directions, therefore a downward motion of the effort (P) will raise the load (W).
- Since the wheel as well as the axle is keyed to the same shaft, therefore when the wheel rotates through one revolution, the axle will also rotate through one revolution.
- We know that displacement of the effort in one revolution of effort wheel A $= \pi D$……………….. (i)
  And displacement of the load in one revolution $= \pi d$……………….. (ii)

$$\therefore \boldsymbol{V.R.} = \frac{\boldsymbol{Distance\ moved\ by\ the\ effort}}{\boldsymbol{Distance\ moved\ by\ the\ load}} = \frac{\boldsymbol{\pi D}}{\boldsymbol{\pi d}} = \frac{\boldsymbol{D}}{\boldsymbol{d}}$$

$$M.A. = \frac{Load\ lifted}{Effort\ applied} = \frac{W}{P}$$

$$\eta = \frac{M.A.}{V.R.}$$

Example – 6: ***A simple wheel and axle has wheel and axle of diameters of 300 mm and 30 mm respectively. What is the efficiency of the machine, if it can lift a load of 900 N by an effort of 100 N?***

Solution. Given: Diameter of wheel (D) = 300 mm; Diameter of axle (d) = 30 mm; Load lifted by the machine (W) = 900 N and effort applied to lift the load (P) = 100 N

We know that velocity ratio of the simple wheel and axle,

$$V.R. = \frac{D}{d} = \frac{300}{30} = 10$$

and mechanical advantage

$$M.A. = \frac{W}{P} = \frac{900}{100} = 9$$

$\therefore$ Efficiency,

$$\eta = \frac{M.A.}{V.R.} = \frac{9}{10} = 0.9 = 90\%$$

Example 5.8: A drum weighing 60 N and holding 420N of water is to be raised from a well by means of wheel and axle. The axle is 100 mm diameter and the wheel is 500 mm diameter. If a force of 120 N has to be applied to the wheel, find (i) mechanical advantage, (ii) velocity ratio and (iii) efficiency of the machine.

Solution. Given: Total load to be lifted (W) = 60 + 420 = 480 N; Diameter of the load axle (d) = 100 mm; Diameter of effort wheel (D) = 500 mm and effort (P) = 120 N.

Mechanical advantage

We know that mechanical advantage

$$M.A. = \frac{W}{P} = \frac{480}{120} = 4$$

Velocity ratio

We know that velocity ratio

$$V.R. = \frac{D}{d} = \frac{500}{100} = 5$$

Efficiency of the machine

We also know that efficiency of the machine,

$$\eta = \frac{M.A.}{V.R.} = \frac{4}{5} = 0.8 = 80\%$$

Single Purchase Crab Winch:

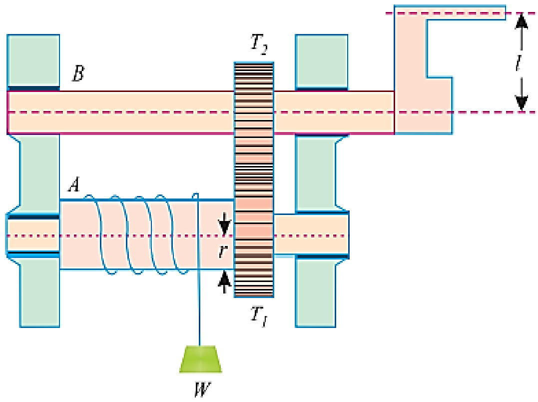


Fig. Single purchase crab winch

- In single purchase crab winch, a rope is fixed to the drum and is wound a few turns round it. The free end of the rope carries the load W. A toothed wheel A is rigidly mounted on the load drum. Another toothed wheel B, called pinion, is geared with the toothed wheel A as shown in the Figure. The effort is applied at the end of the handle to rotate it.

- Let, T_1 = No. of teeth on the main gear (or spur wheel) A,
  T_2 = No. of teeth on the pinion B,
  l = Length of the handle,
  r = Radius of the load drum.
  W = Load lifted, and
  P = Effort applied to lift the load.

- $We\ know\ that\ distance\ moved\ by\ the\ effort\ in\ one\ revolution\ of\ the\ handle = 2\pi l$

  $\therefore\ No.of\ revolutions\ made\ by\ the\ pinion\ B\ = 1$

  $$And\ no.of\ revolutions\ made\ by\ the\ wheel\ A = \frac{T_2}{T_1}$$

  $$\therefore\ No.of\ revolutions\ made\ by\ the\ load\ drum = \frac{T_2}{T_1}$$

  $$And\ distance\ moved\ by\ the\ load = 2\pi r \times \frac{T_2}{T_1}$$

$$\therefore \boldsymbol{V.R.} = \frac{\boldsymbol{Distance\ moved\ by\ the\ effort}}{\boldsymbol{Distance\ moved\ by\ the\ load}} = \frac{\boldsymbol{2\pi l}}{\boldsymbol{2\pi r \times \frac{T_2}{T_1}}} = \frac{\boldsymbol{l}}{\boldsymbol{r}} \times \frac{\boldsymbol{T_1}}{\boldsymbol{T_2}}$$

$$M.A. = \frac{Load\ lifted}{Effort\ applied} = \frac{W}{P}$$

$$\eta = \frac{M.A.}{V.R.}$$

Double Purchase Crab Winch:

- A double purchase crab winch is an improved form of a single purchase crab winch, in which the velocity ratio is intensified with the help of one more spur wheel and a pinion. In a double purchase crab winch, there are two spur wheels of teeth T_1 and T_2 and T_3 as well as two pinions of teeth T_2 and T_4.

- The arrangement of spur wheels and pinions are such that the spur wheel with T_1 gears with the pinion of teeth T_2. Similarly, the spur wheel with teeth T_3 gears with the pinion of the teeth T_4, the effort is applied to a handle as shown in the figure below.

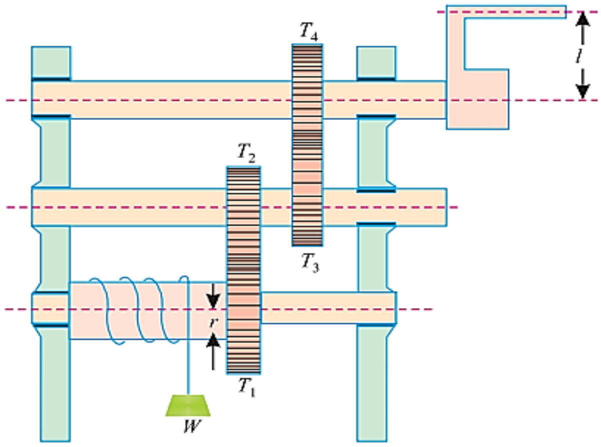


Fig. Double purchase crab winch

- Let, T_1 and T_3 = No. of teeth of spur wheels,
  T_2 and T_4 = No. of teeth of the pinions,
  l = Length of the handle,
  r = Radius of the load drum,
  W = Load lifted, and
  P = Effort applied to lift the load, at the end of the handle.
- $We\ know\ that\ distance\ moved\ by\ the\ effort\ in\ one\ revolution\ of\ the\ handle = 2\pi l$

$\therefore\ No.of\ revolutions\ made\ by\ the\ pinion\ 4 = 1$

$and\ no.of\ revolutions\ made\ by\ the\ wheel\ 3 = \frac{T_4}{T_3}$

$\therefore\ No.of\ revolutions\ made\ by\ the\ pinion\ 2 = \frac{T_4}{T_3}$

$and\ no.of\ revolutions\ made\ by\ the\ wheel\ 1 = \frac{T_2}{T_1} \times \frac{T_4}{T_3}$

$\therefore\ Distance\ moved\ by\ the\ load = 2\pi r \times \frac{T_2}{T_1} \times \frac{T_4}{T_3}$

$$\therefore \boldsymbol{V.R.} = \frac{\boldsymbol{Distance\ moved\ by\ the\ effort}}{\boldsymbol{Distance\ moved\ by\ the\ load}}$$

$$\Rightarrow \boldsymbol{V.R.} = \frac{\boldsymbol{2\pi l}}{\boldsymbol{2\pi r \times \frac{T_2}{T_1} \times \frac{T_4}{T_3}}} = \frac{\boldsymbol{l}}{\boldsymbol{r}}\left(\frac{\boldsymbol{T_1}}{\boldsymbol{T_2}} \times \frac{\boldsymbol{T_3}}{\boldsymbol{T_4}}\right)$$

$$M.A. = \frac{Load\ lifted}{Effort\ applied} = \frac{W}{P}$$

$$\eta = \frac{M.A.}{V.R.}$$

Worm And Worm Wheel:

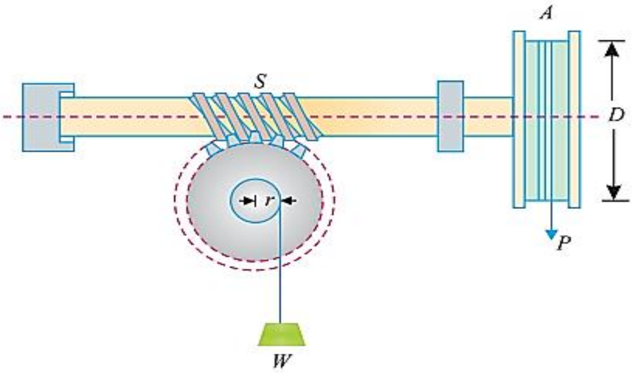


Fig. Worm and worm wheel

- It consists of a square threaded screw, S (known as worm) and a toothed wheel (known as worm wheel) geared with each other, as shown in the above figure. A wheel '**A**' is attached to the worm, over which passes a rope as shown in the figure. Sometimes, a handle is also fixed to the worm (instead of the wheel). A load drum is securely mounted on the worm wheel.

- Let, D = Diameter of the effort wheel,
  r = Radius of the load drum
  W = Load lifted,
  P = Effort applied to lift the load, and
  T = No. of teeth on the worm wheel.

- We know that distance moved by the effort in one revolution of the wheel (or handle) $= \pi D$

- If the worm is single-threaded (i.e., for one revolution of the wheel A, the screw S pushes the worm wheel through one teeth), then the load drum will move through

$$= \frac{1}{T}$$

and distance, through which the load will move

$$= \frac{2\pi r}{T}$$

$$\therefore \boldsymbol{V.R. = \frac{Distance\ moved\ by\ the\ effort}{Distance\ moved\ by\ the\ load} = \frac{\pi D}{\frac{2\pi r}{T}} = \frac{DT}{2r}}$$

$$M.A. = \frac{Load\ lifted}{Effort\ applied} = \frac{W}{P}$$

$$\eta = \frac{M.A.}{V.R.}$$

Notes:

- If the worm is double-threaded i.e., for one revolution of wheel A, the screw S pushes the worm wheel through two teeth, then

$$\boldsymbol{V.R. = \frac{DT}{2 \times 2r} = \frac{DT}{4r}}$$

- In general, if the worm is n threaded, then

$$\boldsymbol{V.R. = \frac{DT}{2nr}}$$

Simple Screw Jack:

- It consists of a screw, fitted in a nut, which forms the body of the jack. The principle, on which a screw jack works, is similar to that of an inclined plane.

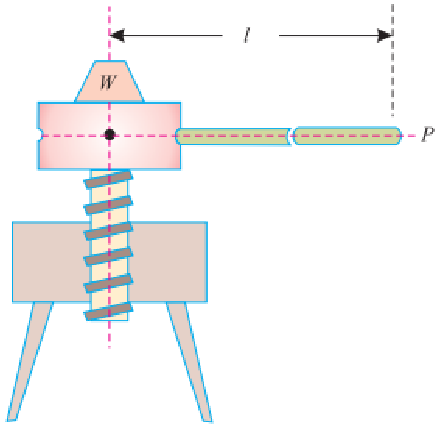


Fig. Simple screw Jack

- The figure shows a simple screw jack, which is rotated by the application of an effort at the end of the lever, for lifting the load. Now consider a single threaded simple screw jack.
- Let, l = Length of the effort arm,
  p = Pitch of the screw,
  W = Load lifted, and
  P = Effort applied to lift the load at the end of the lever

- We know that distance moved by the effort in one revolution of screw $= 2\pi l$

- Distance moved by the load = p

$$\therefore \boldsymbol{V.R.} = \frac{\textbf{Distance moved by the effort}}{\textbf{Distance moved by the load}} = \frac{\boldsymbol{2\pi l}}{\boldsymbol{p}}$$

$$M.A. = \frac{Load\ lifted}{Effort\ applied} = \frac{W}{P}$$

$$\eta = \frac{M.A.}{V.R.}$$

5.3 Types of hoisting machine like derricks etc, their use and working principle. No problems

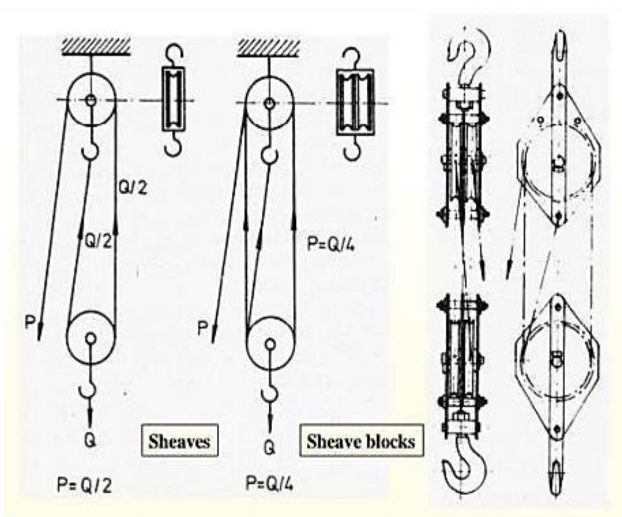
Hoisting Machine:

The hoisting is the lifting of the material against gravity.

Common equipment for hoisting:

- Pulley and sheave block
- Chain hoists
- Cranes
- Winches

Pulley and sheave block:



- A pulley is a wheel on an axle or shaft that is designed to support movement and change of direction of a cable or belt along its circumference. Pulleys are used in a variety of ways to lift loads, apply forces, and to transmit power. In nautical contexts, the assembly of wheel, axle, and supporting shell is referred to as a ‘block’.
- A pulley may also be called a sheave or drum and may have a groove between two flanges around its circumference. The drive element of a pulley system can be a rope, cable, belt, or chain that runs over the pulley inside the groove.
- The pulley and sheave blocks are suitable for lifting rough surface and heavy loads. For this purpose, the chains and wire ropes are used.

Chain Hoists:

- The chain hoists are the popular mechanism for lifting loads up to tones.
- The system consists of two sets of chains, namely the hand and load chain.
- The hand chains are particularly useful for the isolated location, where an electric motor or other types of mechanical equipment are not available.
- The pull applied through the hand chain is transmitted to the load chain with a multiplication factor of over 20.
- The load to be lifted is held by a load hook while another hook (called support hook) at the top, support the mechanism.

Cranes:

A crane is a type of machine, generally equipped with a Hoist rope or chain, and sheaves, that can be used both to lift and lower the materials and to move them horizontally. It is mainly used for lifting heavy Things and transporting them to other places.

Types of cranes:

- Mobile crane
- Truck mounted crane
- Tower crane
- Overhead crane
- Derrick crane

Mobile Crane:

- A mobile crane is a cable-controlled crane Mounted on crawlers or ribbed tired carries or a hydraulic powered crane with a telescopic Boom mounted on truck type carriers or as self-propelled Models.

Truck Mounted Crane:

- Truck mounted crane is a self-propelled loading unloading Machine mounted on a truck Body, with a working section consisting of a Rotating cantilevered boom.
- These cranes are supported (outriggers) while lifting cargo, in order to increase their stability.

Tower Crane:

- These are the crane of swing job type and are mounted on high steel towers.
- The height of tower maybe 25 to 30 m and these cranes are found to be suitable in the construction of tall buildings in congested areas.
- The ground area required for such cranes is very small.

Overhead Crane:

- An overhead crane, commonly called a bridge Crane, is a type of crane found in industrial Environments. An overhead crane consists of Parallel runways with a travelling bridge spanning the gap.
- A hoist, the lifting component of a crane, travels along a bridge.

Derrick Cranes:

- The derrick cranes are of two types, namely
    1. Guy derrick
    2. Stiff leg derrick
- The guy derrick consists of a vertical mast. This mast is supported by the number of guys and can revolve through 360^0.
- While revolving, the radius of revolution should be such that the revolving structure is not obstructed by the guy wires.
- This derrick can be constructed up to 200 tonnes capacity.
- In stiff leg type derricks, the guy wires are replaced by trussed structure.
- The power is supplied by a diesel engine or by an electric motor.

POSSIBLE SHORT TYPE QUESTIONS WITH ANSWER:

1. Define simple lifting machine. (W – 2016)
Ans: It is a device, which enables us to lift a heavy load ($\boldsymbol{W}$) by applying a comparatively smaller effort ($\boldsymbol{P}$). A simple machine may be defined as a device, which enables us to do some useful work at some point or to overcome some resistance, when an effort or force is applied to it, at some other convenient point.

2. Define mechanical advantage of a machine. (Possible)

- Ans: The mechanical advantage (briefly written as M.A.) is the ratio of weight lifted ($\boldsymbol{W}$) to the effort applied ($\boldsymbol{P}$) and is always expressed in pure number.
- Mathematically, mechanical advantage,

$$\boldsymbol{M.A.} = \frac{\boldsymbol{W}}{\boldsymbol{P}}$$

3. Define velocity ratio of a machine. (Possible)

- Ans: The velocity ratio (briefly written as V.R.) is the ratio of distance moved by the effort ($\boldsymbol{y}$) to the distance moved by the load ($\boldsymbol{x}$) and is always expressed in pure number.
- Mathematically, velocity ratio,

$$\boldsymbol{V.R.} = \frac{\boldsymbol{y}}{\boldsymbol{x}}$$

4. What is reversible machine. (S – 2018)
Ans: When a machine is capable of doing some work in the reverse direction even on removal of effort, it is called as reversible machine and its action is known as reversibility of the machine.

5. What is the condition of reversibility of a lifting machine? (W – 2017)
Ans: The condition for a machine, to be reversible, is that its **efficiency should be more than 50%.**

$$\boldsymbol{i.e.\ \eta > 50\%}$$

6. Define self-locking machine. (Possible)
Ans: When a machine is not capable of doing some work in the reverse direction even on removal of effort, it is called as irreversible machine or non-reversible machine or self-locking machine.

7. Write the expression for velocity ratio of a simple wheel and axle. (S – 2018)
Ans:

$$V.R. = \frac{Distance\ moved\ by\ the\ effort}{Distance\ moved\ by\ the\ load} = \frac{\pi D}{\pi d} = \frac{D}{d}$$

8. What is law of machine? (S – 2019)

- Ans: The equation which gives the relation between load lifted and effort applied in the form of a slope and intercept of a straight line is called as Law of a machine.
- Mathematically, the law of a lifting machine is given by the relation:

$$\boldsymbol{P = mW + C}$$

9. State the relation between M.A., V.R. and efficiency of a simple lifting machine. (S – 2019)
Ans:

$$\eta = \frac{Output}{Input} = \frac{W \times x}{P \times y} = \frac{W/P}{y/x} = \frac{M.A.}{V.R.}$$

POSSIBLE LONG TYPE QUESTIONS:

1. Derive the velocity ratio of a compound gear train. **(W – 2016, 2017 & S – 2018, 2019)**

2. Define mechanical advantage, velocity ratio and efficiency of a lifting machine and derive their relationship. **(S – 2018)**

3. Derive the condition for reversibility of a simple lifting machine. **(S – 2019)**

4. In a weight lifting machine, an effort of 40 N can lift a load of 1000 N and an effort of 55 N can lift a load of 1500 N. Find the law of the machine. Also find maximum mechanical advantage and maximum efficiency of the machine. Take velocity ratio of the machine as 48. (**W – 2017)**

[Ans. P = 0.03 W + 10; 33.3; 69.4%]

5. In a certain weight lifting machine, an effort of 15 N can lift a load of 300 N and an effort of 20 N can lift a load of 500 N. Find the law of the machine. Also find the effort required to lift a load of 880 N.
(S – 2018, 2019)

[Ans. P = 0.025 W + 7.5; 29.5 N]

6. In a simple wheel and axle, radii of effort wheel and axle is 240 mm and 40 mm respectively. Find the efficiency of the machine, if a load of 600 N can be lifted by an effort of 120 N. **(Possible)**

[Ans. 83.3%]

CHAPTER NO. – 06

DYNAMICS

LEARNING OBJECTIVES

6.1 Kinematics & Kinetics, Principles of Dynamics, Newton's Laws of Motion, Motion of Particle acted upon by a constant force, Equations of motion, De-Alembert's Principle.

6.2 Work, Power, Energy & its Engineering Applications, Kinetic & Potential energy & its application.

6.3 Momentum & impulse, conservation of energy & linear momentum, collision of elastic bodies, and Coefficient of Restitution.

6.1 Kinematics & Kinetics, Principles of Dynamics, Newton's Laws of Motion, Motion of Particle acted upon by a constant force, Equations of motion, De-Alembert's Principle

Dynamics:

- It is the branch of engineering mechanics which deals with the study of forces and their effects while acting upon the bodies in motion.
- It may be further sub divided into two branches:
  i) Kinetics
  ii) Kinematics

Kinetics: It is that branch of Dynamics, which deals with motion of bodies without considering the forces causing motion.

Kinematics: It is that branch of Dynamics, which deals with motion of bodies and the forces causing the motion. It predicts the type of motion by a given force system.

Principles of Dynamics:

Newton's Laws of Motion:

Newton's First Law of Motion: It states ***"Everybody continues in its state of rest or of uniform motion, in a straight line, unless it is acted upon by some external force."*** It is also called the law of inertia.

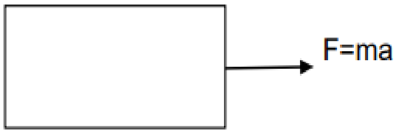
Newton's Second Law of Motion: It states, ***"The rate of change of momentum is directly proportional to the impressed force and takes place, in the same direction in which the force acts."*** This law enables us to measure a force, and establishes the fundamental equation of dynamics.
Mathematically,

$$\mathbf{F = ma = Mass \times Acceleration.}$$

In **S.I.** system of units, the **unit of force** is called ***newton*** briefly written as ***N***.
A Newton may be defined as the force while acting upon a mass of 1 kg, produces an acceleration of 1 m/s^2 in the direction of which it acts.

Newton's Third Law of Motion: It states, ***"To every action, there is always an equal and opposite reaction."***
If a body exerts a force P on another body, the second body will exert the same force P on the first body in the opposite direction. The force exerted by first body is called action whereas the force exerted by the second body is called reaction.

Motion of Particle Acted Upon by A Constant Force:



The motion of a particle acted upon by a constant force is governed by Newton's second law of motion.
If a constant force, $\boldsymbol{F= ma}$ is applied on a particle of mass 'm', then the particle will move with a uniform acceleration 'a'.

Equations of motion:

Let, $\boldsymbol{u}$ = initial velocity of the body
$\boldsymbol{v}$ = final velocity of the body
$\boldsymbol{s}$ = distance travelled by the body in motion
$\boldsymbol{a}$ = acceleration of the body
$\boldsymbol{t}$ = time taken by the body
$\boldsymbol{g}$ = acceleration due to gravity

For Linear Motion:

- $\boldsymbol{v = u \pm at}$
- $\boldsymbol{v^2 = u^2 \pm 2as}$
- $\boldsymbol{s = ut \pm \frac{1}{2}at^2}$

$$(where, +a = for\ acceleration\ and\ -a = for\ retardation)$$

For Motion Under Gravity:

- $\boldsymbol{v = u \pm gt}$
- $\boldsymbol{v^2 = u^2 \pm 2gh}$
- $\boldsymbol{s = ut \pm \frac{1}{2}gt^2}$

$$(where, +g = for\ downward\ motion\ and\ -a = for\ upward\ motion)$$

De-Alembert's Principle: It states, ***"If a rigid body is acted upon by a system of forces, this system may be reduced to a single resultant force whose magnitude, direction and the line of action may be found out by the methods of graphic statics."***

We know that,

$$\boldsymbol{P = ma} \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (i)$$

Where, $\boldsymbol{m}$ = mass of the body, and
$\boldsymbol{a}$ = Acceleration of the body.

The equation (i) may also be written as:

$$\boldsymbol{P - ma = 0} \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (ii)$$

It may be noted that equation (i) is the equation of dynamics whereas the equation (ii) is the equation of statics. The equation (ii) is also known as the equation of dynamic equilibrium under the action of the real force P. This principle is known as D' Alembert's principle.

Example – 1: ***A body of mass 7.5 kg is moving with a velocity of 1.2 m/s. If a force of 15 N is applied on the body, determine its velocity after 2 s.***

Given: Mass of body = 7.5kg; Velocity (u) =1.2m/s, Force (F)=15N and time (t) = 2 sec

We know that the acceleration of the body

$$a = \frac{F}{m} = \frac{15}{7.5} = 2\, m/s^2$$

Velocity of the body after 2 seconds

$$v = u + at = 1.2 + (2 \times 2) = 5.2\, m/s \quad (\boldsymbol{Ans})$$

Example – 2: ***A constant retarding force of 50 N is applied to a body of mass 20 kg moving initially with a velocity of 15 m/s. How long the body will take to stop?***

Given: Retarding force (F) = 50N; Mass of the body (m) = 20kg; initial velocity (u) = 15m/s and final velocity (v) = 0 (because it stops)

Let t = Time taken by the body to stop

We know that the retardation of the body

$$a = \frac{F}{m} = \frac{50}{20} = 2.5\, m/s^2$$

And final velocity of the body,

$$0 = u + at = 15 - 2.5t$$

$$\Rightarrow t = \frac{15}{2.5} = 6\ s\ (\boldsymbol{Ans})$$

Example – 3: ***A multiple unit electric train has 800 tonnes mass. The resistance to motion is 100 N per tonne of the train mass. If the electric motors can provide 200 kN tractive force, how long does it take to accelerate the train to a speed of 90 km/hr from rest.***

Solution. Given: Mass of electric train (m) = 800 t; Resistance to motion = 100 N/t = 100 × 800 = 80000 N = 80 kN; Tractive force = 200 kN; Final velocity (v) = 90 km/hr = 25 m/s and initial velocity (u) = 0 (because it starts from rest)

Let t = Time taken by the electric train

We know that net force available to move the train,

$$F = Tractive\ force - Resistance\ to\ motion$$
$$= 200 - 80 = 120\ kN$$

And acceleration of the train

$$a = \frac{F}{m} = \frac{120}{800} = 0.15\, m/s^2$$

We also know that the final velocity of the body (v)

$$25 = u + at = 0 + 0.15t$$

$$\Rightarrow t = \frac{25}{0.15} = 166.7\ s\ (\boldsymbol{Ans})$$

Example – 4: ***A stone is thrown vertically upwards, from the ground, with a velocity 49 m/s. After 2 seconds, another stone is thrown vertically upwards from the same place. If both the stone strike the ground at the same time, find the velocity, with which the second stone was thrown upwards*.**

Solution. First of all, consider the upwards motion of the first stone. In this case, initial velocity (u) = – 49 m/s (Minus sign due to upward motion) and final velocity (v) = 0 (because stone is at maximum height)

Let t = Time taken by the stone to reach the maximum height

We know that the final velocity of the stone (v)

$$0 = u + gt = -49 + 9.8\, t$$

$$\ldots\ldots\ldots (Minus\ sign\ due\ to\ upwards\ motion)$$

$$\Rightarrow t = \frac{49}{9.8} = 5\ s$$

It means that the tone will take 5 s to reach the maximum height and another 5 s to come back to the ground

$\therefore$ Total time of flight $= 5 + 5 = 10\,s$

Now consider the motion of second stone. We know that the time taken by the second tone for going upward and coming back to the earth $= 10 - 2 = 8s$

And time taken by the second tone to reach maximum height

$$= \frac{8}{2} = 4s$$

Now consider the upward motion of the second tone. We know that final velocity of the stone (v)

$$0 = u + gt = -u + 9.8 \times 4 = -u + 39.2$$

$$\Rightarrow u = 39.2\,m/s\ \ (\boldsymbol{Ans})$$

Example – 5: ***A stone is dropped from the top of a tower 50 m high. At the same time, another stone is thrown upwards from the foot of the tower with a velocity of 25 m/s. When and where the two stones cross each other?***

Solution. Given: Height of the tower = 50m

Time taken by the stone to cross each other

First of all, consider downward motion of the first stone. In this case, initial velocity (u) =0 (Because it is dropped)

Let t = Time taken for the stone to cross each other

We know that the distance traversed by the stone

$$s = ut + \frac{1}{2}gt^2 = 0 + \frac{1}{2}gt^2 = 0.5gt^2 \dots\dots\dots\dots\dots\dots (i)$$

Now consider upward motion of the second stone. In this case, initial velocity = – 25 m/s (Minus sign due to upward) and distance traversed = 50 – s,

We know that the distance traversed

$$50 - s = ut + \frac{1}{2}gt^2 = -25t + 0.5gt^2 \dots\dots\dots\dots\dots\dots (ii)$$

Adding equations (*i*) and (*ii*),

$$50 = 25t \qquad or \qquad t = \frac{50}{25} = 2\,s\ (\boldsymbol{Ans})$$

Point where the stone cross each other

Substituting the value of t = 2 in equation (*i*),

$$s = 0.5 \times 9.8\,(2)^2 = 19.6\,m\ (\boldsymbol{Ans})$$

<u>Recoil of gun</u>: According to Newton's Third Law of Motion, when a bullet is fired from a gun, the opposite reaction of the bullet is known as the recoil of gun.

Let, ***M*** = Mass of the gun,
V = Velocity of the gun with which it recoils,
m = mass of the bullet, and
v = Velocity of the bullet after explosion.

$\therefore$ Momentum of the bullet after explosion = *mv* …………………. (*i*)
And momentum of the gun = *MV* …………………… (*ii*)

Equating the equations (*i*) and (*ii*),

MV = mv

Note. This relation is popularly known as Law of Conservation of Momentum.

Example – 6: ***A machine gun of mass 25 kg fires a bullet of mass 30 gram with a velocity of 250 m/s. Find the velocity with which the machine gun will recoil.***

Solution. Given: Mass of the machine gun (M) = 25 kg; Mass of the bullet (m) = 30 g = 0.03kg and velocity of firing (v) = 250 m/s

Let V = Velocity with which the machine gun will recoil

We know that

$$MV = mv$$

$$\Rightarrow 25 \times V = 0.03 \times 250 = 7.5$$

$$\Rightarrow V = \frac{7.5}{25} = 0.3\, m/s \quad (Ans)$$

Example – 7: ***A bullet of mass 20 g is fired horizontally with a velocity of 300 m/s, from a gun carried in a carriage; which together with the gun has mass of 100 kg. The resistance to sliding of the carriage over the ice on which it rests is 20 N. Find (a) velocities, with which the gun will recoil, (b) distance, in which it comes to rest, and (c) time taken to do so.***

Solution. Given: Mass of the bullet (m) = 20 g = 0.02 kg; Velocity of bullet (v) = 300 m/s; Mass of the carriage with gun (M) = 100 kg and resistance to sliding (F) = 20 N

(a) Velocity, with which the gun will recoil

Let V = velocity with which the gun will recoil
We know that

$$MV = mv$$

$$\Rightarrow 100 \times \mathrm{V} = 0.02 \times 300 = 6$$

$$\Rightarrow v = \frac{6}{100} = 0.06\ m/s\ (\boldsymbol{Ans})$$

(b) Distance, in which the gun comes to rest

Now consider motion of the gun. In this case, initial velocity (u) = 0.06 m/s and final velocity (v) = 0 (because it comes to rest)

Let a = Retardation of the gun, and
s = Distance in which the gun comes to rest

We know that resisting force to sliding of carriage (F)

$$20 = Ma = 100\, a$$

$$\Rightarrow a = \frac{20}{100} = 0.2\, m/s^2$$

We also know that

$$v^2 = u^2 - 2as$$

$$\ldots\ldots\ldots (Minus\ sign\ due\ to\ retardation)$$

$$\Rightarrow 0 = (0.06)^2 - 2 \times 0.2\ \mathrm{s} = 0.0036 - 0.4\ \mathrm{s}$$

$$\Rightarrow s = \frac{0.0036}{0.4} = 0.009m = 9mm\ (\boldsymbol{Ans})$$

(c) Time taken by the gun in coming to rest

Let t = Time taken by the gun in coming to rest

We know that final velocity of the gun (v),

$$0 = u + at = 0.06 - 0.2\,t$$

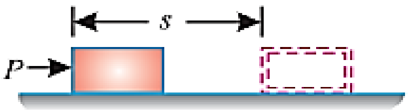
$$\ldots\ldots\ldots(Minus\ sign\ due\ to\ retardation)$$

$$\Rightarrow t = \frac{0.06}{0.2} = 0.3\ s \quad (\boldsymbol{Ans})$$

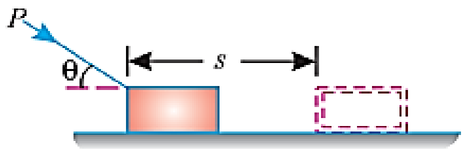
6.2 Work, Power, Energy & its Engineering Applications, Kinetic & Potential energy & its application

Work: Whenever a force acts on a body, and the body undergoes some displacement, then work is said to be done.
e.g., if a force ***P***, acting on a body, causes it to move through a distance ***s*** as shown in Fig. (a).
Then ***work done by the force = Force × Distance = P × s***

Sometimes, the force ***P*** does not act in the direction of motion of the body, or in other words, the body does not move in the direction of the force as shown in Fig. (b).
Then ***work done by the force = Component of the force in the direction of motion × Distance***
= P cos θ × s



(*a*) Body moving in the direction of force



(*b*) Body not moving in the direction of force

S.I. Unit of Work: ***N-m or Joule (J)***

Power: The rate of doing work is called power.
Mathematically,

$$\boldsymbol{Power = \frac{Work}{Time}}$$

$$\boldsymbol{\Rightarrow P = \frac{w}{t}}$$

S.I. Unit of Power: ***J/s or watt (w)***

Energy: The capacity of doing work is called energy.
S.I. Unit: ***Joule (J)***

Potential Energy: The energy possessed by a body for doing work by virtue of its position is called potential energy.
Mathematically,

$$\boldsymbol{Potential\ Energy = Mass\ of\ the\ body \times Acceleration\ due\ to\ gravity \times Height\ of\ the\ body\ from\ earth's\ surface}$$

$$\boldsymbol{\Rightarrow P.E. = mgh}$$

Kinetic Energy: The energy possessed by the body for doing work by virtue of its mass and velocity of motion is called kinetic energy.
Mathematically,

$$\boldsymbol{Kinetic\ Energy = \frac{1}{2} \times Mass \times Velocity^2}$$

$$\boldsymbol{\Rightarrow K.E. = \frac{1}{2} \times m \times v^2}$$

6.3 Momentum & impulse, conservation of energy & linear momentum, collision of elastic bodies, and Coefficient of Restitution

Momentum:

- It is the quantity of motion possessed by a body.
- Or it is the product of mass and velocity of a body.
- Mathematically,

  Momentum = Mass × Velocity = m × v
- **S.I. Unit:** ***kg-m/sec or N-s***

Impulse:

- The change of momentum of one object when the object is acted upon by a force for an interval of time is called Impulse.
- Or it is defined as the product of force and time during which the force acts on the body.
- Mathematically,

  I = F × dt

  Where, ***I*** = impulse, ***F*** = force acting on the object, ***dt*** = time interval.
- **S.I. Unit:** ***N-s***

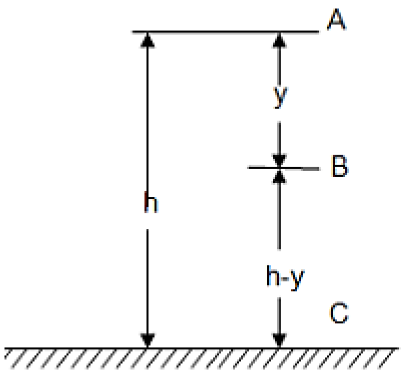
Law of Conservation of Energy: It states ***"The energy can neither be created nor destroyed, though it can be transformed from one form into any of the forms, in which the energy can exist."***
The sum of total energy in this Universe is always constant.

Examples:

1. In an electrical heater, the electrical energy is converted into heat energy.
2. In an electric bulb, the electrical energy is converted into light energy.
3. In a dynamo, the mechanical energy is converted into electrical energy.
4. In a diesel engine chemical energy of the diesel is converted into heat energy, which is then converted into mechanical energy.
5. In a hydro power plant, the hydraulic energy of the water is converted into mechanical energy, which is then converted into electrical energy.

Explanation of Law of Conservation of Energy for a freely falling body:
Consider a body is freely falling from a certain height on the ground. Let us consider the ground level as datum or reference level.

Let ***m*** = mass of the body.
h = height from which the body is falling.



Position 'A'
Velocity at A, u = 0
So kinetic energy at A, $(K.E.)_A = \frac{1}{2}mu^2 = 0$
And potential energy at A, $(P.E.)_A = mgh$

$\therefore$ Total Energy at A = $(K.E.)_A + (P.E.)_A$
$= 0 + mgh = mgh$

Position 'B'
Height from the ground = h – y
Velocity at B = v_1

We know that,

$$\begin{aligned} v_1^2 &= u^2 + 2gy \\ &= 0 + 2gy \\ &= 2gy \end{aligned}$$

So kinetic energy at B, $(K.E.)_B = \frac{1}{2}mv_1^2$
$= \frac{1}{2}m \times 2gy$
$= mgy$

And potential energy at B, $(P.E.)_B = mg(h - y)$
$= mgh - mgy$

∴ Total Energy at B = $(K.E.)_B + (P.E.)_B$
$= mgy + mgh - mgy$
$= mgh$

Position 'C'
Height from the ground (h) = 0
Velocity at C = v
We know that,

$$v^2 = u^2 + 2gh$$
$$= 0 + 2gh$$
$$= 2gh$$

So kinetic energy at C, $(K.E.)_C = \frac{1}{2}mv^2$
$= \frac{1}{2}m \times 2gh$
$= mgh$

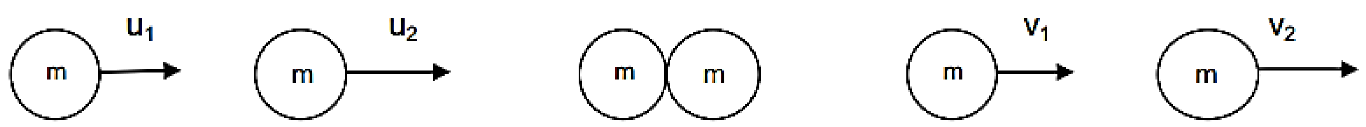
And potential energy at C, $(P.E.)_C = mgh = 0$

∴ Total Energy at C = $(K.E.)_C + (P.E.)_C$
$= mgh + 0$
$= mgh$

Thus, we see that, Total energy at all points remain constant.

Law of Conservation of Linear Momentum: *It states that* ***"the total momentum of two bodies remains constant after their collision or any other mutual action''.***

Explanation:
Momentum along a straight line is called linear momentum.



If a body of mass m_1 moving with velocity u_1 collides with another body of mass m_2 moving with velocity u_2.
Let v_1 and v_2 be the velocities of the bodies after collision.
We have:

Total momentum before collision = $\boldsymbol{m_1 u_1 + m_2 u_2}$
Total momentum after collision = $\boldsymbol{m_1 v_1 + m_2 v_2}$

Now, according to the law of conservation of linear momentum,
Momentum before collision = momentum after collision
$\Rightarrow \boldsymbol{m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2}$

Collision of Elastic Bodies:

- A collision is said to have taken place, if two bodies interact with each other for a short period of time and undergo a change in momentum and kinetic energy.

Newton's Law of Collision of Elastic Bodies:

Newton's law of collision of elastic bodies states that ***"when two moving bodies collide with each other, their velocity of separation bears a constant ratio to their velocity of approach".***

Explanation:

Let us consider two bodies A and B of masses m_1 and m_2 respectively move along the same line and produce direct impact.

Let $\boldsymbol{u_1}$ = initial velocity of body A
$\boldsymbol{u_2}$ = initial velocity of body B
$\boldsymbol{v_1}$ = final velocity of body A after collision
$\boldsymbol{v_2}$ = final velocity of body B after collision

The impact will take place when $u_1 > u_2$
Hence the velocity of approach = $u_1 - u_2$
After impact, the separation of the two bodies will take place if $v_2 > v_1$
Hence the velocity of separation = $v_2 - v_1$

According to Newton's law of Collision of Elastic bodies,

$$(\boldsymbol{v_2} - \boldsymbol{v_1})\ \boldsymbol{\alpha}\ (\boldsymbol{u_1} - \boldsymbol{u_2})$$

$$\Rightarrow\ (\boldsymbol{v_2} - \boldsymbol{v_1})\ =\ \boldsymbol{e}\,(\boldsymbol{u_1} - \boldsymbol{u_2})$$

Where, e = a constant of proportionality known as coefficient of restitution.
The value of 'e' lies between 0 and 1.
If e = 0, it indicates that the two bodies are inelastic.
If e =1, it indicates that the two bodies are perfectly elastic.

Coefficient of restitution:

- **Definition:** The ratio of velocity of separation to the velocity of approach of two elastic bodies is known as coefficient of restitution.
- Mathematically,

$$\boldsymbol{e} = \frac{\boldsymbol{v_2} - \boldsymbol{v_1}}{\boldsymbol{u_1} - \boldsymbol{u_2}}$$

Example – 8: ***A ball of mass 1 kg moving with a velocity of 2 m/s impinges directly on a ball of mass 2 kg at rest. The first ball, after impinging, comes to rest. Find the velocity of the second ball after the impact and the coefficient of restitution.***

Solution. Given: Mass of first ball (m_1) = 1 kg; Initial velocity of first ball (u_1) = 2 m/s; Mass of second ball (m_2) = 2 kg; Initial velocity of second ball (u_2) = 0 (because it is at rest) and final Velocity of first ball after impact (v_1) = 0 (because, it comes to rest)

Velocity of the second ball after impact.

Let v_2 = Velocity of the second ball after impact.
We know from the law of conservation of momentum that

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$\Rightarrow (1 \times 2) + (2 \times 0) = (1 \times 0) + (2 \times v_2)$$

$$\Rightarrow 2 = 2v_2 \qquad \Rightarrow v_2 = 1\,m/s \quad (\boldsymbol{Ans})$$

Coefficient of restitution

Let e = Coefficient of restitution.
We also know from the law of collision of elastic bodies that

$$v_2 - v_1 = e\,(u_1 - u_2)$$

$$\Rightarrow (1 - 0) = e(2 - 0)$$

$$\Rightarrow e = \frac{1}{2} = 0.5\ \ (\boldsymbol{Ans})$$

Example – 9: ***A ball of mass 2 kg impinges directly with a ball of mass 1 kg, which is at rest. If the velocity of the smaller mass after impact be the same as that of the first ball before impact, find the coefficient of restitution.***

Solution. Given: $m_1 = 2$ kg, $m_2 = 1$ kg, $u_2 = 0$, $v_2 = u_1$

We know that,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$\Rightarrow\ (2 \times v_2) + (1 \times 0) = (2 \times v_1) + (1 \times v_2)$$

$$\Rightarrow\ 2v_2 = 2v_1\ +\ v_2$$

$$\Rightarrow\ 2v_2 - v_2 = 2v_1$$

$$\Rightarrow\ v_2 = 2v_1$$

$$\Rightarrow\ {}^{v_2}/_{v_1}\ =\ {}^{1}/_{2} = 0.5$$

We know that,

$$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{2v_1 - v_1}{v_2 - 0} = \frac{v_1}{v_2} = 0.5$$

POSSIBLE SHORT TYPE QUESTIONS WITH ANSWER

1. State Newton's first Law of motion. (W – 2017)

Ans: It states ***"Everybody continues in its state of rest or of uniform motion, in a straight line, unless it is acted upon by some external force."***

2. State Newton's second Law of motion. (S – 2019)

Ans: It states, ***"The rate of change of momentum is directly proportional to the impressed force and takes place, in the same direction in which the force acts."***

3. State Newton's third Law of motion. (Possible)

Ans: It states, ***"To every action, there is always an equal and opposite reaction."***

4. State Law of conservation of energy. (Possible)

Ans: It states ***"The energy can neither be created nor destroyed, though it can be transformed from one form into any of the forms, in which the energy can exist."***

5. State Law of conservation of Linear momentum. (W – 2017 & S – 2018)

Ans: *It states that* ***"the total momentum of two bodies remains constant after their collision or any other mutual action.***

6. Define Coefficient of Restitution. (W – 2017 & S – 2018)

Ans: The ratio of velocity of separation to the velocity of approach of two elastic bodies is known as coefficient of restitution.

Mathematically,

$$\boldsymbol{e = \frac{v_2 - v_1}{u_1 - u_2}}$$

7. Define power and write down its S.I. unit. (S – 2019)

Ans: The rate of doing work is called power.

Mathematically,

$$\boldsymbol{Power = \frac{Work}{Time} \qquad \Rightarrow P = \frac{w}{t}}$$

S.I. Unit of Power: ***J/s or watt (w)***

POSSIBLE LONG TYPE QUESTIONS:

1. State and explain Law of conservation of energy. **(S – 2019)**

2. State and explain De - Alembert's Principle. **(W – 2016 & S – 2019)**

3. Two balls of masses 2 kg and 3 kg are moving with velocities 2 m/s and 3 m/s towards each other. If the coefficient of restitution is 0.5, find the velocity of the two balls after impact. **(S – 2018)**

(Ans. 2.5 m/s; 0)

4. A bullet of 10 gm mass is fired horizontally with a velocity of 1000 m/s from a gun of mass 50 kg. Find (a) velocity with which the gun will recoil, and (b) force necessary to be ring the gun to rest in 250 mm. **(S – 2018)**

(Ans. 0.2 m/s; 4 N)

5. A constant force acting on a body of mass 20 kg changes its speed from 2.5 m/s to 10 m/s in 15 sec. What is the magnitude of the force? **(S – 2019)**

6. A ball is dropped from a height h_0 = 1 m on a smooth floor. Knowing that the height of the first bounce is h_1 = 81cm, determine

(a) coefficient of restitution, and

(b) expected height h_2 after the second bounce. **(S – 2019)**