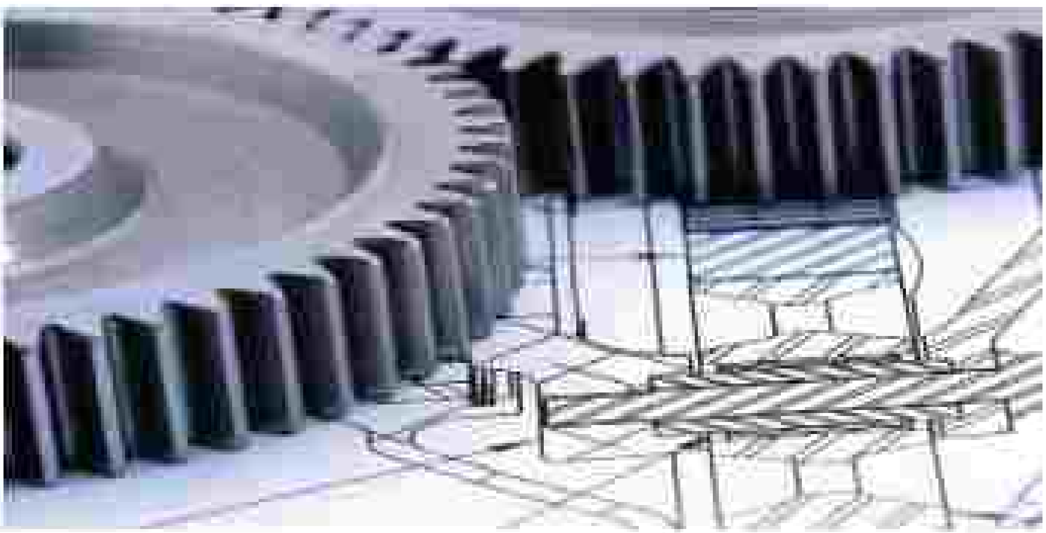
BHADRAK ENGINEERING SCHOOL & TECHNOLOGY
(BEST), ASURALI, BHADRAK

Engineering Mechanics

(Th- 04)

(As per the 2024-25 syllabus of the SCTE&VT, Bhubaneswar, Odisha)



First/Second Semester

Mechanical Engg.

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ENGINEERING MECHANICS

TOPIC WISE DISTRIBUTION PERIODS

SL No.	Name of the chapter as per the syllabus	No of Periods Actually Needed	Expected marks
01	Basics of mechanics and force system	17	24
02	Equilibrium	17	22
03	Friction	08	17
04	Centroid & Centre of Gravity	10	20
05	Simple Lifting Machine	08	17
	TOTAL	**60**	**100**

CHAPTER NO. – 01

BASICS OF MECHANICS AND FORCE SYSTEM

Significance and relevance of Mechanics

Mechanics can be defined as branch of science, which deals with behavior of a body under the action of forces. Engineering mechanics refer to practical applications of principles of mechanics to engineering problem. Engineering mechanics is also called as applied mechanics. In practice we encounter three types of bodies namely (a) Rigid body (b) Deformable body (c) Fluid.

Divisions of engineering mechanics:

The subject of Engineering Mechanics may be divided into the following two main groups:

1. Statics and
2. Dynamics

Statics:

- It is the branch of Engineering Mechanics, which deals with the forces and their effects, while acting upon the bodies at rest.

Dynamics:

- It is the branch of Engineering Mechanics, which deals with the forces and their effects, while acting upon the bodies in motion. Dynamics may be further sub-divided into the following two branches:
    1. Kinematics
    2. Kinetics
- Kinetic deals with the forces acting on moving bodies, whereas kinematics deals with the motion of the bodies without any reference to forces responsible for the motion.

Fundamental concepts

Before we study the mechanics, certain Basic concept should be clearly understood.

Space: It is a region, which extends in all direction and contains everything in it. Examples: Sun, moon, star etc. In space position of body is located with respect to a reference system. The position of an aircraft in space found with respect to earth.

Time: It is measure of succession of events. The time is measured in second(s) and other related units. An event can be described by position of point.

Mass: It is an indication of the quantity of matter present in a system. The more mass means more matter.

Flexible body: A body, which deform, under the action of applied force, is call flexible body.

Rigid body: A body, which does not deform, under the action of applied forces, is call rigid body.

Scalar and Vector

The physical quantities in mechanics can be Express mathematically as follows:

Scalar Quantity: Quantities, which described by their magnitude known as scalar quantity.

Examples are mass, length, time, volume, temperature etc.

Vector Quantity: Quantities, which described by their magnitude and direction (both) known as vector quantity.

Examples are velocity, force, acceleration, momentum etc.

A vector quantity can be represented by straight line with an arrow head. The length of straight line represents the magnitude while direction of line represents the direction of vector and arrow head indicate the sense of direction.

Units of measurement [SI units]

Fundamental units: Length, Mass and Time are the basic fundamental quantities and unit of these quantities are known as fundamental units.

Derived units: Units of other than fundamental quantities may be deriving from the basic units referred as derived units. Examples: (1) Area is result of multiplication of two lengths quantity as L^2. (2) Velocity is length divided by time as L/T. (3) Force is product of mass & acceleration as kg . m/sec² [N].

SI units: By international agreement in in 1960, the international system of units known as S.I. Unit is accepted and used all over the worldwide. The symbols and notation of SI units and their derivatives are standardizing to avoid any possibility of confusion.

Fundamental SI units:

Sr. No.	Fundamental Quantity	Name of SI unit	Symbol
1	Length	Meter	m
2	Mass	Kilogram	kg
3	Time	Second	s
4	Electrical current	Ampere	A
5	Temperature	Kelvin	K
6	Luminous intensity	Candela	cd

Force:

- Force is that which changes or tends to change the state of rest of uniform motion of a body along a straight line. It may also deform a body changing its dimensions.
- The force may be broadly defined as an agent which produces or tends to produce, destroys or tends to destroy motion. It has a magnitude and direction.
- Mathematically,

  Force = Mass × Acceleration.

  Where, F = force, M = mass and A = acceleration.

Units of force:

- **In C.G.S. System**: In this system, there are two units of force. (I) Dyne and (ii) Gram force (gmf). Dyne is the absolute unit of force in the C.G.S. system. One dyne is that force which acting on a mass of one gram produces in it an acceleration of one centimeter per second².
- **In M.K.S. System**: In this system, unit of force is kilogram force (kgf). One kilogram force is that force which acting on a mass of one kilogram produces in it an acceleration of 9.81 m/sec².
- **In S.I. Unit**: In this system, unit of force is Newton (N). One Newton is that force which acting on a mass of one kilogram produces in it an acceleration of one m/sec².

  1 Newton = 10^5 Dyne

Representation of a Force:

Since force is a vector quantity, it can be represented by a straight line. The length of the line represents magnitude of the force, the line itself represents the direction and an arrow put on the head of the straight line indicates the sense in which the force acts.

Denoting A Force by Bow's Notation:

Fig 1.10

- In Bow's notation for denoting a force, two English capital letters are placed, one on each side of the line of action of the force. In figure 1.10 AB denotes the force F.

Effect of force:

A force may produce the following effects in a body, on which it acts:

- It may change the motion of a body. i.e., if a body is at rest, the force may set it in motion. And if the body is already in motion, the force may accelerate or decelerate it.
- It may retard the forces, already acting on a body, thus bringing it to rest or in equilibrium.
- It may give rise to the internal stresses in the body, on which it acts.
- A force can change the direction of a moving object.
- A force can change the shape and size of an object.

Characteristics of a Force:

In order to determine the effects of a force, acting on a body, we must know the following characteristics of a force:

- Magnitude of the force (i.e., 50 N, 30 N, 20N etc.)
- The direction of the line, along which the force acts (i.e., along West, at 30° North of East etc.). It is also known as line of action of the force.
- Natures of the force (push or pull).
- The point at which (or through which) the force acts on the body.

System of forces:

When two or more forces act on a body, they are called to form a system of forces. Force system is basically classified into following types.

- Collinear forces
- Coplanar forces
- Concurrent forces
- Coplanar concurrent forces
- Coplanar non- concurrent forces
- Non-coplanar concurrent forces
- Non- coplanar non- concurrent force

Collinear forces:

- The forces, whose lines of action lie on the same line, are known as collinear forces. They act along the same line. Collinear forces may act in the opposite directions or in the same direction.

Collinear forces.

Fig 1.1

Coplanar forces:

- The forces, whose lines of action lie on the same plane, are known as coplanar forces.

Concurrent forces:

- The forces, whose lines of action pass through a common point, are known as concurrent forces. The concurrent forces may or may not be collinear.

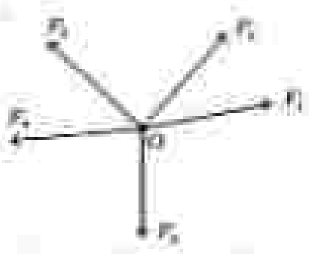


Fig Concurrent forces

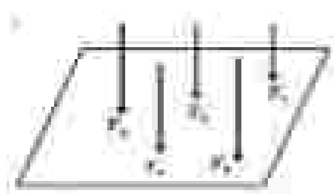


Fig Coplanar forces

Coplanar concurrent forces:

- The forces, whose lines of action lie in the same plane and at the same time pass through a common point, are known as coplanar concurrent forces.

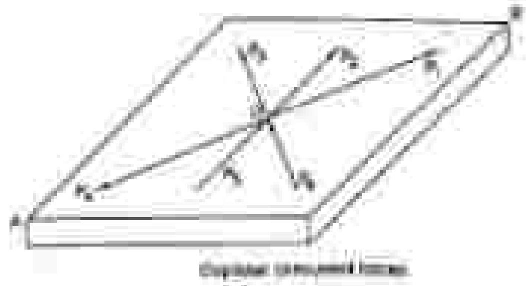


Fig 1.4

Coplanar non-concurrent forces:

- The forces, which do not meet at one point, but their lines of action lie on the same plane, are known as coplanar non-concurrent forces.

Coplanar non-concurrent forces.

Fig 1.5

Non-coplanar concurrent forces:

- The forces, which meet at one point, but their lines of action do not lie on the same plane, are known as non-coplanar concurrent forces.

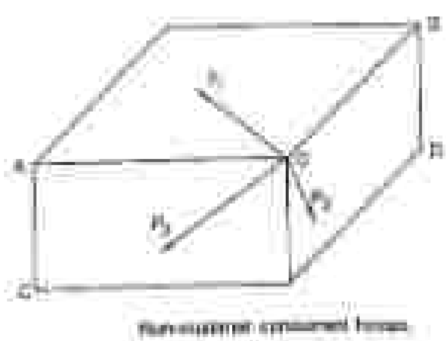


Fig 1.6

Non-coplanar non-concurrent forces:

- The forces, which do not meet at one point and their lines of action do not lie on the same plane, are called non-coplanar non-concurrent forces.

Principle of Transmissibility of Forces:

- It states, "If a force acts at any point on a rigid body, it may also be considered to act at any other point on its line of action, provided this point is rigidly connected with the body". That means the point of application of a force can be moved anywhere along its line of action without changing the external reaction forces on a rigid body.

Fig 1.11

- Here, force at point A = force at B (the magnitude of force in the body at any point along the line of action are same)

Principle of Superposition of Forces:

- This principle states that the combined effect of force system acting on a particle or a rigid body is the sum of effects of individual forces.
- Consider two forces P and Q acting at A on a boat as shown in Fig 1.12. Let R be the resultant of these two forces P and Q. According to Newton's second law of motion, the boat will move in the direction of resultant force R with acceleration proportional to R. The same motion can be obtained when P and Q are applied simultaneously.

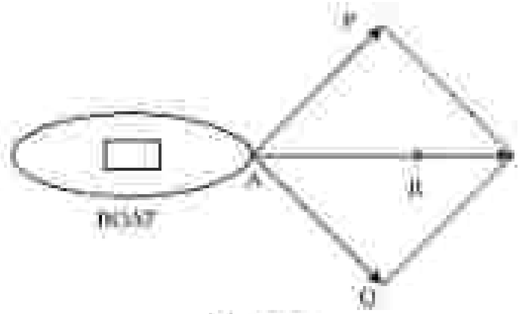


Fig 1.12

Resolution of a Force:

- The process of splitting up the given force into a number of components, without changing its effect on the body is called resolution of a force. A force is, generally, resolved along two mutually perpendicular directions.

Method of resolution:

- Resolved all the forces horizontally and find the algebraic sum of all the horizontal components (i.e., $\sum H$).
- Resolved all the forces vertically and find the algebraic sum of all the vertical components (i.e., $\sum V$).
- The resultant R of the given forces will be given by the equation:

$$R = \sqrt{\left(\sum H\right)^2 + \left(\sum V\right)^2}$$

- The resultant force will be inclined at an angle θ, with the horizontal, such that

$$\tan\theta = \frac{\sum V}{\sum H}$$

Types of Component Forces:

Generally, a force is resolved into two types of components

- Mutually perpendicular components
- Non perpendicular components

Mutually Perpendicular Components:

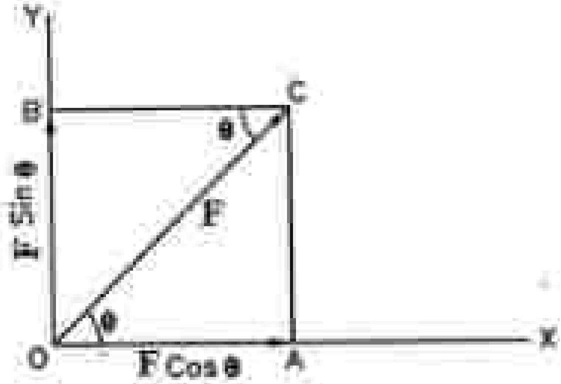


Fig 1.13

- Resolved parts of a force means components of the force along two mutually perpendicular directions.
- Let a force F represented in magnitude and direction by OC make an angle θ with OX. Line OY is drawn through O at right angles to OX as shown in figure 1.13.
- Through C, lines CA and CB are drawn parallel to OY and OX respectively. Then the resolved parts of the force F along OX and OY are represented in magnitude and direction by OA and OB respectively.
- Now in the right angled Δ AOC,

$$\cos\theta = \frac{OA}{OC} = \frac{OA}{F}$$
$$\Rightarrow OA = F\cos\theta$$

$$\sin\theta = \frac{AC}{OC} = \frac{AC}{F}$$
$$\Rightarrow AC = OB = F\sin\theta$$

- Thus, the resolved parts of F along OX and OY are $F\cos\theta$ and $F\sin\theta$ respectively.

Non perpendicular components:

- Let P be the given force represented in magnitude and direction by OB as shown in Fig 1.14. Also let OX and OY be two given directions along which the components of P are to be found out.

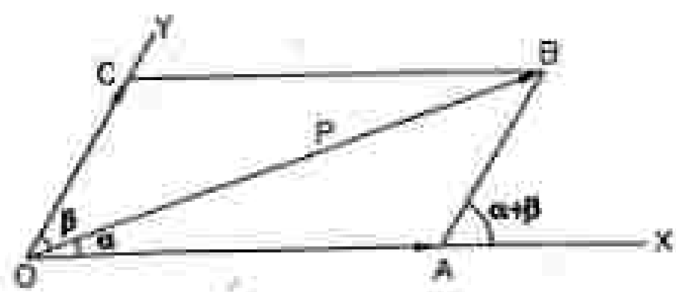


Fig 1.14

- Let ∠BOX = α and ∠BOY = β
- From B, lines BA and BC are drawn parallel to OY and OX respectively. Then the required components of the given force P along OX and OY are represented in magnitude and direction by OA and OC respectively.
- Since AB is parallel to OC, $\angle BAX = \angle AOC = \alpha + \beta$ And $\angle OAB = 180^0 - (\alpha + \beta)$

- Now, in Δ OAB

$$\frac{OA}{\sin\beta}=\frac{AB}{\sin\alpha}=\frac{OB}{\sin 180^0-(\alpha+\beta)}$$

$$\Rightarrow\frac{OA}{\sin\beta}=\frac{AB}{\sin\alpha}=\frac{P}{\sin(\alpha+\beta)}$$

$$\therefore OA=\frac{P\sin\beta}{\sin(\alpha+\beta)}\quad And\quad AB=OC=\frac{P\sin\alpha}{\sin(\alpha+\beta)}$$

Composition of Forces:

- The process of finding out the resultant force of a number of given forces is called composition of forces or compounding of forces.

Resultant Force:

- It is a single force replaced by number of forces acting upon a rigid body, whose effect on it is same as the combined effect of all forces.

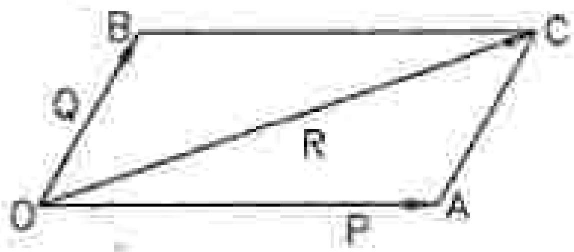


Fig 1.15

- In figure 1.15, ***R*** is the resultant of forces ***P*** and ***Q***. If R is the resultant of two forces P and Q, it means forces P and Q can be replaced by R. Similarly, R can be replaced by two forces P and Q whose joint effect on a body will be the same as R on the body. Then these two forces P and Q are called components of R.

Equilibriant:

- Equilibrant of a system of forces is a single force which will keep the given forces in equilibrium. Evidently, equilibrant is equal and opposite to the resultant of the given forces.

Method of Composition of Forces:

There are two important methods of composition of forces.

- Analytical method
- Graphical method

Analytical Method:

Parallelogram law of forces:

- It states, *"If two forces, acting simultaneously on a particle, be represented in magnitude and direction by the two adjacent sides of a parallelogram; their resultant may be represented in magnitude and direction by the diagonal of the parallelogram, which passes through their point of intersection."*

- **Explanation:** Let two forces P and Q are acting at a point O be represented in magnitude and direction by OA and OB respectively as shown in Fig 1.16. Then, according to the theorem of parallelogram of forces, the diagonal OC drawn through O represents the resultant of P and Q in magnitude and direction.

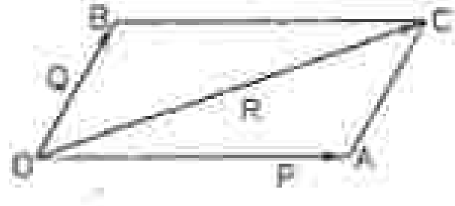


Fig 1.16

- **Proof:**

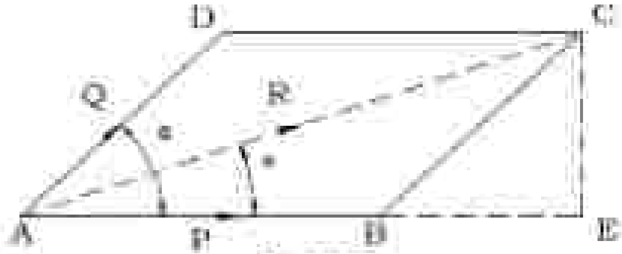


Fig 1.17

- Consider two forces 'P' and 'Q' acting at and away from point 'A' as shown in figure 1.17.
- Let, the forces P and Q are represented by the two adjacent sides of a parallelogram AD and AB respectively as shown in fig. Let, θ be the angle between the force P and Q and α be the angle between R and P. Extend line AB and drop perpendicular from point C on the extended line AB to meet at point E.

- *Consider Right angle triangle ACE,*
  $AC^2 = AE^2 + CE^2$
  $= (AB + BE)^2 + CE^2$
  $= AB^2 + BE^2 + 2.AB.BE + CE^2$
  $= AB^2 + BE^2 + CE^2 + 2.AB.BE$ ………………………………(i)

- *Consider right angle triangle BCE,*
  $BC^2 = BE^2 + CE^2$
  $Cos\,\theta = BE / BC$
  $\Rightarrow BE = BC.Cos\,\theta$

- *Putting* $BE^2 + CE^2 = BC^2$ & $BE = BC.\,Cos\,\theta$ *in equation (i), we get*
  $AC^2 = AB^2 + BC^2 + 2.AB.BC.Cos\,\theta$
  But, $AB = P, BC = Q$ *and* $AC = R$

- *So, magnitude of the resultant,*

$$R = \sqrt{P^2 + Q^2 + 2PQ\,Cos\,\theta}$$

- *In triangle BCE,*
  $Sin\,\theta = CE / BC \Rightarrow CE = BC.Sin\,\theta = Q\,Sin\,\theta$

- *In triangle ACE,*

$$\tan\alpha = \frac{CE}{AE} = \frac{CE}{AB + BE}$$

- *So, direction of the resultant,*

$$\therefore \tan\alpha = \frac{Q \sin\theta}{P + Q \cos\theta}$$

Now let us consider two forces F_1 and F_2 are represented by the two adjacent sides of a parallelogram

F_1 and F_2 = Forces whose resultant is required to be found out,
θ = Angle between the forces F_1 and F_2, and
α = Angle which the resultant force makes with one of the forces (say F_1).

- Then magnitude of resultant,

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos\theta}$$

- And direction of the resultant,

$$\tan\alpha = \frac{F_2 \sin\theta}{F_1 + F_2 \cos\theta}$$

- If (α) is the angle which the resultant force makes with the other force F_2, then

$$\tan\alpha = \frac{F_1 \sin\theta}{F_2 + F_1 \cos\theta}$$

CASES:

1. If $\theta = 0$ i.e., when the forces act along the same line, then

$$R_{Max} = F_1 + F_2$$

2. If $\theta = 90°$ i.e., when the forces act at right angle, then

$$R = \sqrt{F_1^2 + F_2^2}$$

3. If $\theta = 180°$ i.e., when the forces act along the same straight line but in opposite directions, then

$$R_{Min} = F_1 - F_2$$

4. If the two forces are equal i.e., when $F_1 = F_2 = F$ then

$$R = \sqrt{F^2 + F^2 + 2F^2 \cos\theta}$$
$$= \sqrt{2F^2(1 + \cos\theta)}$$
$$= \sqrt{2F^2 \times 2\cos^2(\theta/2)}$$
$$= \sqrt{4F^2\cos^2(\theta/2)}$$
$$= 2F \cos\theta/2$$

Example – 1: ***Two forces of 100 N and 150 N are acting simultaneously at a point. What is the resultant of these two forces, if the angle between them is 45°?***
Solution. Given: First force (F_1) = 100 N; Second force (F_2) = 150 N and angle between F_1 and F_2 (θ) = 45°.
We know that the resultant force,

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos\theta}$$
$$= \sqrt{(100)^2 + (150)^2 + 2 \times 100 \times 150 \cos 45°}$$
$$= \sqrt{10000 + 22500 + (30000 \times 0.707)} = 232\ N$$

Example – 2: ***Two forces act at an angle of 120°. The bigger force is of 40 N and the resultant is perpendicular to the smaller one. Find the smaller force.***

Solution. Given: Angle between the forces ∠AOC = 120°, Bigger force (F_1) = 40 N and angle between the resultant and F_2 (∠BOC) = 90°;

Let F_2 = Smaller force in N

From the geometry of the figure, we find that ∠AOB,

$\alpha = 120° - 90° = 30°$

We know that,

$$\tan\alpha = \frac{F_2 \sin\theta}{F_1 + F_2 \cos\theta}$$

$$\Rightarrow \tan 30^0 = \frac{F_2 \sin 120^0}{40 + F_2 \cos 120^0} = \frac{F_2 \sin 60^0}{40 + F_2(-\cos 60^0)}$$

$$\Rightarrow 0.577 = \frac{F_2 \times 0.866}{40 - F_2 \times 0.5} = \frac{0.866\, F_2}{40 - 0.5\, F_2}$$

$$\Rightarrow 40 - 0.5\, F_2 = \frac{0.866\, F_2}{0.577} = 1.5\, F_2$$

$$\Rightarrow 2F_2 = 40$$

$$\Rightarrow \boldsymbol{F_2 = 20\, N}$$

Example – 3: ***Find the magnitude of the two forces, such that if they act at right angles, their resultant is $\sqrt{10}N$. But if they Act at 60°, their resultant is $\sqrt{13}N$.***

Solution. Given: Two forces = F_1 and F_2.

First of all, consider the two forces acting at right angles. We know that when the angle between the two given forces is 90°, then the resultant force (R)

$$\sqrt{10} = \sqrt{F_1^{\,2} + F_2^{\,2}}$$

$$\Rightarrow 10 = F_1^{\,2} + F_2^{\,2}$$

…(Squaring both sides)

Similarly, when the angle between the two forces is 60°, then the resultant force (R)

$$\sqrt{13} = \sqrt{F_1^{\,2} + F_2^{\,2} + 2\, F_1 F_2 \cos 60^0}$$

$$\Rightarrow 13 = F_1^{\,2} + F_2^{\,2} + 2\, F_1 F_2 \times 0.5 \qquad \ldots(Squaring\ both\ sides)$$

$$F_1 F_2 = 13 - 10 = 3$$

…(Substituting $F_1^{\,2} + F_2^{\,2} = 10$)

We know that,

$$(F_1 + F_2)^2 = F_1^{\,2} + F_2^{\,2} + 2\, F_1 F_2 = 10 + 6 = 16$$

$$\Rightarrow F_1 + F_2 = \sqrt{16} = 4 \ldots\ldots\ldots\ldots\ldots\ldots(i)$$

Similarly,

$$(F_1 - F_2)^2 = F_1^{\,2} - F_2^{\,2} + 2\, F_1 F_2 = 10 - 6 = 4$$

$$\Rightarrow F_1 - F_2 = \sqrt{4} = 2 \ldots\ldots\ldots\ldots\ldots(ii)$$

Solving equations (*i*) and (ii),

$$F_1 = 3\,N \text{ and } F_2 = 1\,N$$

Method Of Resolution:

- Resolved all the forces horizontally and find the algebraic sum of all the horizontal components (i.e., $\sum H$).
- Resolved all the forces vertically and find the algebraic sum of all the vertical components (i.e., $\sum V$).
- The resultant R of the given forces will be given by the equation:

$$R = \sqrt{\left(\sum H\right)^2 + \left(\sum V\right)^2}$$

- The resultant force will be inclined at an angle θ, with the horizontal, such that

$$\tan\theta = \frac{\sum V}{\sum H}$$

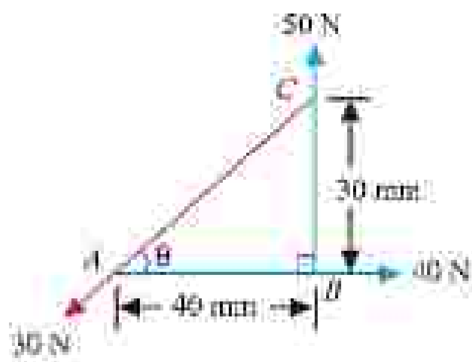
Notes: The value of the angle θ will vary depending upon the values of ∑V and ∑H, as discussed below:

1. When ∑V is +ve, the resultant makes an angle between 0° and 180°. But when ∑V is –ve, the resultant makes an angle between 180° and 360°.
2. When ∑H is +ve, the resultant makes an angle between 0° to 90° or 270° to 360°. But when ∑H is –ve, the resultant makes an angle between 90° to 270°.

Example – 4: ***A triangle ABC has its side AB = 40 mm along positive x-axis and side BC = 30 mm along positive y-axis. Three forces of 40 N, 50 N and 30 N act along the sides AB, BC and CA respectively. Determine magnitude of the resultant of such a system of forces.***

Solution. The system of given forces is shown in Figure.

From the geometry of the figure, we find that the triangle ABC is a right-angled triangle, in which the side AC = 50 mm. Therefore



$$\sin\theta = \frac{30}{50} = 0.6$$

$$\cos\theta = \frac{40}{50} = 0.8$$

Resolving all the forces horizontally (i.e., along AB),

$$\sum H = 40 - 30\cos\theta$$
$$= 40 - (30 \times 0.8) = 16\,N$$

and now resolving all the forces vertically (i.e., along BC)

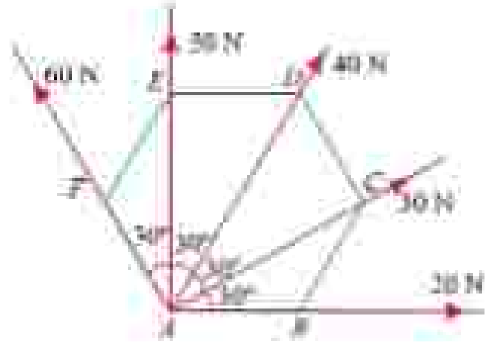
$$\sum V = 50 - 30\sin\theta$$
$$= 50 - (30 \times 0.6) = 32\,N$$

We know that magnitude of the resultant force,

$$R = \sqrt{(\sum H)^2 + (\sum V)^2} = \sqrt{16^2 + 32^2} = 35.8\,N$$

Example – 5: ***The forces 20 N, 30 N, 40 N, 50 N and 60 N are acting at one of the angular points of a regular hexagon, towards the other five angular points, taken in order. Find the magnitude and direction of the resultant force.***

Solution. The system of given forces is shown in Figure.



Magnitude of the resultant force
Resolving all the forces horizontally (i.e., along AB),

$$\sum H = 20\cos 0^\circ + 30\cos 30^\circ + 40\cos 60^\circ + 50\cos 90^\circ + 60\cos 120^\circ\ N$$
$$= (20\times 1) + (30\times 0.866) + (40\times 0.5) + (50\times 0) + 60\,(-0.5)\ N = 36\ N$$

and now resolving the all forces vertically (i.e., at right angles to AB),

$$\sum V = 20\sin 0^\circ + 30\sin 30^\circ + 40\sin 60^\circ + 50\sin 90^\circ + 60\sin 120^\circ\ N$$
$$= (20\times 0) + (30\times 0.5) + (40\times 0.866) + (50\times 1) + (60\times 0.866)\ N = 151.6\ N$$

We know that magnitude of the resultant force,

$$R = \sqrt{(\sum H)^2 + (\sum V)^2} = \sqrt{36^2 + 151.6^2} = 155.8\ N$$

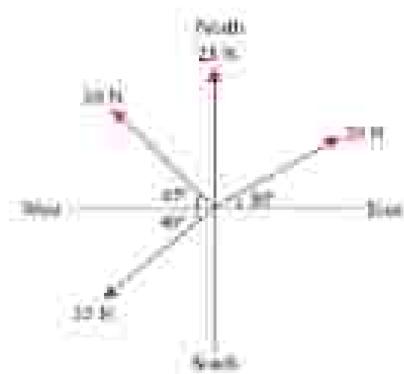
Direction of the resultant force
Let θ = Angle, which the resultant force makes with the horizontal (i.e., AB).
We know that,

$$\tan\theta = \frac{\sum V}{\sum H} = \frac{151.6}{36} = 4.211$$

$$\Rightarrow \theta = 76.6^\circ$$

Note. Since both the values of ∑H and ∑V are positive, therefore actual angle of resultant force lies between 0° and 90°.

Example – 6: *The following forces act at a point:*
(i) 20 N inclined at 30° towards North of East,
(ii) 25 N towards North,
(iii) 30 N towards North West, and
(iv) 35 N inclined at 40° towards South of West.
Find the magnitude and direction of the resultant force.
Solution. The system of given forces is shown in Figure.



Magnitude of the resultant force

Resolving all the forces horizontally i.e., along East-West line,

$$\sum H = 20\cos 30^\circ + 25\cos 90^\circ + 30\cos 135^\circ + 35\cos 220^\circ\ N$$
$$= (20 \times 0.866) + (25 \times 0) + 30\,(-0.707) + 35\,(-0.766)N = -30.7\ N$$

and now resolving all the forces vertically i.e., along North-South line,

$$\sum V = 20\sin 30^\circ + 25\sin 90^\circ + 30\sin 135^\circ + 35\sin 220^\circ\ N$$
$$= (20 \times 0.5) + (25 \times 1.0) + (30 \times 0.707) + 35\,(-0.6428)N = 33.7\ N$$

We know that magnitude of the resultant force,

$$R = \sqrt{(\sum H)^2 + (\sum V)^2} = \sqrt{(-30.7)^2 + (33.7)^2} = 45.6\ N$$

Direction of the resultant force

Let θ = Angle, which the resultant force makes with the East.

We know that,

$$\tan\theta = \frac{\sum V}{\sum H} = \frac{33.7}{-30.7} = -1.098$$

$$\Rightarrow \theta = 47.7^\circ$$

Since $\sum H$ is negative and $\sum V$ is positive, therefore resultant lies between 90° and 180°. Thus, actual angle of the resultant = 180° – 47.7° = 132.3°.

Graphical Method:

Triangle Law of Forces:

- It states, ***"If two forces acting simultaneously on a particle be represented in magnitude and direction by the two sides of a triangle, taken in order, their resultant may be represented in magnitude and direction by the third side of the triangle, taken in opposite order."***
- **Explanation:** Let two forces P and Q acting at O be such that they can be represented in magnitude and direction by the sides AB and BC of the triangle ABC. Then, according to the theorem of triangle of forces, their resultant R will be represented in magnitude and direction by AC which is the third side of the triangle ABC taken in the reverse order of CA.

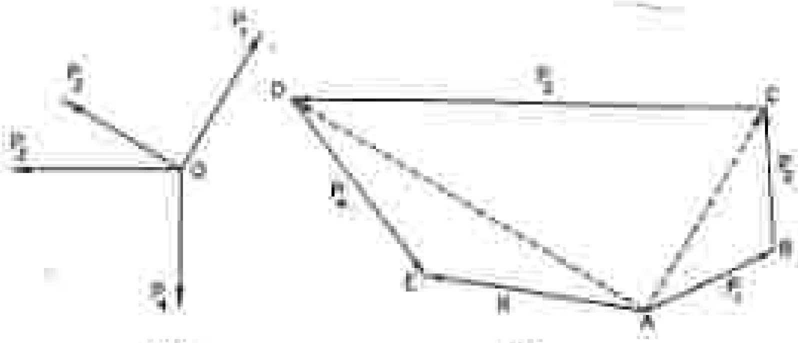
- **Proof:**

Fig. 1.18

- In Fig 1.18 the parallelogram ABCD is completed with sides AB and BC of the triangle ABC. Side AD is equal and parallel to BC. So, force Q is also represented in magnitude and direction by AD. Now, the resultant of P (represented by AB) and Q (represented by AD) is represented in magnitude and direction by the diagonal AC of the parallelogram ABCD. Thus, the resultant of P and Q is represented in magnitude and direction by the third side AC of the triangle ABC taken in the reverse order.

Polygon Law of Forces:

- It is an extension of Triangle Law of Forces for more than two forces, which states, *"If a number of forces acting simultaneously on a particle, be represented in magnitude and direction, by the sides of a polygon taken in order then the resultant of all these forces may be represented, in magnitude and direction, by the closing side of the polygon, taken in opposite order."*
- **Proof**



(a) (b)

Fig. 1.19

- Let forces P_1, P_2, P_3 and P_4, acting at a point O be such that they can be represented in magnitude and direction by the sides AB, BC, CD and DE of a polygon ABCDE as shown in fig. 1.19.
- We are to prove that the resultant of these forces is represented in magnitude and direction by the side AE in the direction from A towards E.
- According to the triangle law of forces, AC represents the resultant R_1 of P_1 and P_2. AD represents the resultant R_2 of R_1 and P_3. Thus, AD represents the resultant of P_1, P_2 and P_3.
- According to the same law, AE represents the resultant R_3 of R_2 and P_4. Thus, AE represents the resultant of P_1, P_2, P_3 and P_4.

Parallel Forces:

- The forces whose lines of action are parallel to each other, then the forces are known as parallel forces.

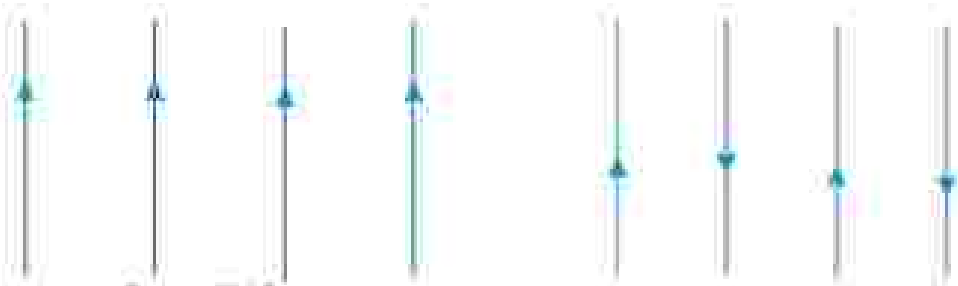
Classification Of Parallel Forces:

The parallel forces may be, broadly, classified into the following two categories, depending upon their directions.

- Like parallel forces
- Unlike parallel forces

Like Parallel Forces:

- The forces, whose lines of action are parallel to each other and all of them act in the same direction as shown in Fig. 1.23 (a) are known as like parallel forces.



(a) Like parallel forces (b) Unlike parallel forces.

Fig. 1.23

Unlike Parallel Forces:

- The forces, whose lines of action are parallel to each other and all of them do not act in the same direction as shown in Fig. 1.23 (b) are known as like parallel forces.

Methods For Magnitude and Position of The Resultant of Parallel Forces:

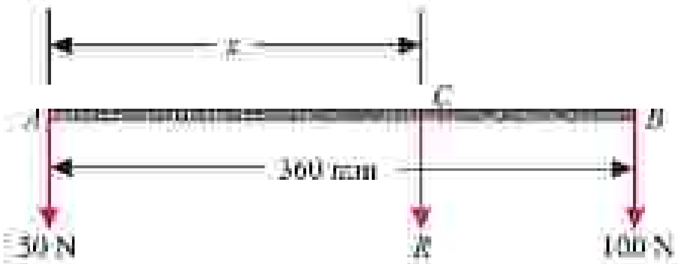
- The magnitude and position of the resultant force, of a given system of parallel forces (like or unlike) may be found out analytically or graphically. Here we shall discuss both the methods one by one.

Analytical Method for The Resultant of Parallel Forces:

- In this method, the sum of clockwise moments is equated with the sum of anticlockwise moments about a point.

Example – 7: ***Two like parallel forces of 50 N and 100 N act at the ends of a rod 360 mm long. Find the magnitude of the resultant force and the point where it acts.***

Solution. Given: The system of given forces is shown in Figure below.



Magnitude of the resultant force

Since the given forces are like and parallel, therefore magnitude of the resultant force,

$$R = 50 + 100 = 150\ N$$

Point where the resultant force acts

Let x = Distance between the line of action of the resultant force (R) and A (i.e., AC) in mm.

Now taking clockwise and anticlockwise moments of the forces about C and equating the same,

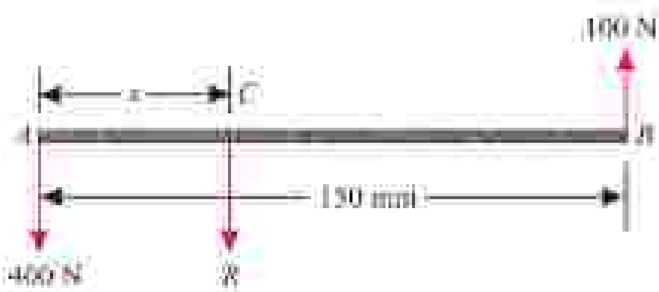
$$50 \times x = 100\,(360 - x) = 36\,000 - 100\,x$$

$$\Rightarrow 150\,x = 36\,000$$

$$\Rightarrow x = \frac{36000}{150} = 240\ mm$$

Example – 8: ***Two unlike parallel forces of magnitude 400 N and 100 N are acting in such a way that their lines of action are 150 mm apart. Determine the magnitude of the resultant force and the point at which it acts.***

Solution. Given: The system of given force is shown in Figure below.



Magnitude of the resultant force

Since the given forces are unlike and parallel, therefore magnitude of the resultant force,

$$R = 400 - 100 = \mathbf{300\ N}$$

Point where the resultant force acts

Let x = Distance between the lines of action of the resultant force and A in mm.

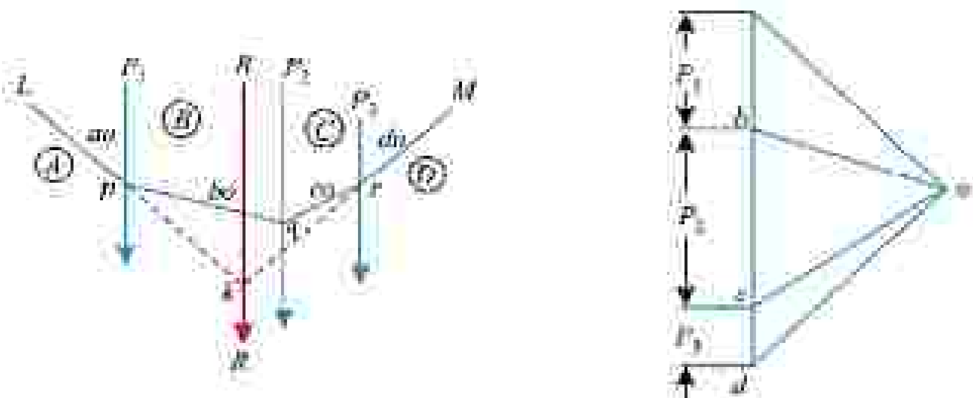
Now taking clockwise and anticlockwise moments about A and equating the same,

$$300 \times x = 100 \times 150 = 15\,000$$

$$\Rightarrow x = \frac{15000}{300} = \mathbf{50\ mm}$$

Graphical Method for The Resultant of Parallel Forces:

Consider a number of parallel forces (say three like parallel forces) P1, P2 and P3 whose resultant is required to be found out as shown in Fig. 4.6 (a).



(a) Space diagram (b) Vector diagram

Fig. 1.24

First of all, draw the space diagram of the given system of forces and name them according to Bow's notations as shown in Fig. 1.24 (a). Now draw the vector diagram for the given forces as shown in Fig. 1.24 (b) and as discussed below:

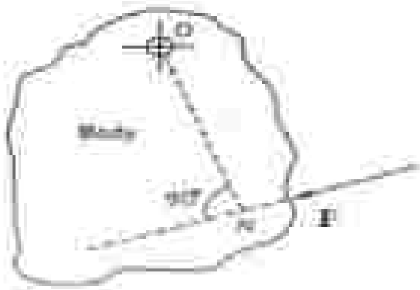
- Select some suitable point *a*, and draw *ab* equal to the force *AB* (P_1) and parallel to it to some suitable scale.
- Similarly draw *bc* and *cd* equal to and parallel to the forces *BC* (P_2) and *CD* (P_3) respectively.
- Now take some convenient point *o* and joint *oa*, *ob*, *oc* and *od*.
- Select some point *p*, on the line of action of the force *AB* of the space diagram and through it draw a line *Lp* parallel to *ao*. Now through *p* draw *pq parallel* to *bo* meeting the line of action of the force *BC* at *q*.
- Similarly draw *qr* and *rM* parallel to *co* and *do* respectively.
- Now extend *Lp* and *Mr* to meet at *k*. Through *k*, draw a line parallel to *ad*, which gives the required position of the resultant force.
- The magnitude of the resultant force is given by *ad* to the scale.

Note. This method for the position of the resultant force may also be used for any system of forces i.e. parallel, like, unlike or even inclined.

Moment Of Force:

- It is the turning effect produced by a force, on the body, on which it acts.
- The moment of a force is equal to the product of the force and the perpendicular distance of the point, about which the moment is required and the line of action of the force.
- Mathematically, moment,
  M = P × *l*
  Where, P = Force acting on the body, *l* = Perpendicular distance between the point, about which the moment is required and the line of action of the force.

- **Example:** Moment of a force about a point is the product of the force and the perpendicular distance of the point from the line of action of the force.



- Let a force F act on a body which is hinged at O. Then, moment of F about the point O in the body is = F × ON, Where: ON = perpendicular distance of O from the line of action of the force F.
- Unit of moment: Newton meter (N-m), kilo Newton meter (kN-m), N-mm.

Types of Moments:

Broadly speaking, the moments are of the following two types:
1. Clockwise moments 2. Anticlockwise moments

Clockwise Moment:

- It is the moment of a force, whose effect is to turn or rotate the body, about the point in the same direction in which hands of a clock move as shown in Fig. 1.25 (a).

Anticlockwise Moment:

- It is the moment of a force, whose effect is to turn or rotate the body, about the point in the opposite direction in which the hands of a clock move as shown in Fig. 1.25 (b).

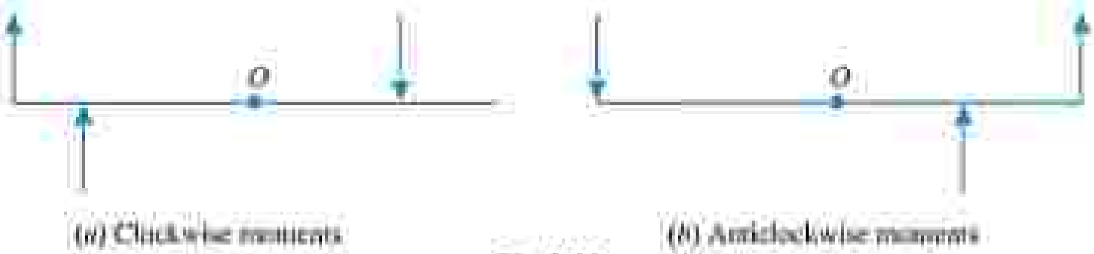


Fig. 1.25

- **Sign convention:** The general convention is to take clockwise moment as positive and anticlockwise moment as negative.

Geometrical representation of moment of a force:

- Consider a force P represented, in magnitude and direction, by the line AB. Let O be a point, about which the moment of this force is required to be found out, as shown in Fig. 1.26 from O, draw OC perpendicular to AB. Join OA and OB.

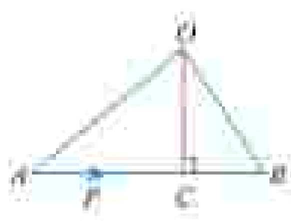


Fig. 1.26

- Now moment of the force P about O $= P \times OC$

$$= AB \times OC$$
$$= 2 \times (1/2 \times AB \times OC)$$
$$= 2 \times Area\ of\ \Delta\ AOB$$

- Thus, the moment of a force, about any point, is equal to twice the area of the triangle, whose base is the line to some scale representing the force and whose vertex is the point about which the moment is taken.

Varignon's theorem:

Varignon's theorem states that the algebraic sum of the moments of two forces about any point in their plane is equal to the moment of their resultant about the same point.

Proof

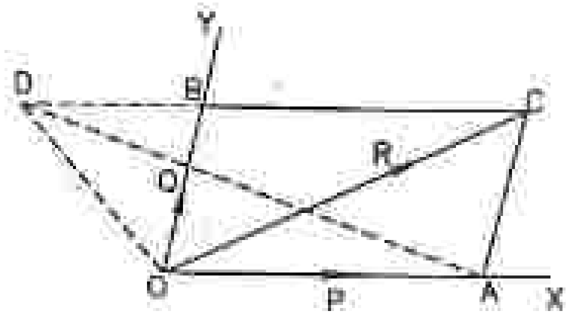


Fig. 1.27

Let P and Q be any two forces acting at a point O along lines OX and OY respectively and let D be any point in their plane as shown in Fig 1.27.

Line DC is drawn parallel to OX to meet OY at B. Let in some suitable scale, line OB represent the force Q in magnitude and direction and let in the same scale, OA represent the force P in magnitude and direction.

With OA and OB as the adjacent sides, parallelogram OACB is completed and OC is joined. Let R be the resultant of forces P and Q. Then, according to the "Theorem of parallelogram of forces", R is represented in magnitude and direction by the diagonal OC of the parallelogram OACB.

The point D is joined with points O and A. The moments of P, Q and R about D are given by 2 × area of Δ AOD, 2 × area of Δ OBD and 2 × area of Δ OCD respectively.

From the geometry of the figure,

$$Area\ of\ \Delta\ OCD = area\ of\ \Delta\ OBC + area\ of\ \Delta\ OBD$$
$$\Rightarrow Area\ of\ \Delta\ OCD = area\ of\ \Delta\ AOC + area\ of\ \Delta\ OBD$$
$$\Rightarrow Area\ of\ \Delta\ OCD = area\ of\ \Delta\ AOD + area\ of\ \Delta\ OBD$$
$$\Rightarrow 2 \times Area\ of\ \Delta\ OCD = 2 \times Area\ of\ \Delta\ AOD + 2 \times Area\ of\ \Delta\ OBD$$
$$\Rightarrow Moment\ of\ R\ about\ D = Moment\ of\ P\ about\ D + Moment\ of\ Q\ about\ D$$

[Note: As Δ AOC and Δ AOD are on the same base and have the same altitude, Δ AOC = Δ AOD. Again, As AOC and OBC have equal bases and equal altitudes, ΔAOC = ΔOBC]

Example – 9: ***A force of 15 N is applied perpendicular to the edge of a door 0.8 m wide as shown in Fig. (a). Find the moment of the force about the hinge. If this force is applied at an angle of 60° to the edge of the same door, as shown in Fig. (b), find the moment of this force.***

Solution. Given: Force applied (P) = 15 N and width of the door (l) = 0.8 m

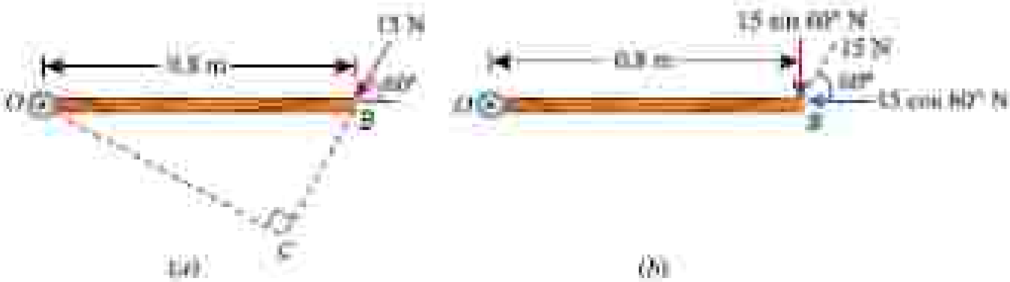
Moment when the force acts perpendicular to the door

We know that the moment of the force about the hinge,

$$Moment = P \times l = 15 \times 0.8 = \mathbf{12.0\ N-m}$$

Moment when the force acts at an angle of 60° to the door

This part of the example may be solved either by finding out the perpendicular distance between the hinge and the line of action of the force as shown in Fig. (a) or by finding out the vertical component of the force as shown in Fig. (b).



From the geometry of Fig. (a), we find that the perpendicular distance between the line of action of the force and hinge,

$$OC = OB \sin 60^\circ = 0.8 \times 0.866 = 0.693\ m$$

$$\therefore Moment = 15 \times 0.693 = \mathbf{10.4\ N-m}$$

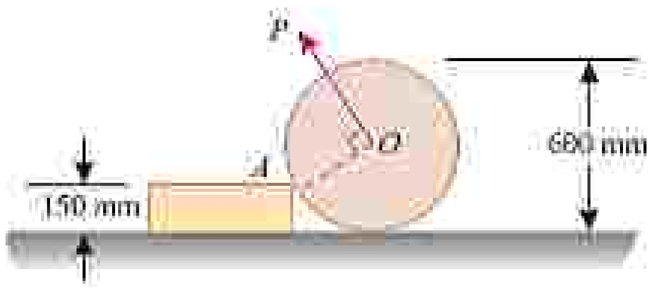
In the second case, we know that the vertical component of the force

$$= 15 \sin 60^\circ = 15 \times 0.866 = 13.0\ N$$

$$\therefore Moment = 13 \times 0.8 = \mathbf{10.4\ N-m}$$

Note. Since distance between the horizontal component of force (15 cos 60°) and the hinge is zero, therefore moment of horizontal component of the force about the hinge is also zero.

Example – 10: ***A uniform wheel of 600 mm diameter, weighing 5 kN rests against a rigid rectangular block of 150 mm height as shown in Figure. Find the least pull, through the centre of the wheel, required just to turn the wheel over the corner A of the block. Also find the reaction on the block. Take all the surfaces to be smooth.***



Solution. Given: Diameter of wheel = 600 mm; Weight of wheel = 5 kN and height of the block = 150 mm.

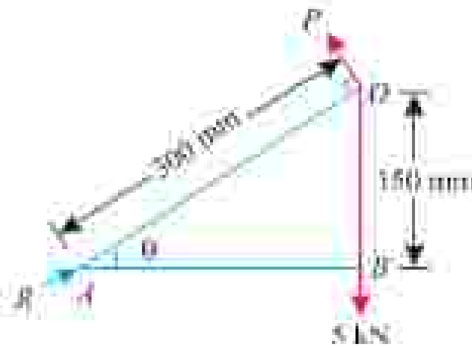
Least pull required just to turn the wheel over the corner.

Let P = Least pull required just to turn the wheel in kN.

A little consideration will show that for the least pull, it must be applied normal to AO. The system of forces is shown in Figure. From the geometry of the figure, we find that

$$\sin\theta = \frac{150}{300} = 0.5 \text{ or } \theta = 30^0$$

$$AB = \sqrt{(300)^2 - (150)^2} = 260\ mm$$



Now taking moments about A and equating the same,

$$P \times 300 = 5 \times 260 = 1300$$

$$\Rightarrow P = \frac{1300}{300} = 4.33\ kN$$

Reaction on the block

Let R = Reaction on the block in kN.

Resolving the forces horizontally and equating the same,

$$R\cos 30^0 = P\sin 30^0$$

$$\Rightarrow R = \frac{P\sin 30^0}{\cos 30^0} = \frac{4.33 \times 0.5}{0.866} = 2.5\ kN$$

Couple:

- **Definition**: A couple is a pair of two equal and unlike parallel forces acting on a body in such a way that the lines of action of the two forces are not in the same straight line.
- As a matter of fact, a couple is unable to produce any translatory motion (i.e., motion in a straight line). But it produces a motion of rotation in the body, on which it acts.
- The simplest example of a couple is the forces applied to the key of a lock, while locking or unlocking it.

Arm of a Couple:

- The perpendicular distance (a), between the lines of action of the two equal and opposite parallel forces, is known as arm of the couple as shown in Fig. 1.28.

Fig 1.28

Moment of a Couple:

- The moment of a couple is the product of the force (i.e., one of the forces of the two equal and opposite parallel forces) and the arm of the couple.
- Mathematically:

$$\textbf{Moment of a couple} = P \times a$$

$$Where\ P = Magnitude\ of\ the\ force.\ and\ a = Arm\ of\ the\ couple$$

Units of Couple:

- The SI unit of couple will be Newton-meter (briefly written as N-m). Similarly, the units of couple may also be kN-m (i.e., kN × m), N-mm (i.e., N × mm) etc.

Classification of couples:

The couples may be, broadly, classified into the following two categories, depending upon their direction, in which the couple tends to rotate the body, on which it acts:
1. Clockwise couple; and 2. Anticlockwise couple.

Clockwise couple:

- A couple, whose tendency is to rotate the body, on which it acts, in a clockwise direction, is known as a clockwise couple as shown in Fig. 1.29 (a). Such a couple is also called positive couple.

Anticlockwise couple:

- A couple, whose tendency is to rotate the body, on which it acts, in an anticlockwise direction, is known as an anticlockwise couple as shown in Fig. 1.29 (b). Such a couple is also called a negative couple.

Fig 1.29

Properties of a Couple:

A couple (whether clockwise or anticlockwise) has the following characteristics:

- The algebraic sum of the forces, constituting the couple, is zero.
- The algebraic sum of the moments of the forces, constituting the couple, about any point is the same, and equal to the moment of the couple itself.
- A couple cannot be balanced by a single force. But it can be balanced only by a couple of opposite sense.
- Any no. of coplanar couples can be reduced to a single couple, whose magnitude will be equal to the algebraic sum of the moments of all the couples.

POSSIBLE SHORT TYPE QUESTIONS WITH ANSWER

1. Define force & state its unit in S.I. system. (W – 2016 & 2017)

Ans. The force may be broadly defined as an agent which produces or tends to produce, destroys or tends to destroy motion. S.I. Unit: Newton (N)

2. Define a rigid body. (S – 2019 Old)

Ans. A body is said to be rigid if it does not undergo deformation whatever force may be applied to the body. In actual practice, there is no body which can be said to be rigid in true sense of terms.

3. What is free body diagram? (S – 2019 New)

Ans. The representation of reaction force on the body by removing all the support or forces act from the body is called free body diagram.

4. What do you mean by concurrent forces? (W – 2017, S – 2019 Old)

Ans. The forces, whose lines of action pass through a common point, are known as concurrent forces. The concurrent forces may or may not be collinear.

5. What do you mean by coplanar forces? (S – 2018)

Ans. The forces, whose lines of action lie on the same plane, are known as coplanar forces.

6. State parallelogram law of forces. (W – 2016)

Ans. It states, "If two forces, acting simultaneously on a particle, be represented in magnitude and direction by the two adjacent sides of a parallelogram; their resultant may be represented in magnitude and direction by the diagonal of the parallelogram, which passes through their point of intersection."

7. State polygon law of forces. (S – 2018 & 2019)
Ans. It is an extension of Triangle Law of Forces for more than two forces, which states, "If a number of forces acting simultaneously on a particle, be represented in magnitude and direction, by the sides of a polygon taken in order then the resultant of all these forces may be represented, in magnitude and direction, by the closing side of the polygon, taken in opposite order."

8. Define moment of a force & state its unit in S.I. (W – 2016)
Ans. It is the turning effect produced by a force, on the body, on which it acts. The moment of a force is equal to the product of the force and the perpendicular distance of the point, about which the moment is required and the line of action of the force.
S.I. Unit: Newton meter (N-m), kilo Newton meter (kN-m), N-mm

9. State varignon's theorem. (W – 2017)
Ans. Varignon's theorem states that the algebraic sum of the moments of two forces about any point in their plane is equal to the moment of their resultant about the same point.

10. Define couple. (W – 2017 & S – 2019 Old)
Ans. A couple is a pair of two equal and unlike parallel forces acting on a body in such a way that the lines of action of the two forces are not in the same straight line.

POSSIBLE LONG TYPE QUESTIONS

1. Two forces act at an angle of 120°. The bigger force is of 40 N and the resultant is perpendicular to the smaller one. Find the smaller force. **(W – 2016)**
Hints: refer page - 13, example-2

2. Explain principles of transmissibility & superposition. **(W – 2016)**
Hints: refer page - 07

3. The resultant of two forces ***P*** and 15 N is 20 N inclined at 60° to the 15 N force, find the magnitude and direction of ***P***. **(W – 2017)**

4. Find the angle between two equal forces of magnitude P, when their resultant is (i) P and (ii) P/2. **(S – 2018)**

5. A uniform wheel of 600 mm diameter, weighing 5 kN rests against a rigid rectangular block of 150 mm height as shown in Fig. below. Find the least pull force through the center of the wheel required just to turn the wheel over the corner A of the block. (Assume surfaces to be smooth) **(W – 2016)**

Hints: refer page – 24, example – 10

6. State and explain varignon's theorem. **(W – 2016 & S – 2019 Old)**
Hints: refer page – 22 & 23

7. A uniform rod AB of weight 75 N and 3 m long is simply supported at its ends. Downward forces of 25 N and 60 N are acting at a distance of 0.5 m and 1.2 m from the end A. Find the reactions at A and B. **(S – 2018)**

8. ABCD is a rectangle, in which AB = CD = 150 mm and BC = DA = 75 mm. Forces of 320 N each act

along AB and CD and forces of 150 N each act along BC and DA. Find the resultant moment of the two couples. **(S – 2018)**

9. A particle is acted on by three forces 2, $2\sqrt{2}$, and 1 kN. The first force is horizontal and towards the right; the second acts at 45° to the horizontal and inclined right upwards and the third is vertical. Determine the resultant of the given forces. **(S – 2019 New)**

10. What do you mean by force? Mention four effects of a force. **(S – 2019 Old)**

11. State and explain triangle law of forces. **(S – 2019 Old)**

CHAPTER NO. – 02

EQUILIBRIUM

Equilibrium and Equilibrant

Definition:

- If the resultant of a number of forces, acting on a particle is zero, the particle will be in equilibrium. Such a set of forces, whose resultant is zero, are called equilibrium forces.
- The force, which brings the set of forces in equilibrium, is called an equilibrant.
- As a matter of fact, the equilibrant is equal to the resultant force in magnitude, but opposite in direction.

Principles Of Equilibrium:

- Though there are many principles of equilibrium, yet the following three are important from the subject point of view.
  1. **Two force principle:** As per this principle, if a body in equilibrium is acted upon by two forces, then they must be equal, opposite and collinear.
  2. **Three force principle:** As per this principle, if a body in equilibrium is acted upon by three forces, then the resultant of any two forces must be equal, opposite and collinear with the third force.
  3. **Four force principle:** As per this principle, if a body in equilibrium is acted upon by four forces, then the resultant of any two forces must be equal, opposite and collinear with the resultant of the other two forces.

Conditions Of Equilibrium:

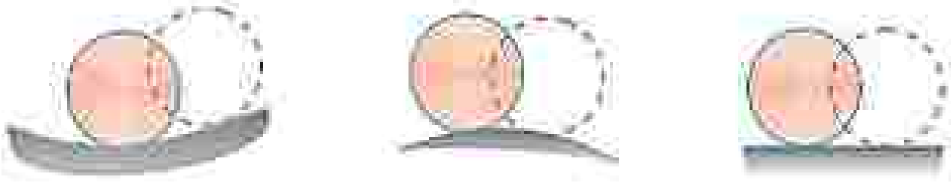
Analytical conditions of equilibrium for concurrent, non-concurrent forces:

- The algebric sum of horizontal components of the forces must be zero. i.e., $\sum H = 0$
- The algebric sum of vertical components of the forces must be zero. i.e., $\sum V = 0$
- The algebric sum of moment of forces about any point in their plane is equal to zero. i.e., $\sum M = 0$

Graphical conditions of equilibrium for concurrent, non-concurrent forces:

1. Stable equilibrium:

- A body is said to be in stable equilibrium, if it returns back to its original position, after it is slightly displaced from its position of rest. This happens when some additional force sets up due to displacement and brings the body back to its original position. A smooth cylinder, lying in a curved surface, is in stable equilibrium. If we slightly displace the cylinder from its position of rest (as shown by dotted lines), it will tend to return back to its original position in order to bring its weight normal to horizontal axis as shown in Fig. (a).



(a) Stable (b) Unstable (c) Neutral

2. Unstable equilibrium:

- A body is said to be in an unstable equilibrium, if it does not return back to its original position, and heels farther away, after slightly displaced from its position of rest. This happens when the additional force moves the body away from its position of rest. This happens when the additional force moves the body away from its position of rest. A smooth cylinder lying on a convex surface is in unstable equilibrium. If we slightly displace the cylinder from its position of rest (as shown by dotted lines) the body will tend to move away from its original position as shown in Fig. (b).

3. Neutral equilibrium:

- A body is said to be in a neutral equilibrium, if it occupies a new position (and remains at rest in this position) after slightly displaced from its position of rest. This happens when no additional force sets up due to the displacement. A smooth cylinder lying on a horizontal plane is in neutral equilibrium as shown in Fig. (c).

Free Body Diagram:

- Free body diagram of a body is the diagram drawn by showing all the external forces and reactions on the body and by removing the contact surfaces.
- Steps to be followed in drawing a free body diagram.
  1. Isolate the body from all other bodies.
  2. Indicate the external forces on the free body. (The weight of the body should also be included. It should be applied at the Centre of gravity of the body)
  3. The magnitude and direction of the known external forces should be mentioned.
  4. The reactions exerted by the supports on the body should be clearly indicated.
  5. Clearly mark the dimensions in the free body diagram.

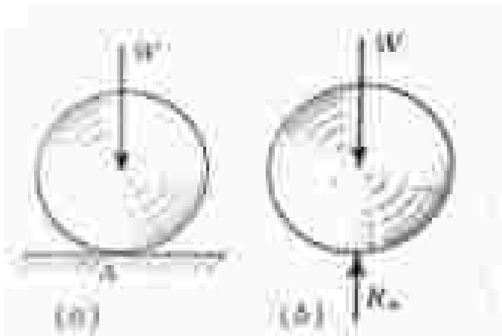


Figure 2.1 (a)

A spherical ball is rested upon a surface as shown in figure 2.1 (a). By following the necessary steps, we can draw the free body diagram for this force system as shown in figure 2.1(b). Similarly, fig 2.2 (b) represents free body diagram for the force system shown in figure 2.2(a).

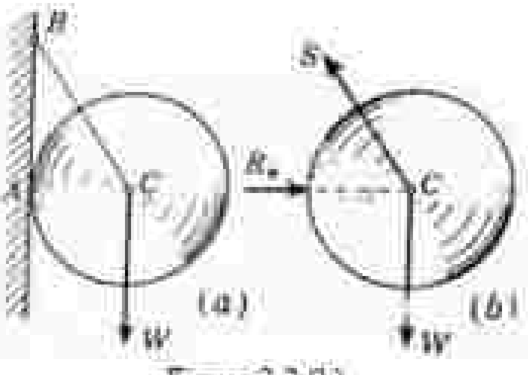


Figure 2.2 (b)

Lami's Theorem:

- It states, ***"If three coplanar forces acting at a point be in equilibrium, then each force is proportional to the sine of the angle between the other two."***
- Mathematically,

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

Where, P, Q, and R are three forces and α, β, γ are the angles as shown in Figure.

Proof:

- Consider three coplanar forces P, Q, and R acting at a point O. Let the opposite angles to three forces be α, β and γ as shown in Figure. Now let us complete the parallelogram OACB with OA and OB as adjacent sides as shown in the figure. We know that the resultant of two forces P and Q will be given by the diagonal OC both in magnitude and direction of the parallelogram OACB. Since these forces are in equilibrium, therefore the resultant of the forces P and Q must be in line with OD and equal to R, but in opposite direction.

From the geometry of the figure, we find

$BC = P$ and $AC = Q$

$\therefore \angle AOC = (180° - \beta)$

and $\angle ACO = \angle BOC = (180° - \alpha)$

$$\begin{aligned}\therefore \angle CAO &= 180° - (\angle AOC + \angle ACO)\\ &= 180° - [(180° - \beta) + (180° - \alpha)]\\ &= 180° - 180° + \beta - 180° + \alpha\\ &= \alpha + \beta - 180°\end{aligned}$$

But $\alpha + \beta + \gamma = 360°$

Subtracting 180° from both sides of the above equation,

$(\alpha + \beta - 180°) + \gamma = 360° - 180° = 180°$

or $\angle CAO = 180° - \gamma$

We know that in triangle AOC,

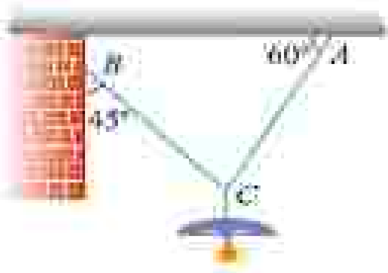
$$\frac{OA}{\sin\angle ACO} = \frac{AC}{\sin\angle AOC} = \frac{OC}{\sin\angle CAO}$$

$$\Rightarrow \frac{OA}{\sin(180° - \alpha)} = \frac{AC}{\sin(180° - \beta)} = \frac{OC}{\sin(180° - \gamma)}$$

$$\Rightarrow \frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

$\ldots[\because \sin(180° - \theta) = \sin\theta]$

Example – 1: ***An electric light fixture weighting 15 N hangs from a point C, by two strings AC and BC. The string AC is inclined at 60° to the horizontal and BC at 45° to the horizontal as shown in Figure. Using Lami's theorem, or otherwise, determine the forces in the strings AC and BC.***



Solution. Given: Weight at C = 15 N

Let T_{AC} = Force in the string AC, and

T_{BC} = Force in the string BC.
The system of forces is shown in Figure. From the geometry of the figure, we find that angle between T_{AC} and 15 N is 150° and angle between T_{BC} and 15 N is 135°.

$$\therefore \angle ACB = 180° - (45° + 60°) = 75°$$

Applying Lami's equation at C,

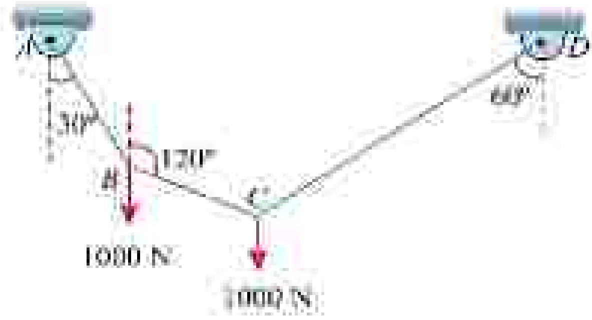
$$\frac{15}{\sin 75^0} = \frac{T_{AC}}{\sin 135^0} = \frac{T_{BC}}{\sin 150^0}$$

$$\Rightarrow \frac{15}{\sin 75^0} = \frac{T_{AC}}{\sin 45^0} = \frac{T_{BC}}{\sin 30^0}$$

$$\Rightarrow T_{AC} = \frac{15 \sin 45^0}{\sin 75^0} = \frac{15 \times 0.707}{0.9659} = \mathbf{10.98\ N}$$

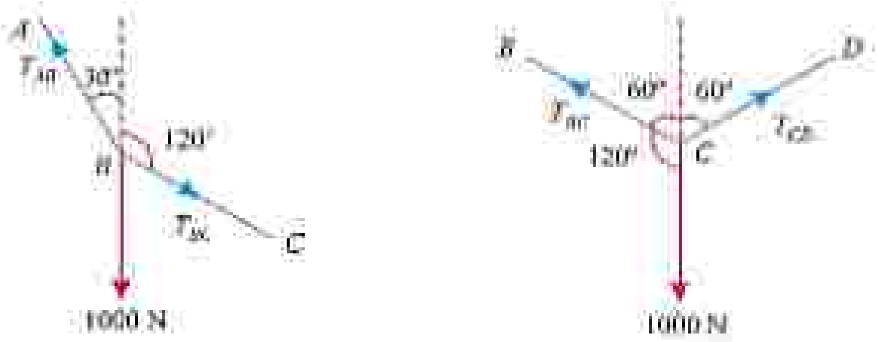
$$\Rightarrow T_{BC} = \frac{15 \sin 30^0}{\sin 75^0} = \frac{15 \times 0.5}{0.9659} = \mathbf{7.76\ N}$$

Example – 2: ***A string ABCD, attached to fixed points A and D has two equal weights of 1000 N attached to it at B and C. The weights rest with the portions AB and CD inclined at angles as shown in Figure. Find the tensions in the portions AB, BC and CD of the string, if the inclination of the portion BC with the vertical is 120°.***



Solution. Given: Load at B = Load at C = 1000 N For the sake of convenience, let us split up the string ABCD into two parts. The system of forces at joints B and is shown in Fig. (a) and (b).

Let T_{AB} = Tension in the portion AB of the string,
T_{BC} = Tension in the portion BC of the string, and
T_{CD} = Tension in the portion CD of the string.



(a) Joint B (b) Joint C

Applying Lami's equation at joint B,

$$\frac{T_{AB}}{\sin 60^0} = \frac{T_{BC}}{\sin 150^0} = \frac{1000}{\sin 150^0}$$

$$\Rightarrow \frac{T_{AB}}{\sin 60^0} = \frac{T_{BC}}{\sin 30^0} = \frac{1000}{\sin 30^0}$$

$$\Rightarrow T_{AB} = \frac{1000 \sin 60^0}{\sin 30^0} = \frac{1000 \times 0.866}{0.5} = \mathbf{1732\ N}$$

$$\Rightarrow T_{BC} = \frac{1000 \sin 30^0}{\sin 30^0} = \mathbf{1000\ N}$$

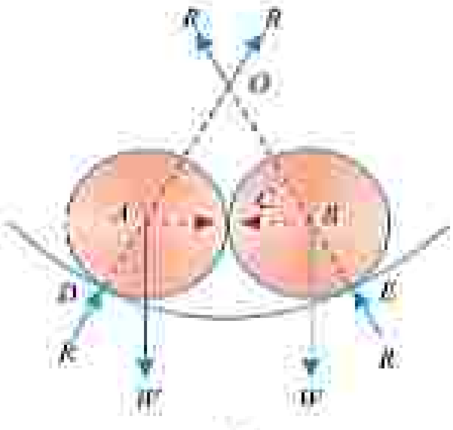
Applying Lami's equation at joint C,

$$\frac{T_{CD}}{\sin 120^0} = \frac{T_{BC}}{\sin 120^0} = \frac{1000}{\sin 120^0}$$

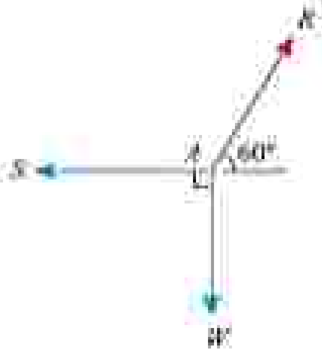
$$\Rightarrow T_{CD} = \mathbf{1000\ N}$$

Example – 3: ***Two equal heavy spheres of 50 mm radius are in equilibrium within a smooth cup of 150 mm radius. Show that the reaction between the cup of one sphere is double than that between the two spheres.***

Solution. Given: Radius of spheres = 50 mm and radius of the cup = 150 mm.



(a)



(b)

The two spheres with centres A and B, lying in equilibrium, in the cup with O as centre are shown in Fig.(a). Let the two spheres touch each other at C and touch the cup at D and E respectively.

Let R = Reactions between the spheres and cup; and
S = Reaction between the two spheres at C.

From the geometry of the figure, we find that OD = 150 mm and AD = 50 mm. Therefore OA = 100 mm. Similarly, OB = 100 mm. We also find that AB = 100 mm. Therefore, OAB is an equilateral triangle. The system of forces at A is shown in Figure (b).

Applying Lami's equation at A,

$$\frac{R}{\sin 90^0} = \frac{W}{\sin 120^0} = \frac{S}{\sin 150^0}$$

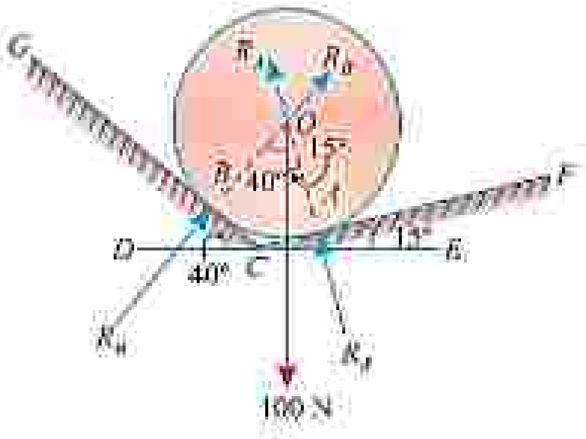
$$\Rightarrow \frac{R}{1} = \frac{W}{\sin 60^0} = \frac{S}{\sin 30^0}$$

$$\Rightarrow R = \frac{S}{\sin 30^0} = \frac{S}{0.5} = 2S$$

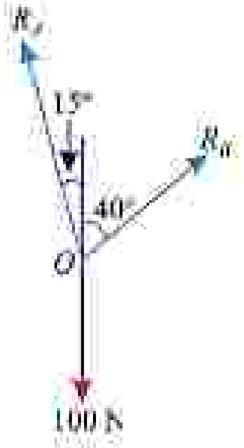
Hence the reaction between the cup and the sphere is double than that between the two spheres.

Example – 4: ***A smooth circular cylinder of radius 1.5 meter is lying in a triangular groove, one side of which makes 15° angle and the other 40° angle with the horizontal. Find the reactions at the surfaces of contact, if there is no friction and the cylinder weighes 100 N.***

Solution. Given: Weight of cylinder = 100 N



(a)



(b)

Let R_A = Reaction at A, and
R_B = Reaction at B.

The smooth cylinder lying in the groove is shown in Fig. (a). In order to keep the system in equilibrium, three forces i.e., R_A, R_B and weight of cylinder (100 N) must pass through the centre of the cylinder. Moreover, as there is no friction, the reactions R_A and R_B must be normal to the surfaces as shown in Fig (a). The system of forces is shown in Fig. (b).

Applying Lami's equation, at O,

$$\frac{R_A}{\sin(180^0 - 40^0)} = \frac{R_B}{\sin(180^0 - 15^0)} = \frac{100}{\sin(15^0 + 40^0)}$$

$$\Rightarrow \frac{R_A}{\sin 40^0} = \frac{R_B}{\sin 15^0} = \frac{100}{\sin 55^0}$$

$$\Rightarrow R_A = \frac{100 \times \sin 40^0}{\sin 55^0} = \frac{100 \times 0.6428}{0.8192} = \mathbf{78.5\ N}$$

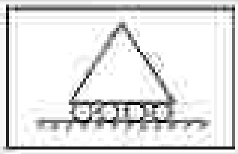
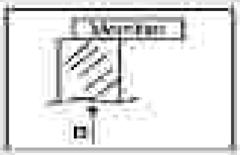
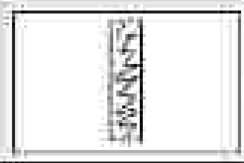
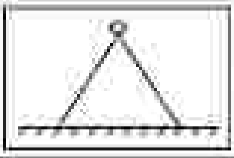
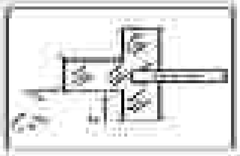
$$\Rightarrow R_B = \frac{100 \times \sin 15^0}{\sin 55^0} = \frac{100 \times 0.2588}{0.8192} = \mathbf{31.6\ N}$$

Types of supports, loading & beam

Beam is a structural element which is taken as specimen for studying the effects of loads, on the structure. It's carrying transverse load. The function of beam is to carry loads. It rest on supports which can offer reaction to keep system in equilibrium. Beams are classified according to their type of supports.

Types of supports

Structure or their components can be supported on different types of supports which can be classified depending upon the reaction offered by them as following.

Sr. No.	Name of Support	Description for reaction	Diagram with reaction	Symbol & Nos. of reaction
1	Roller	It provides the resistance to movement in the direction perpendicular to supporting surface. Ex. Skating roller		 (01)
2	Simple	It Supports without any type of joint or connection & hence reaction is always acting along the direction of support.		 (01)
3	Hinge	It provides resistance to movement in any direction by offering inclined reaction. Ex: Door hinge		 (02)
4	Fixed	It provides resistance to rotation & it effectively held in position & restrained against rotation. Ex. Nail in the wall		 (03)

Types of loading

Loads which act on structural components can be external or due to self-weight of body. These load act as forces on structure. Following are important types of loading. (A) Concentrated or Point load (B) Uniformly distributed load (C) Uniformly varying load (D) Moment (E) Couple.

(A) Concentrated or Point load [fig. 2.9 (a)]

Load concentrated on a very small length compared to length of beam is known as concentrated or point load. It is practically assumed to be acting through a point. Example of point load is car standing on ground. In this contact area of wheel on ground is very small and hence load on ground is a point load. A person standing on a beam is also an example of the point load.

(B) Uniformly distributed load (UDL) [fig. 2.9(b)]

Load uniformly spread over the length of a beam is known as uniformly distributed load (UDL). In this type of loading, Weight of load per unit length is known as intensity of load; and is same along the length which denoted by w with units N/cm or kN/cm or N/m or KN/m. A truck loaded with sand of equal height & compound wall transferring load on the ground & person sleeping on bed are examples of UDL. For analysis, total load is taken as(w × l) acting as point load P at mid-point of length of UDL as equivalent value of UDL to find support reaction of beam as shown in fig. b(ii).

(a) Point Load (b) Uniformly distributed load

Fig. 2.9: Types of load

(C) Uniformly varying load (UVL) [fig. 2.9 (c)]

If the intensity of load is not same along the length but if uniformly increasing or decreasing from one end to another is known as uniformly varying load. If intensity increase from 0 to any value w1 at the other end, then UVL known as triangular load and if intensity increase or decrease from w1 value at one end to w2 value at other and then UVL known as trapezoidal load as shown in fig (c).

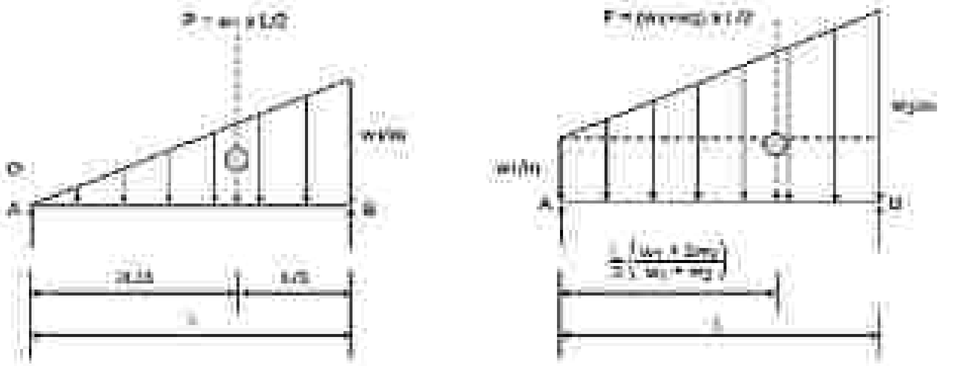
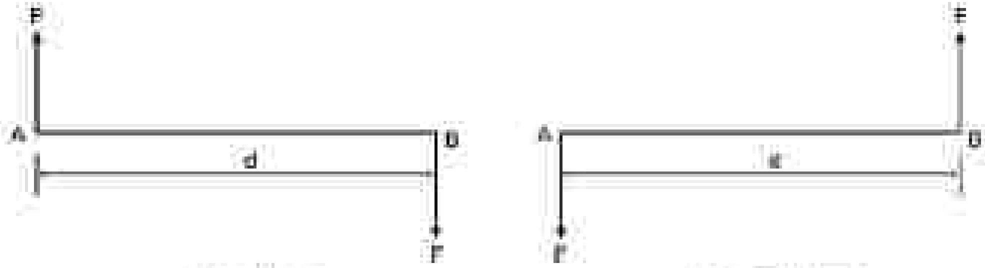


Fig. 2.9: (c) Types of Load - Uniformly Varying Load (UVL)

A truck loaded with sand with top surface as inclined is the Example of UVL. In this type of load, total load is to be acting at C.G. of load diagram & value is total area of load diagram as shown in fig. (c)

(D) Couple: A couple is defined as two parallel forces that have the same magnitude, opposite direction & are separated by a perpendicular distance d as shown in fig (d). The resultant force in this case will be zero but body will not be in equilibrium as these forces will tend to rotate the body. Hence, we can say that effect of couple is to produce a pure moment or tendency of rotation in specified direction. Examples are (i) to open or close a water tap; (ii) rotating steering wheel of vehicle (iii) to wind the spring of the clock. The plane in which the forces constituting the couple act is called plane of the couple & perpendicular distance between the lines of action of force constituting couple is called arm 'd' of the couple as shown in fig.(d). Moment of couple is multiplication of force F and arm d.



(i) Clockwise (ii) Anti-Clockwise

Fig. 2.9: (d) Types of Load - Couple

Type of couple: According to rotation of the body due to couple, it classified as clockwise couple & anticlockwise couple as shown in fig. (d) (i) & (ii) respectively.

Types of Beam

Beams are broadly classified in to two groups.

(A) Statically determinate beam & (B) Statically indeterminate beam.

Analysis of statically indeterminate beams is not in scope for you at this stage.

(A) Statically determinate beams

A beam is said to be statically determinate beam if the number of unknown reactions are not more than the number of equilibrium conditions. There are three equations from equilibrium condition which are (i) $\Sigma H = 0$ (ii) $\Sigma V = 0$ (iii) $\Sigma M = 0$. Hence according to types of supports of beam maximum three unknown reaction can be solved. Following are statically determinate beams.

(i) Simply supported beam: It is supported on two simple supports at each end of the beam. In this case supports offer only reaction force and not moment. Usually one support is hinge & other is roller or both support is simple support. Nos. of unknown supports reaction are not more than 3 in any case as shown in below fig.

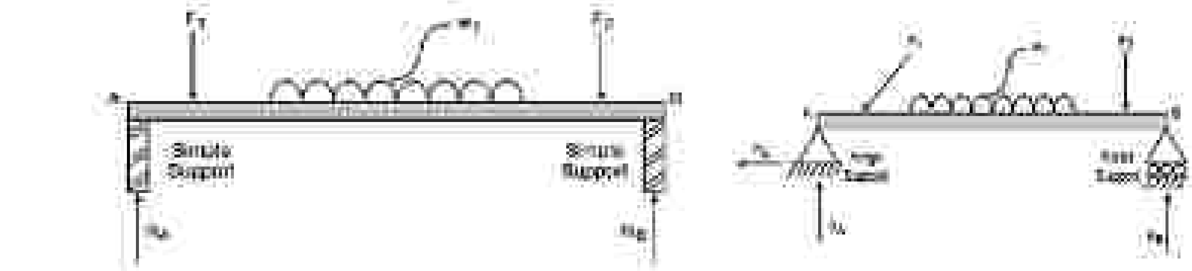
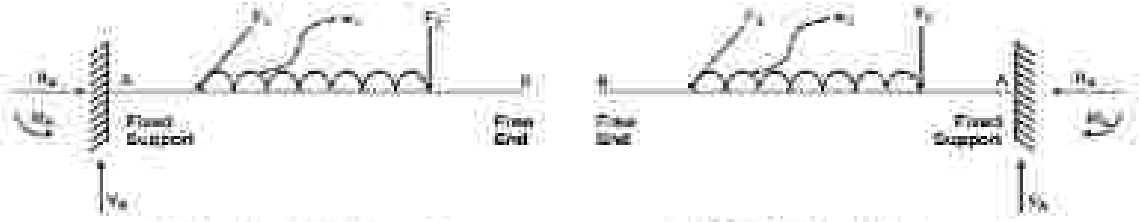


Fig. 2.10: Simply supported beam

(ii) Cantilever beam: In this beam, one end is fixed support and other end is free i.e., no support. In practice such beams are used when it is not possible to provide support at one end of the ends of the beam. Nos. of unknowns are not more than 3 in this beams as shown in below fig. Fixed support may be on left end or right end, as shown in fig. (a) & (b) respectively.



(a) Left hand end fixed support (b) Right hand end fixed support

Fig. 2.11: Cantilever beam

(iii) Overhang beam: If the one portion or two portions of the simply supported beam are extended beyond the support, then it's known as overhang beam. Depending upon the overhang, they are classified as single overhanging beam or double overhanging beam as shown in below fig. We can say that its special type of simply supported beam.

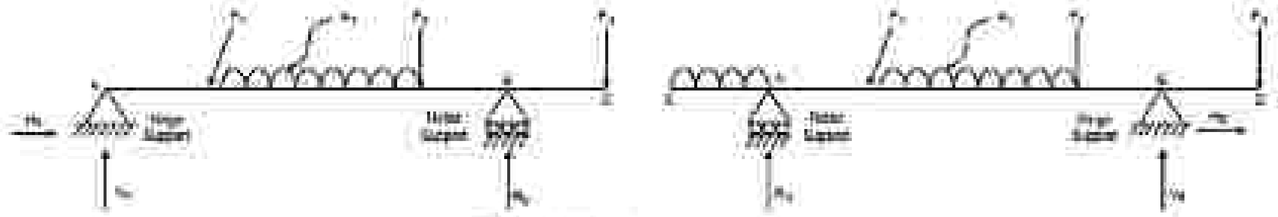
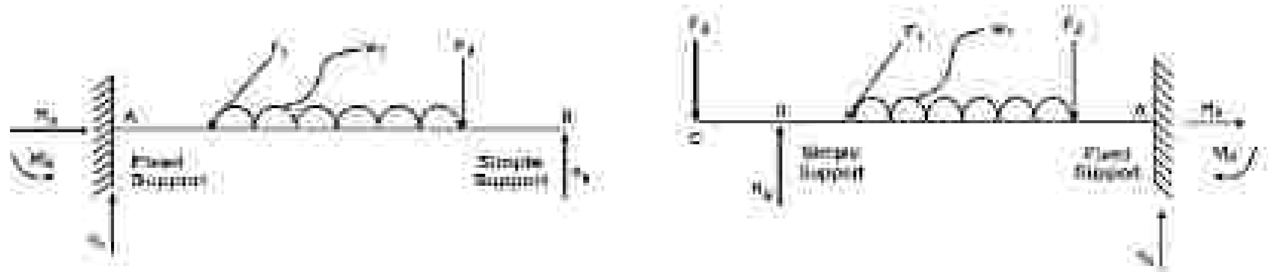


Fig. 2.12: Overhang beam

(B) Statically indeterminate beams

If nos. of unknown reaction is more than the equilibrium condition then such type of beam is known as statically indeterminate beam. Following are different statically indeterminate beams.

(i) Propped cantilever beam: In this beam one end is fixed support and other end is simple support with overhang or no overhang as shown in fig.



(a) Without overhang (b) With overhang

Fig. 2.13: Propped cantilever

(ii) Continuous beam: In this beam nos. of support are more than two as shown in below fig.

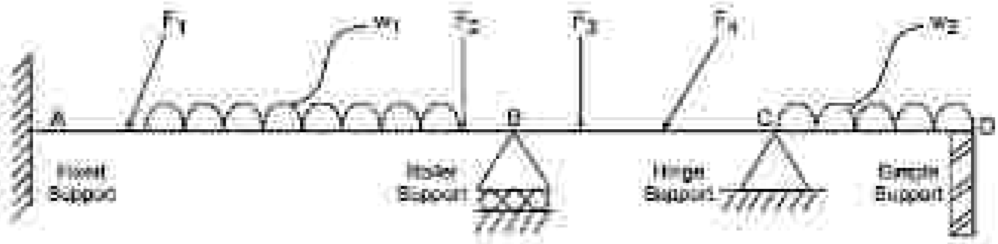


Fig. 2.14: Continuous beam

(iii) Fixed beam: In this beam both ends are with fixed support as shown in below fig.

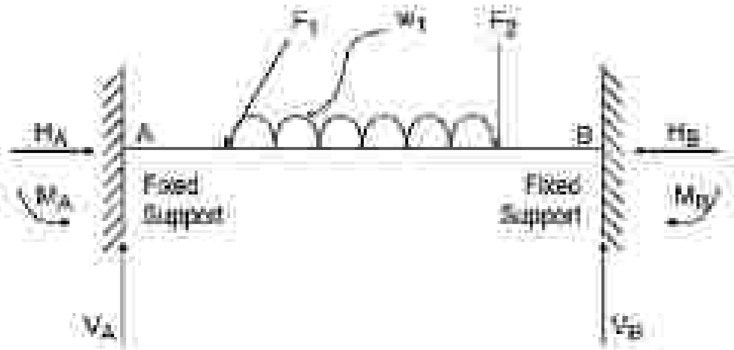


Fig. 2.15: Fixed beam

Beam reactions

Beam supports reaction can be finding by two methods. (I) Analytical method (II) Graphical Method. In analytical method, we have to use conditions of equilibrium to solve unknown support reaction as discuss in this topic. Graphical method will discuss in next topic 2.5. The beam we have to consider are: (A) Cantilever beam (B) Simply supported beam & (C) Overhang beam.

Beam reaction for cantilever beam

As we know the cantilever beam have one end fixed support and other end as free i.e., no support. We can find beam support reactions by using conditions of equilibrium by taking some examples.

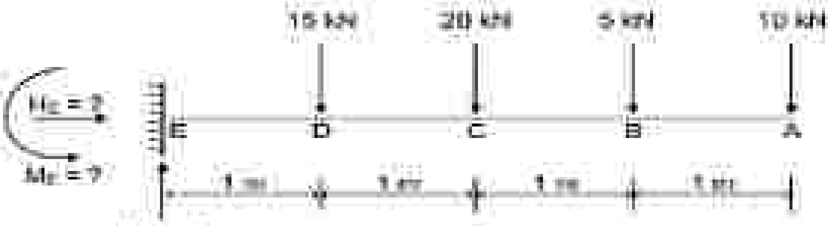


Fig. 2.16

Solution:

First draw the beam and loading on the beam as per given data as shown in fig.

(a) Using equilibrium condition $\Sigma V = 0$ with + ve sign for ↑ upward force. Assume vertical reaction at V_E as upward load.

$\therefore V_E - 15 - 20 - 5 - 10 = 0$

$\therefore V_E = 50$ kN ↑ (Answer)

(b) Using equilibrium condition $\Sigma H = 0$ with +ve sign for → eastward force. As no horizontal load is acting on beam, $H_E = 0$ kN → (Answer)

(c) Using equilibrium condition $\Sigma M = 0$ with +ve sign for clockwise. ↻

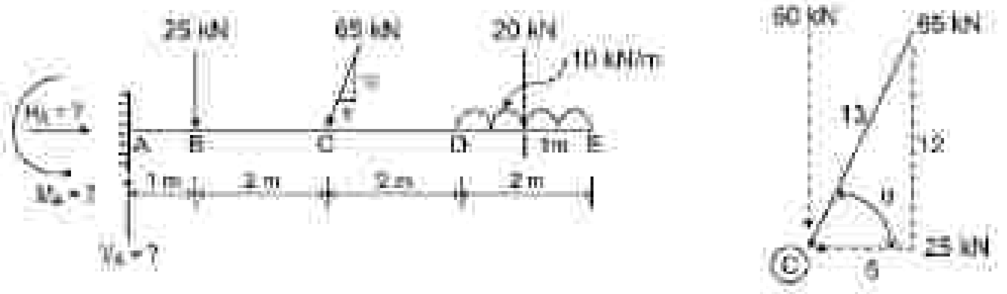
Consider moment at fixed support E, we get

$\Sigma M_E = (15 \times 1) + (20 \times 2) + (5 \times 3) + (10 \times 4) - M_E = 0$

$\therefore M_E = 15 + 40 + 15 + 40 = 110$ kN m ↺ anticlockwise (Answer)

Note : If calculation get +ve sign our assumed sign is perfect otherwise we have to revers it.

Example 5. Determine the support reaction of a cantilever beam as shown in fig.



(a) Data (b) components of inclined force

Fig. 2.17

Solution:

(i) First, we have to find horizontal and vertical components of inclined force 65 kN acting at point C as shown in fig. (b).

Here $\tan\theta = \frac{12}{5}$; $\sin\theta = \frac{12}{13}$ & $\cos\theta = \frac{5}{13}$

$\therefore$ Horizontal component at point C = $65 \times \cos\theta = 65 \times \frac{5}{13} = 25$ kN →

& Vertical component at point C = $65 \times \sin\theta = 65 \times \frac{12}{13} = 60$ kN ↓

(ii) Also find equivalent load for UDL on DE portion of the beam.

(I) Total load as point load = P = w × l = 10 ×2 = 20 kN ↓

(II) Point of application of point load is at mid-point of DE i.e. 1m from point E as shown by dotted line in fig.(a)

Now applying three equilibrium conditions one by one, to get reactions.

(a) ΣV = 0 with + ve sign as ↑ upward and assuming V_A as upward

$\therefore V_A - 25 - 60 - (10 \times 2) = 0$

$\therefore V_A = 25 + 60 + 20 = 105$ kN ↑ (Answer)

(b) Σ H = 0 with + ve sign as → eastward and assuming HA as eastward.

$\therefore H_A - 25 = 0$

$\therefore H_A = 25$ kN → (Answer)

(c) Σ M_A = 0 with + ve sign as ↻ clockwise and assuming M_A as anticlockwise.

Note : Horizontal component of 60 kN force will pass from point A, hence moment due to this force will be zero.

$\therefore 25 \times 1 + 60 \times 3 + 25 \times 0 + (10 \times 2) \times 6 - M_A = 0$

$\therefore M_A = 25 + 180 + 0 + 120 = 325$ kN m ↺ anticlockwise **(Answer)**

Example 6. Determine the support reaction of cantilever beam as shown in fig.

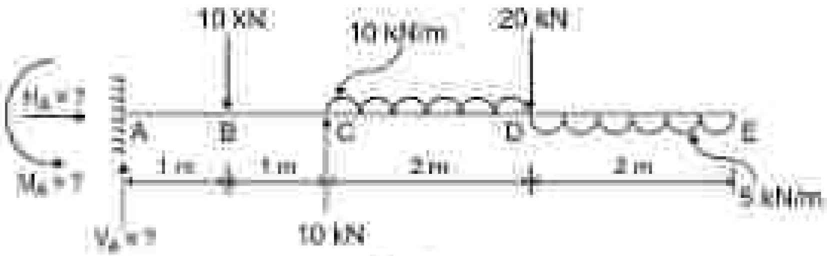


Fig. 2.18

Solution:

Applying three equilibrium conditions one by one to get reaction at support.

(a) $\Sigma H = 0$. As no Horizontal force acting on beam, $H_A = 0$. **(Answer)**

(b) $\Sigma V = 0$ with + ve sign as ↑ upward and assuming V_A as upward.

$\therefore V_A - 10 - 10 - (10 \times 2) - 20 + (5 \times 2) = 0$

$\therefore V_A = 10 - 10 + 20 + 20 - 10 = 30$ kN ↑ (Answer)

(c) $\Sigma M_A = 0$ with + ve sign as ↻ clockwise and assuming M_A as anticlockwise.

$\therefore (10 \times 1) - (20 \times 2) + ([10 \times 2] \times 3) + (20 \times 4) - ([5 \times 2] \times 5) - M_A = 0$

$\therefore M_A = 10 - 40 + 60 + 80 - 50 = 60$ kN-m anticlockwise ↺ **(Answer)**

<u>Beam reaction for simply supported beam:</u>

As we know the simply supported beam have simple support at both ends or one end with roller support and other end with hinge support. We have also studied the support reaction for each support. By using equilibrium condition, we get unknown support reaction. Let to explain above point, take some examples.

Example 7. A simply supported beam of span 10m carries three points loads of 40 kN, 30 kN and 20 kN from left hinge support at the distance 2 m, 5 m and 8 m respectively in downward direction. The right-hand

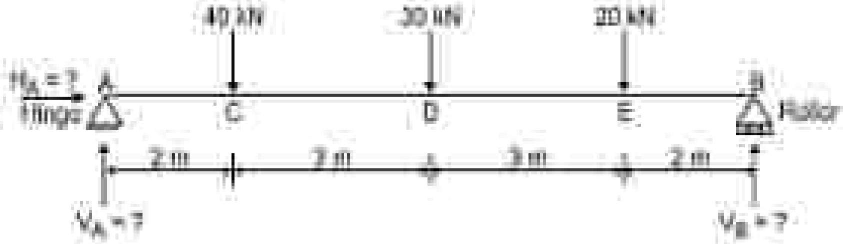


Fig. 2.19

Solution:

First draw space diagram of the beam from given data as shown in fig. Now applying three equilibrium condition for the beam.

(a) $\Sigma H = 0$ since there is no horizontal load on the beam. $\therefore H_A = 0$ kN **(Answer)**

(b) $\Sigma V = 0$ with + ve sign ↑ upward and assume V_A and V_B both upward.

$\therefore V_A - 40 - 30 - 20 + V_B = 0.$

$\therefore V_A + V_B = 90$ kN. We have to use this equation as check point of our calculation.

(c) $\Sigma M = 0$ with + ve sign as ↻ clockwise moment.

(i) Consider moment at support point A.

$\Sigma M_A = (40 \times 2) + (30 \times 5) + (20 \times 8) - (V_B \times 10) + (V_A \times 0) = 0$

$\therefore 10\ V_B = 80 + 150 + 160 = 390$

$\therefore V_B = 390\ 10 = 39$ kN ↑ **(Answer)**

(ii) Consider moment at other support point B.

$\Sigma M_B = (V_A \times 10) - (40 \times 8) - (30 \times 5) - (20 \times 2) + (V_B \times 0) = 0$

$\therefore 10 V_A = 320 + 150 + 40 = 510$

$\therefore V_A = 510\ 10 = 51$ kN ↑ **(Answer)**

(d) Now we can check our calculation for perfectness in equation obtain in (b). If it fulfills, our calculation has no error.

$\therefore V_A + V_B = 90$ put values of V_A and V_B, we get

LHS = 51 + 39 = 90 = RHS. **OK.** Means our answer is perfect.

If not satisfied in any case, you have done some mistake in calculation, recalculate it until it's satisfied the check point equation.

Example 8. Find the reaction for the beam shown below.

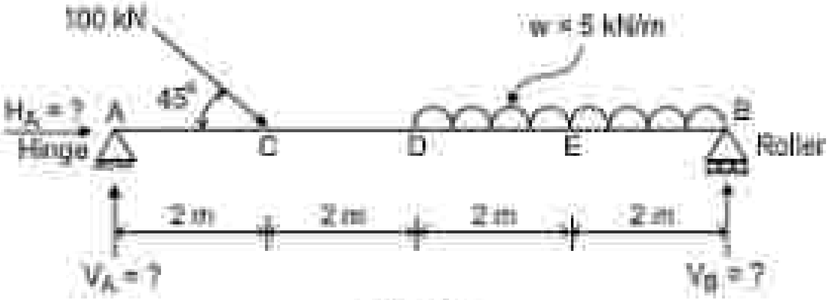


Fig. 2.30

Solution:

First, we have to find horizontal & vertical components of inclined force 100 kN acting at point C at angle 45° with beam alignment.

∴ Horizontal components of force 100 kN at C = 100 × cos 45° = 70.71 kN → eastward

& Vertical components of force 100 kN at C = 100 × sin 45° = 70.71 kN ↓ downward

Secondly, we have to find equivalent load for UDL as point load at mid-point of DB i.e., acting at point E as shown in fig. with dotted line.

Equivalent point load of UDL = P = (5 × 4) = 20 kN at point E.

Now applying conditions of equilibrium for the given beam.

(a) Σ H = 0 with + ve sign as → eastward and assuming H_A as eastward.

$\therefore H_A + 70.71 = 0$

∴ H_A = −70.71 kN ← westward. As we have get – Ve value, we have to reverse the assumed direction. **(Answer)**

(b) Σ V = 0 with + ve sign ↑ upward and assume V_A and V_B both upward.

$\therefore V_A - 70.71 - (5 \times 4) + V_B = 0$

∴ $V_A + V_B = 90.71$ kN. We have to use this equation as check point of our calculation.

(c) Σ M = 0 with + ve sign as ↻ clockwise moment.

(i) Consider moment at support point A.

$\Sigma M_A = V_A \times 0 + H_A \times 0 + 70.71 \times 2 + 70.71 \times 0 + (5 \times 4) \times 6 - V_B \times 8 = 0$

$\therefore 8\ V_B = 0 + 0 + 141.42 + 0 + 120 = 261.42$

$\therefore V_B = \frac{261.42}{8} = 32.68$ kN ↑ **(Answer)**

(ii) Consider moment at other support point B.

$\Sigma M_B = V_B \times 0 - (5 \times 4) \times 2 - 70.71 \times 6 + 70.71 \times 0 + H_A \times 0 + V_A \times 8 = 0$

$\therefore 8\ V_A = 0 + 40 + 424.26 - 0 - 0 = 464.26$

∴ V_A = 58.03 kN ↑ **(Answer)**

(d) Now we can get self-check of our calculation for perfectness in equation obtain in (b). If it fulfills, our calculation has no error.

$\therefore V_A + V_B = 90.71$ kN

Put values of V_A and V_B, we get

LHS = 58.03 + 32.68 = 90.71 kN = RHS. **OK.** Means our answer is perfect.

If not satisfied in any case, you have done some mistake in calculation, recalculate it until it' satisfied the check point equation.

Example 9.

A beam AB 10m long is hinge at left hand support A and supported on roller support surface inclined at 30° with horizontal at right hand support B. The beam is carrying load as shown in fig. Find the reactions at supports of the beam.

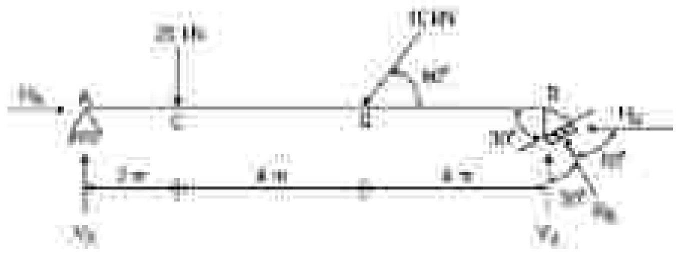


Fig. 2.21

Solution:

(i) In this case, the beam is supported on roller surface inclined at 30° to horizontal. Since roller support provides reaction perpendicular to the surface, the reaction R_B makes an angle of 60° with horizontal as shown in fig. Horizontal & vertical components of R_B, can found as

$\therefore H_B = R_B \times \cos 60° = 0.5\, R_B \leftarrow$ westward

& $V_B = R_B \times \sin 60° = 0.866\, R_B \uparrow$ upward

(ii) We have to find horizontal & vertical components of inclined force 10 kN acting at point D at angle 60° with beam alignment.

Horizontal components of force 10 kN at D = $10 \times \cos 60° = 5.0$ kN ← westward

&Vertical components of force 10 kN at D = $10 \times \sin 60° = 8.66$ kN ↓ downward

Now applying conditions of equilibrium for the given beam.

(a) $\Sigma H = 0$ with +ve sign as → eastward and assuming H_A as eastward.

$\therefore H_A - 5.0 - 0.5R_B = 0$

$\therefore H_A - 0.5\, R_B = 5.0$ kN

(b) $\Sigma V = 0$ with +ve sign ↑ upward and assume V_A and R_B both upward.

$\therefore V_A - 20 - 8.66 + 0.866\, R_B = 0$

$\therefore V_A + 0.866\, R_B = 28.66$ kN

(c) $\Sigma M = 0$ with +ve sign as ↻ clockwise moment & Consider moment at support point A.

$\Sigma M_A = V_A \times 0 + H_A \times 0 + 20 \times 2 + 8.66 \times 6 + 5 \times 0 - V_B \times 10 + H_B \times 0 = 0$

$\therefore 0 + 0 + 40 + 51.96 + 0 - 10 \times 0.866\, R_B + 0 = 0$ (As $V_B = 0.866\, R_B$)

$\therefore 8.66\, R_B = 91.96$

$\therefore R_B = \frac{91.96}{8.66} = 10.62$kN ↖ **(Answer)**

(d) Put value of R_B in equation of (a), we get H_A

$H_A - 0.5\, R_B = 5.0$

$\therefore H_A = 5.0 + 0.5 \times 10.62 = 10.31$ kN → eastward **(Answer)**

(e) Put value of R_B in equation of (b), we get V_A

$V_A + 0.866\, R_B = 28.66$

$\therefore V_A = 28.66 - 0.866 \times 10.62 = 19.46$ kN ↑ upward **(Answer)**

Beam reaction for simply supported with overhang:

Advance Beam-II As we know that if the one or two portions of the simply supported beam are extended beyond the support, then it's known as overhang beam. Let us take some examples.

Example 10.

Calculate the reactions at supports of the beam shown in figure.

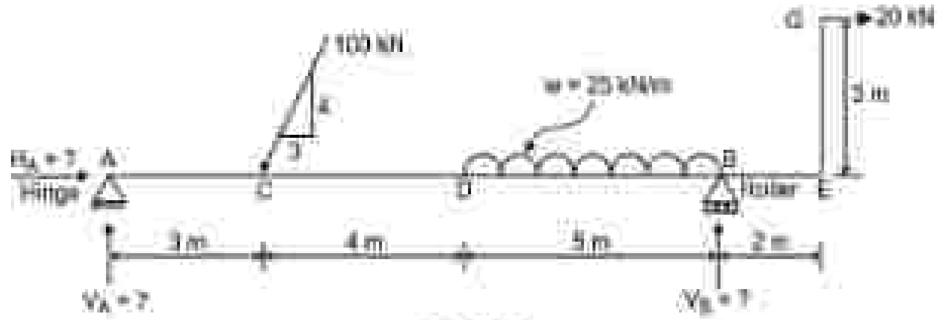


Fig. 2.22

Solution:

(i) We have to find horizontal & vertical components of inclined force 100 kN acting at point C.

$\therefore$ Horizontal components of force 100 kN at C = $\frac{100 \times 3}{5}$ = 60 kN ← westward

& Vertical components of force 100 kN at C = $\frac{100 \times 4}{5}$ = 80 kN ↓ downward

(ii) We have to find equivalent load for UDL as point load at mid-point of DB portion.
Equivalent point load of UDL = P = (25× 5) = 125 kN act at distance of 2.5 m from point D.

(iii) Here one horizontal load on bracket EG is act as 20 kN → eastward, which create also moment that should be consider while applying moment condition.

Now applying conditions of equilibrium for the given beam.

(a) Σ H = 0 with + ve sign as → eastward and assuming H_A as eastward.

$\therefore H_A - 60 + 20 = 0$

$\therefore H_A = 40$ kN ↑ eastward. **(Answer)**

(b) Σ V = 0 with + ve sign ↑ upward and assume V_A and V_B both upward.

$\therefore V_A - 80 - (25 \times 5) + V_B = 0$

$\therefore V_A + V_B = 205$ kN We have to use this equation as check point of our calculation.

(c) Σ M = 0 with + ve sign as ↻ clockwise moment.

(i) Consider moment at support point A, we get

$\Sigma M_A = V_A \times 0 + H_A \times 0 + 60 \times 0 + 80 \times 3 + 125 \times 9.5 - V_B \times 12 + 20 \times 3 = 0$

$\therefore 0 + 0 + 0 + 240 + 1187.50 - 12V_B + 60 = 0$

$\therefore 12\,V_B = 1247.5$

$\therefore V_B = 103.96$ kN ↑ upward **(Answer)**

(ii) Consider moment at other support point B.

$\Sigma M_B = 20 \times 3 + V_B \times 0 - 125 \times 2.5 - 80 \times 9 + 60 \times 0 + H_A \times 0 + V_A \times 12 = 0$

$\therefore 12\,V_A = -60 + 312.5 + 720 = 972.5$

$\therefore V_A = \frac{972.5}{12} = 81.04$ kN ↑ upward **(Answer)**

(c) Now we can get self-check of our calculation for perfectness in equation obtain in (b) as $V_A + V_B = 205$ kN. If it fulfills, our calculation has no error.

We have LHS as $V_A + V_B = 81.04 + 103.96 = 205$ kN = RHS. OK. Means our answer is perfect.

If not satisfied in any case, you have done some mistake in calculation, recalculate it until its satisfied the check point equation.

Example 11.

Find reaction at supports for a double hanging beam shown in below figure.

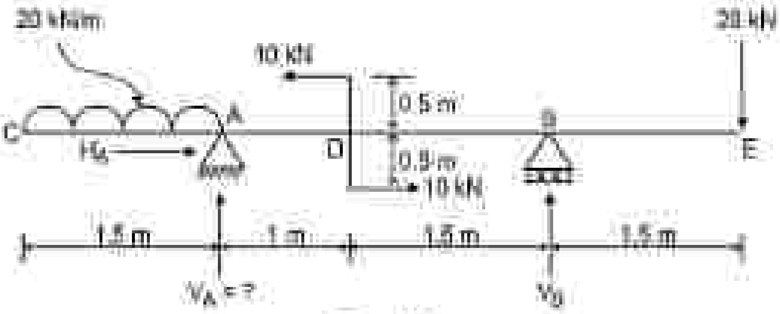


Fig. 2.23

Solution:

(i) As per loading, UDL on AC may have equivalent point load of P as (20 × 1.5) = 30 kN ↓ at mid-point of AC.

(ii) On beam CABE at point D, there are two equal & opposite force of 10 kN, resulting in to a couple of magnitude of 10 × (0.5 + 0.5) = 10 kN m ↺ anticlockwise with net horizontal force as zero.

Now applying conditions of equilibrium for the given beam.

(a) Σ H = 0 with + ve sign as → eastward and assuming H_A as eastward.

$\therefore\ H_A - 10 + 10 = 0$

$\therefore\ H_A = 0$ kN **(Answer)**

(b) Σ V = 0 with + ve sign ↑ upward and assume V_A and V_B both upward.

$\therefore\ V_A - (20 \times 1.5) - 20 + V_B = 0$

$\therefore\ V_A + V_B = 50$ kN. We have to use this equation as check point of our calculation.

(c) Σ M = 0 with + ve sign as ↻ clockwise moment.

(i) Consider moment at support point A, we get

$\Sigma M_A = 20 \times 4 - V_B \times 2.5 - 10 - V_A \times 0 + H_A \times 0 - (20 \times 1.5) \times 0.75 = 0$

$\therefore\ 2.5\,V_B = 80 - 10 + 0 - 0 - 22.5 = 47.5$

$\therefore\ V_B = \frac{47.5}{2.5} = 19.0$ kN ↑ upward **(Answer)**

(ii) Consider moment at other support point B, we get

$\Sigma M_B = 20 \times 1.5 + V_B \times 0 - 10 + H_A \times 0 + V_A \times 2.5 - 30 \times 3.25 = 0$

$\therefore\ 2.5V_A = -30 + 10 + 97.5 = 77.5$

$\therefore\ V_A = \frac{77.5}{2.5} = 31.0$ kN ↑ upward **(Answer)**

(d) Now we can get self-check of our calculation for perfectness in equation obtain in (b) as $V_A + V_B = 50$ kN. If it fulfills, our calculation has no error.

We have LHS = $V_A + V_B$ = 31.0 + 19.0 = 50 kN = RHS. **OK.** Means our answer is perfect. If not satisfied in any case, you have done some mistake in calculation, recalculate it until it's satisfied the check point equation.

Example 12.

A beam is loaded as shown in below fig. If R_A = 5.6 kN, find the intensity of UDL w in kN/m on length AC and reaction R_B.

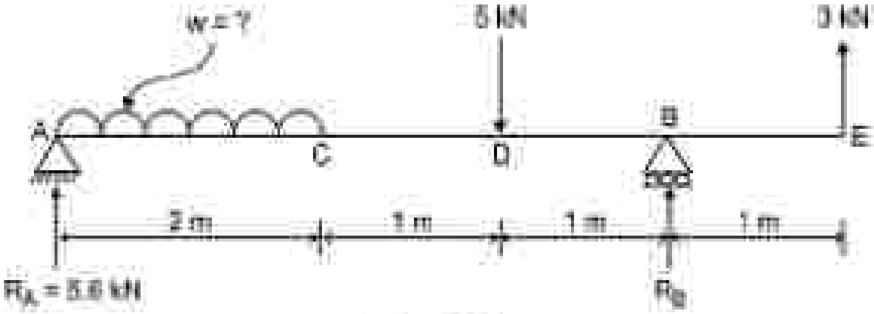


Fig. 2.24

Solution:

Here one reaction R_A at support A is given, but the value of UDL (kN/m) is unknown with other reaction R_B. We have to apply two equilibrium conditions to solve these two unknowns.

(i) $\Sigma M = 0$ with + ve sign as ↻ clockwise moment. Consider moment at support point B, we get

$\Sigma M_B = R_A \times 4 - (w\times 2) \times 3 - 5 \times 1 + R_B \times 0 - 3\times 1 = 0$

$\therefore 5.6 \times 4 - 6w - 5 + 0 - 3 = 0$

$\therefore 6w = 22.4 - 5 - 3 = 14.4$

$\therefore w = \frac{14.4}{6} = 2.4$ kN/m **(Answer)**

(ii) $\Sigma V = 0$ with + ve sign ↑ upward and assume R_B upward.

$\therefore R_A + R_B + 3 - (w\times 2) - 5 = 0$

$\therefore 5.6 + R_B + 3 - (2.4 \times 2) - 5 = 0$

$\therefore R_B = 1.2$ kN ↑ upward **(Answer)**

(iii) We can take check point as $\Sigma MA = 0$ with + ve sign as ↻ clockwise moment.

$\Sigma M_A = (w \times 2) \times 1 + 5 \times 3 - R_B \times 4 - 3 \times 5 = 2.4\times 2 \times 1 + 15 - 1.2\times 4 - 15$
$= 4.8 + 15 - 4.8 - 15 = 0$

So, we obtained that $\Sigma M_A = 0$ means our calculation is error free. Thus, we get self-assessment for TRUE value.

BEAM REACTION BY GRAPHICAL METHOD:

We have discussed analytical method in previous topic 2.4 for different types of beams. Now we are discussing the second method i.e., graphical method to find beam support reactions.

In this method, we have to study only for simply supporting beam carrying only point load as per your syllabus.

Funicular Polygon graphical method:

It is necessary to understand, certain technical terminologies for graphical method. This graphical method is also known as Funicular Polygon Method.

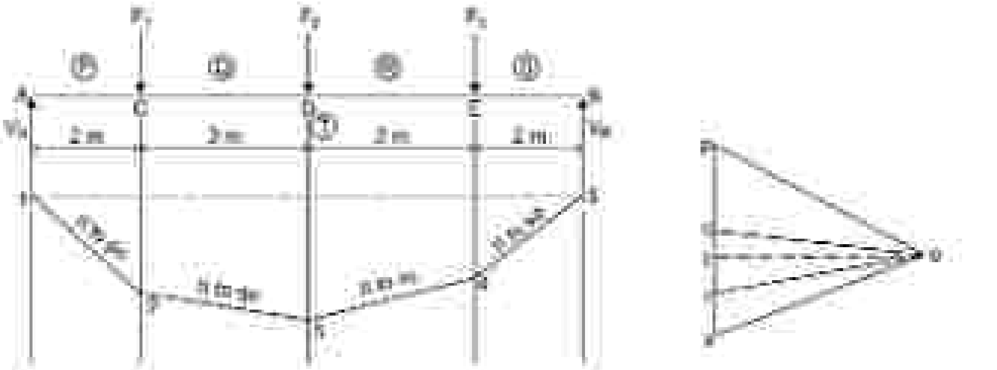
(i) Bow's notatson : Forces are identified by two different identical capital letters, placed on both side (in space) of the force, as shown in figure.

Fig. 2.25 : Bow's notation

In this case, four forces F_1, F_2, F_3 and F_4 are acting on the body. Now placed capital letters of alphabets on either sides of the force direction, we have to put A & B for force F_1. Now as per bow's notation, this force F_1 can identify as F_{AB}. Similarly for other forces F_2, F_3 and F4 fill the space on either sides. For force F_2 on one side B letter is already place on LHS, so on other side (RHS) put another letter as C. So F_2 represent by bow' notation as F_{BC}. Thus force F_3 and F_4 can identify as F_{CD} & F_{DE} respectively by bow's notation.

(ii) **Space diagram :** A diagram showing all the forces in position along with their magnitude and direction acting on a body is known as space diagram. Span & position of load shown as per suitable linear scale i.e., 1 cm = ______ m.

(iii) **Vector diagram :** All the forces on the body is represented one by one in vectorial form by magnitude and direction. The direction is represented by its original direction, while magnitude is represented by some suitable force scale i.e., 1 cm = _____ N or kN. Now, we can understand the steps of drawing funicular polygon. The graphical method to determine support reaction is given in following.



(a) Space diagram with Length scale (b) Vector diagram with Force scale

Fig. 2.26: Funicular polygon graphical method

Step-1: Draw the space diagram which shows position, direction & magnitude of all the forces acting on the body (beam) as shown if fig (a). The distance between forces may be drawn with some length scale i.e. 1 cm = _____ m.

Step-2: Give bow's notations to all forces by placing alphabets on both sides of arrow. Reaction is considered as force acting on the body.

Step-3: Draw vector diagram for the given forces with some suitable force scale i.e. 1 cm = ___ N or kN which represents the magnitude of each force. All the forces were drawn, one by one taken in order, in a vector diagram as shown in fig.(b). II to so 62 | Engineering Mechanics.

Step-4: Take some convenient point O in front of vectorial form of forces & join all the points of vector diagram with this point O.

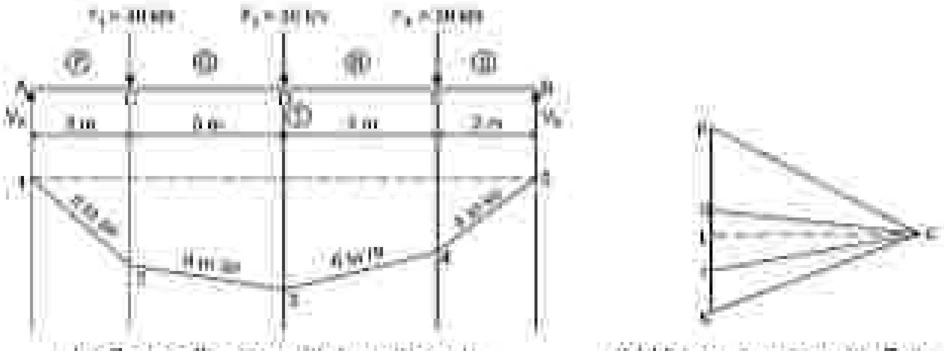
Step-5: Now select a point 1 on the line of action of first force R_A & through it draw a line parallel to "op" which cross at point 2 on force F_1. Now through point 2 draw line 2-3 parallel to "oq". Similarly draw lines 3-4 & 4-5 parallel to vector diagram lines "or" and "os" respectively on space diagram.

Step-6: Now join first start point 1 & last end point 5 obtained on line of force R_B as dotted line 1-5 in space diagram as shown in fig. (a). Draw a parallel line to 1-5 on vector diagram passing through point O as "ot" as dotted line as shown in fig. (b).

Step-7: Now measure "pt" & "ts" length on vector diagram & convert it by force scale as reaction R_A and R_B respectively.

Let to explain steps of graphical method (Funicular Polygon), take one example

Example 13. Solve example 7 as shown in below fig. by graphical method.



(a) Space diagram with Length scale (b) Vector diagram with Forc

Fig. 2.27

Solution:

Step-1: Draw space diagram from given data with length scale as 1 cm = 1 m as shown in fig.(a).

Step-2: Gives bow's notations to all forces including reactions VA & VB by placing alphabets on either side space on line of direction. Here for force F_1 = 40 kN, we have placed "P" & "Q" on either sides of space on line of direction of force F_1 means force F_1 is now Fpq as per bow's notation. Similarly placed "R", "S" & "T" as bow's notation as shown in fig.(a).

Step-3: Draw vector diagram for the forces on the beam with force scale as 1 cm = 20 kN as shown in fig.(b). Select start point "p" & draw parallel line as line of action of force Fpq (in this case vertically downward) and get point "q" on it by converting magnitude of force Fpq as 40 kN as per scale as 2 cm from point "p". Thus line "pq" represent vectorial form of force Fpq. Similarly draw all the forces taken in order in vector form in vector diagram as "qr" & "rs" for force Fqr (F_2) & Frs (F_3) respectively.

Step-4: Take some convenient point "o" in front of vectorial form of forces. Join all points of vector diagram p, q, r & s with o and obtained line po, qo, ro & so as shown in fig.(b).

Step-5: Now on space diagram extend all line of action for all the forces. Select start point "1" on line of reaction R_A. Through this point "1", draw line parallel to line "po" of vector diagram II to so Equilibrium : 63 to get point "2" on line of action of force F_1. Thus line "1-2" is parallel to line "po". Similarly draw "2-3", "3-4" & "4-5" parallel to line "qo", "ro" & "so" respectively.

Step-6: Now join first start point "1" with last end point "5" in this case, as line "1-5" as shown dotted line in space diagram fig.(a). Draw parallel line to "1-5" on vector diagram through point "o" & get line "ot" as shown in dotted line in fig.(b).

Step-7: Now measure length of line "pt" & "ts" and convert it in force magnitude by using force scale, we get

V_A = pt × force scale = 2.6 cm × 20 = 52 kN ↓ (Answer)

& V_B = ts × force scale = 1.9 cm × 20 = 38 kN ↓ (Answer)

Here, the value of reaction by graphical method, may vary by 5 to 10 %, as compared to analytical method and is permitted.

POSSIBLE SHORT TYPE QUESTIONS WITH ANSWER

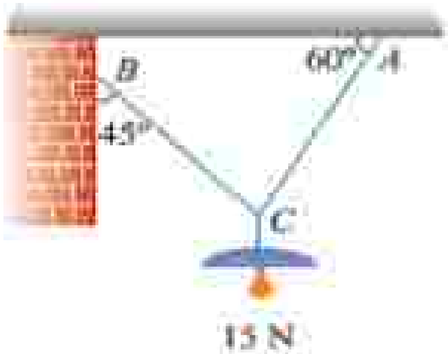
1. **State Lami's theorem. (S – 2018)**
   - It states, *"If three coplanar forces acting at a point be in equilibrium, then each force is proportional to the sine of the angle between the other two."*
   - Mathematically,

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

2. **State the conditions of equilibrium. (S – 2019)**
   - The algebric sum of horizontal components of the forces must be zero. i.e., $\sum H=0$
   - The algebric sum of vertical components of the forces must be zero. i.e., $\sum V=0$
   - The algebric sum of moment of forces about any point in their plane is equal to zero. i.e., $\sum M=0$

POSSIBLE LONG TYPE QUESTIONS

1. State and proof Lami's theorem. **(W – 2016)**
2. An electric light fixture weighting 15 N hangs from a point C, by two strings AC and BC. The string AC is inclined at 60° to the horizontal and BC at 45° to the horizontal as shown in Figure. Using Lami's theorem, or otherwise, determine the forces in the strings AC and BC. **(W – 2016)**



3. Write down the conditions of equilibrium? Also mention it mathematically. **[S – 2019 (O)]**
4. Three forces acting on a particle are in equilibrium, the angles between the first and second is 90° and that between the second and third is 120°. Find the ratio between the forces. **[S – 2019 (O)]**

CHAPTER NO. – 03

FRICTION

Definition of Friction:

- When a rigid body slides over another rigid body, a resisting force is exerted at the surface of contact, this resisting force is called force of friction or simply friction.
- It always acts in a direction opposite to the direction of motion.

Classification of friction:

- Friction may be classified into two types
    1. Static friction
    2. Dynamic friction

1. **Static friction:** It is the friction experienced by a body when it is at rest. Or in other words, it is the friction when the body tends to move.
2. **Dynamic friction:** It is the friction experienced by a body when it is in motion. It is also called kinetic friction. The dynamic friction is of the following two types :
    - Sliding friction: It is the friction, experienced by a body when it slides over another body.
    - Rolling friction: It is the friction, experienced by a body when it rolls over another body.

Limiting Friction:

- It is the maximum force of friction between two contact surfaces when a body tends to move over another body.

Angle of friction:

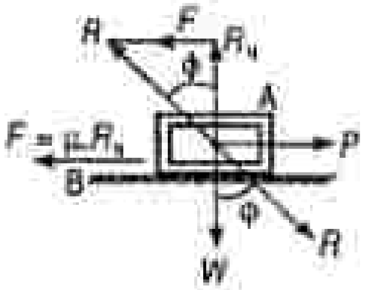
- Consider a body of weight *(W)* is resting on a horizontal plane. If a force ***P*** is applied to the body, no relative motion takes place until the applied force ***P*** is equal to the force of friction ***F***, acting opposite to the direction of motion.

  Let R_N = Normal reaction acting on the body.

  W = Weight of the body.

  F = Maximum force of friction.

  P = Horizontal force acting on the body.



- From the geometry of the figure, we have

$$\tan\varphi = \frac{F}{R_N} \qquad \Rightarrow \varphi = \tan^{-1}\left(\frac{F}{R_N}\right)$$

- Where, $\varphi = Angle\ of\ friction$

- **Definition of φ:** It is the angle between normal reaction (R_N) and resultant (R) of normal reaction and frictional forces.

Coefficient of friction:

- It is the ratio of limiting friction (F) to the normal reaction (R_N) between two bodies.
- It is generally denoted by μ.
- Mathematically,

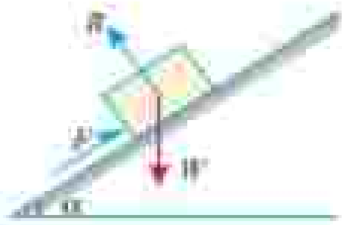
$$\mu = \frac{F}{R_N} = \tan\varphi$$

$$\Rightarrow \varphi = \tan^{-1}\mu$$

Angle of repose:

- It is the maximum angle between the inclined plane and horizontal plane at which the body just tends to slide downwards.
- This angle is generally specified by α.

- Angle of friction (φ) = Angle of repose (α)



Laws of Friction:

Prof. Coulomb, after extensive experiments, gave some laws of friction, which may be grouped under the following heads:

- Laws of static friction, and
- Laws of kinetic or dynamic friction.

Laws of Static Friction:

Following are the laws of static friction:

- The force of friction always acts in a direction, opposite to that in which the body tends to move.
- The magnitude of the force of friction is exactly equal to the force, which tends to move the body.
- The magnitude of the limiting friction (F) bears a constant ratio to the normal reaction (R_N) between the two surfaces. Mathematically:

$$\frac{F}{R_N} = constant$$

- The force of friction is independent of the area of contact between the two surfaces.
- The force of friction depends upon the roughness of the surfaces.

Laws of Dynamic Friction:

Following are the laws of kinetic or dynamic friction:

- The force of friction always acts in a direction, opposite to that in which the body is moving.
- The magnitude of kinetic friction bears a constant ratio to the normal reaction between the two surfaces. But this ratio is slightly less than that in case of limiting friction.
- For moderate speeds, the force of friction remains constant. But it decreases slightly with the increase of speed.

Advantages of Friction:

- Friction is responsible for many types of motion.
- It helps us walk on the ground.
- Brakes in a car make use of friction to stop the car.
- Asteroids are burnt in the atmosphere before reaching Earth due to friction.
- It helps in the generation of heat when we rub our hands.

Disadvantages of Friction:

- Friction produces unnecessary heat leading to the wastage of energy.
- The force of friction acts in the opposite direction of motion, so friction slows down the motion of moving objects.
- Forest fires are caused due to the friction between tree branches.
- A lot of money goes into preventing friction and the usual wear and tear caused by it by using techniques like greasing and oiling.

EQUILIBRIUM OF A BODY ON A ROUGH HORIZONTAL PLANE:

We know that a body, lying on a rough horizontal plane will remain in equilibrium. But when- ever a force is applied on it, the body will tend to move in the direction of the force. In such cases, equilibrium of the body is studied first by resolving the forces horizontally and then vertically.
Now the value of the force of friction is obtained from the relation:

$$F = \mu R$$

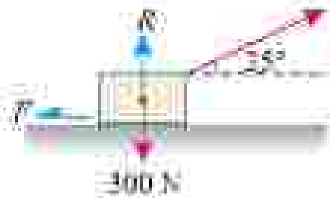
Where, F = Force of friction
μ = coefficient of friction
R = Normal reaction

Example – 1: ***A body of weight 300 N is lying on a rough horizontal plane having a coefficient of friction as 0.3. Find the magnitude of the force, which can move the body, while acting at an angle of 25° with the horizontal.***

Solution. Given: Weight of the body (W) = 300 N, Coefficient of friction (μ) = 0.3 and angle made by the force with the horizontal (α) = 25°

Let P = Magnitude of the force, which can move the body, and
F = Force of friction.

Resolving the forces horizontally,



$$F = P\cos\alpha = P\cos 25° = P \times 0.9063$$

and now resolving the forces vertically,

$$R = W - P\sin\alpha = 300 - P\sin 25° = 300 - P \times 0.4226$$

We know that the force of friction (F),

$$0.9063\,P = \mu R = 0.3 \times (300 - 0.4226\,P) = 90 - 0.1268\,P$$

$$\Rightarrow 90 = 0.9063\,P + 0.1268\,P = 1.0331\,P$$

$$\Rightarrow P = \frac{90}{1.0331} = 87.1\,N$$

Example – 2: ***A body, resting on a rough horizontal plane, required a pull of 180 N inclined at 30° to the plane just to move it. It was found that a push of 220 N inclined at 30° to the plane just moved the body. Determine the weight of the body and the coefficient of friction.***

Solution. Given: Pull = 180 N; Push = 220 N and angle at which force is inclined with horizontal plane (α) = 30°

Let W = Weight of the body
R = Normal reaction, and
μ = Coefficient of friction.

First of all, consider a pull of 180 N acting on the body. We know that in this case, the force of friction (F_1) will act towards left as shown in Fig. (a).

Resolving the forces horizontally,

$$F_1 = 180\cos 30° = 180 \times 0.866 = 155.9\,N$$

and now resolving the forces vertically,

$$R_1 = W - 180\sin 30° = W - 180 \times 0.5 = W - 90\,N$$

We know that the force of friction (F_1),

$$155.9 = \mu R_1 = \mu(W - 90) \ldots\ldots\ldots (i)$$

(a) Pull of 180 N (b) Pull of 220 N

Now consider a push of 220 N acting on the body. We know that in this case, the force of friction (F_2) will act towards right as shown in Fig. (b)

Resolving the forces horizontally,

$$F_2 = 220 \cos 30° = 220 \times 0.866 = 190.5\ N$$

and now resolving the forces horizontally,

$$R_2 = W + 220 \sin 30° = W + 220 \times 0.5 = W + 110\ N$$

We know that the force of friction (F_2),

$$190.5 = \mu . R_2 = \mu\,(W + 110) \ldots\ldots\ldots (ii)$$

Dividing equation (*i*) by (*ii*)

$$\frac{155.9}{190.5} = \frac{\mu\,(W - 90)}{\mu\,(W + 110)} = \frac{(W - 90)}{(W + 110)}$$

$$\Rightarrow 155.9\,W + 17\,149 = 190.5\,W - 17\,145$$

$$\Rightarrow 34.6\,W = 34\,294$$

$$\Rightarrow W = \frac{34\,294}{34.6} = \mathbf{991.2\ N}$$

Now substituting the value of W in equation (*i*),

$$155.9 = \mu\,(991.2 - 90) = 901.2\,\mu$$

$$\Rightarrow \mu = \frac{155.9}{901.2} = \mathbf{0.173}$$

EQUILIBRIUM OF A BODY ON A ROUGH INCLINED PLANE SUBJECTED TO A FORCE ACTING ALONG THE INCLINED PLANE

Consider a body lying on a rough inclined plane subjected force acting along the inclined plane, which keeps it in equilibrium as shown in Fig. (a) and (b).

Let W = Weight of the body,

α = Angle, which the inclined plane makes with the horizontal,

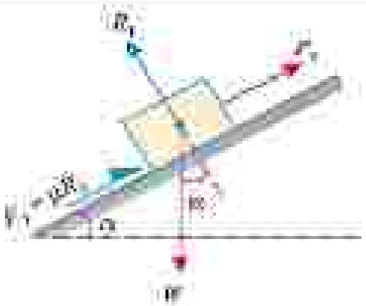
R = Normal reaction,

μ = Coefficient of friction between the body and the inclined plane, and

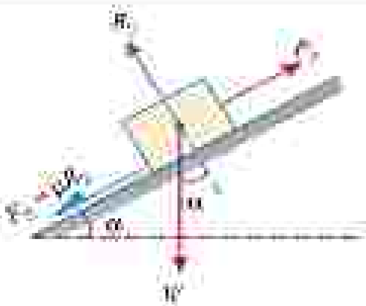
ϕ = Angle of friction, such that $\mu = \tan \phi$.

A little consideration will show that if the force is not there, the body will slide down the plane. Now we shall discuss the following two cases:

1. Minimum force (P_1) which will keep the body in equilibrium, when it is at the point of sliding downwards



(a) Body at the point of sliding downwards



(b) Body at the point of sliding upwards

In this case, the force of friction ($F_1 = \mu.R_1$) will act upwards, as the body is at the point of sliding downwards as shown in Fig. (a). Now resolving the forces along the plane,

$$P_1 = W \sin \alpha - \mu.R_1 \ldots\ldots\ldots\ldots\ldots\ldots (i)$$

And now resolving the forces perpendicular to the plane.

$$R_1 = W \cos \alpha \_\_\_\_\_\_\_\_\_\_ (ii)$$

Substituting the value of R_1 in equation (i),

$$P_1 = W \sin \alpha - \mu\, W \cos \alpha = W (\sin \alpha - \mu \cos \alpha)$$

And now substituting the value of $\mu = \tan \phi$ in the above equation,

$$P_1 = W (\sin \alpha - \tan \phi \cos \alpha)$$

Multiplying both sides of this equation by $\cos \phi$,

$$P_1 \cos \phi = W (\sin \alpha \cos \phi - \sin \phi \cos \alpha) = W \sin (\alpha - \phi)$$

$$\therefore \boldsymbol{P_1 = W \times \frac{\sin(\alpha - \emptyset)}{\cos \emptyset}}$$

2. Maximum force (P_2) which will keep the body in equilibrium, when it is at the point of sliding upwards

In this case, the force of friction ($F_2 = \mu.R_2$) will act downwards as the body is at the point of sliding upwards as shown in Fig. (b). Now resolving the forces along the plane,

$$P_2 = W \sin \alpha + \mu.R_2 \ldots\ldots\ldots\ldots (i)$$

And now resolving the forces perpendicular to the plane,

$$R_2 = W \cos \alpha \_\_\_\_\_\_\_\_ (ii)$$

Substituting the value of R_2 in equation (i),

$$P_2 = W \sin \alpha + \mu\, W \cos \alpha = W (\sin \alpha + \mu \cos \alpha)$$

And now substituting the value of $\mu = \tan \phi$ in the above equation,

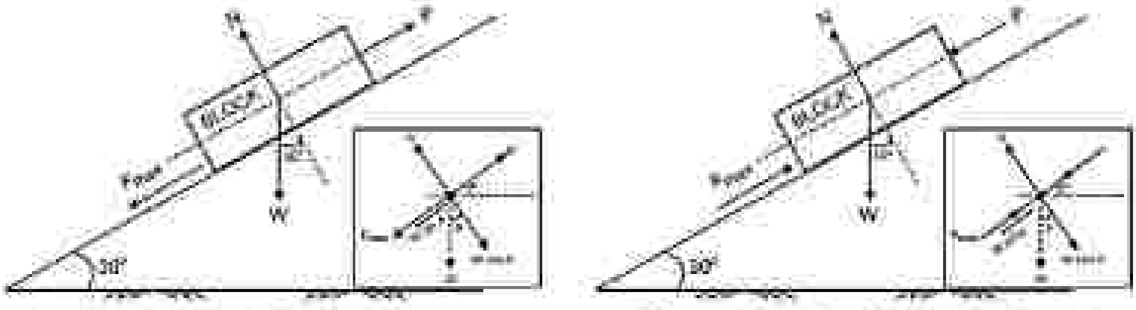
$$P_2 = W (\sin \alpha + \tan \phi \cos \alpha)$$

Multiplying both sides of this equation by $\cos \phi$,

$$P_2 \cos \phi = W (\sin \alpha \cos \phi + \sin \phi \cos \alpha) = W \sin (\alpha + \phi)$$

$$\therefore \boldsymbol{P_2 = W \times \frac{\sin(\alpha + \emptyset)}{\cos \emptyset}}$$

A block of mass 10 kg is resting on the rough inclined plane surface with inclination of 30° to horizontal. If coefficient of friction is 0.25 between two contact surfaces, find the external force to be apply parallel to inclined plane to move the block (i) upward and (ii) downward.



(a) Upward motion of block on inclined plane (b) Downward motion of block on inclined plane

Fig. 3.11

Solution:

Given data : $\theta = 30°$, $\mu = 0.25$ & mass = m = 10 kg., W = mg = 10 × 9.8 N = 98 N

(A) Block motion (move) upward on inclined plane surface : [fig. 3.11(a)]

(i) Resolving all the forces perpendicular (normal) to given an inclined plane surface, we get,

$N = W \cos \theta$

$N = 98 \times \cos 30°$

$N = 84.87$ N ...(i)

(ii) Resolving all the forces parallel (along) to given an inclined plane surface, we get,

$P = F_{max} + W \sin \theta$

$P = \mu N + W \sin \theta$ ...(ii)

Putting values of μ, θ, N & W, we get;

$P = (0.25 \times 84.87) + (98 \times \sin 30)$

$P = 70.22$ N (Answer)

(B) Block motion (move) downward on inclined plane surface : [fig. 3.11(b)]

(i) Resolving all the forces perpendicular (normal) to given an inclined plane surface, we get,

$N = W \cos \theta$

$N = 98 \times \cos 30°$

$N = 84.87$ N ...(i)

(ii) Resolving all the forces parallel (along) to given an inclined plane surface, we get,

$P = F_{max} - W \sin \theta$

$P = \mu N - W \sin \theta$ ...(ii)

Putting values of μ, θ, N & W, we get;

$P = (0.25 \times 84.87) - (98 \times \sin 30)$

$P = -27.78$ N (Push) (Answer)

A block weighing of 500 N just start its motion in downward direction, on rough inclined plane surface, when a pull force of 200 N is apply parallel to the inclined plane surface. The same block is at the point of moving upward, when a pull force of 300 N is apply parallel to the inclined plane surface. Find the inclination of the plane and the coefficient of friction between block and inclined plane surface.

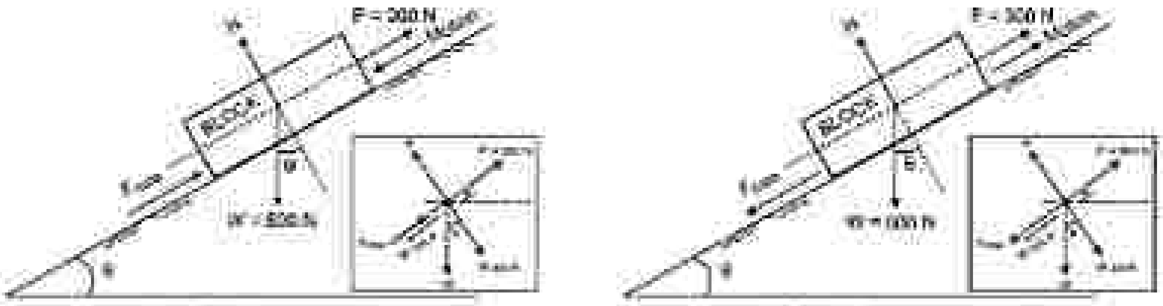
Solution: Free body diagrams of block to move downward & upward as shown in fig. 3.12 (a) & (b) respectively.

We may note that frictional force F_{max} opposing the motion & acting on opposite direction of motion of the body. In both cases $F_{max} = \mu.N$ [as body is about (just) to move]

Given data: W = 500 N, External force (i) 200 N & (ii) 300 N. (A) Block motion (move) downward on inclined plane surface: (i) Resolving all the forces perpendicular (normal) to given an inclined plane surface, we get,

$N = W \cos \theta$

$N = 500 \times \cos \theta$



(a) Block just move downwards (b) Block just move upwards

Fig. 3.12

(ii) Resolving all the forces parallel (along) to the inclined plane surface, we get,

$P + F_{lim} = W \sin\theta$, where $F_{lim} = \mu.N$ ....(ii)

Substituting values of P, N & W in equation (ii), we get;

$200 + (\mu \times 500 \times \cos q) = (500 \times \sin\theta)$

$200 = 500 \times \sin\theta - \mu \times 500 \times \cos\theta$ ....(iii)

(B) Block motion (move) upward on inclined plane surface:

(i) Resolving all the forces perpendicular (normal) to given an inclined plane surface, we get,

$N = W \cos\theta$

$N = 500 \times \cos\theta$ same as case (A)

(ii) Resolving all the forces parallel (along) to given an inclined plane surface, we get,

$P = W \sin\theta + F_{lim}$, where $F_{lim} = \mu.N$

$300 = (500 \times \sin\theta) + (\mu \times 500 \times \cos\theta)$ ....(iv)

(C) Now, Adding equation (iii) & (iv), we get;

$500 = 1000 \times \sin\theta$

$\sin\theta = 0.5$

$\theta = 30°$ (Answer)

POSSIBLE SHORT TYPE QUESTIONS WITH ANSWER

1. Define friction. **(W – 2016)**

Ans: When a rigid body slides over another rigid body, a resisting force is exerted at the surface of contact, this resisting force is called force of friction or simply friction. It always acts in a direction opposite to the direction of motion.

2. Define limiting friction. **(Possible)**

Ans: It is the maximum force of friction between two contact surfaces when a body tends to move over another body.

3. Define angle of friction. **(S – 2019)**

Ans: It is the angle between normal reaction (R_N) and resultant (R) of normal reaction and frictional forces. It is denoted by φ.

$$\Rightarrow \varphi = \tan^{-1}\left(\frac{F}{R_N}\right)$$

4. Define coefficient of friction. **(W – 2016, 2017 & S – 2018)**

Ans:

- It is the ratio of limiting friction (F) to the normal reaction (R_N) between two bodies.
- It is generally denoted by μ.
- Mathematically,

$$\mu = \frac{F}{R_N} = \tan\varphi$$
$$\Rightarrow \varphi = \tan^{-1}\mu$$

5. **Define angle of repose. (S – 2019 Old)**
Ans: It is the maximum angle between the inclined plane and horizontal plane at which the body just tends to slide downwards. This angle is generally specified by α.

6. Write any two disadvantages of friction. **(S – 2019)**
Ans:

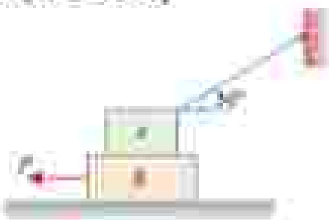
- Friction produces unnecessary heat leading to the wastage of energy.
- The force of friction acts in the opposite direction of motion, so friction slows down the motion of moving objects.

POSSIBLE LONG TYPE QUESTIONS

1. Explain laws of friction? **(W – 2016)**

2. What are the advantages and disadvantages of friction? **(Possible)**

3. Two blocks A and B of weights 1 kN and 2 kN respectively are in equilibrium position as shown in Fig below. If the coefficient of friction between the two blocks as well as the block B and the floor is 0.3, find the force (P) required to move the block B. **(W – 2016)**



4. A body, resting on a rough horizontal plane, required a pull of 18 N inclined at 30° to the plane just to move it. It was found that a push of 22 N inclined at 30° to the plane just moved the body. Determine the weight of the body and the coefficient of friction. **(W – 2017)**

5. A force of 250 N pulls a body of weight 500 N up an inclined plane, the force being applied parallel to the plane. If the inclination of the plane to the horizontal is 15°, find the coefficient of friction. **(S – 2018)**

4. A uniform ladder 3 m long weighs 200 N. It is placed against a wall making an angle of 60° with the floor. The coefficient of friction between the wall and the ladder is 0.25 and that between the floor and ladder is 0.35. The ladder, in addition to its own weight, has to support a man of 1000 N at its top at B. Calculate the horizontal force P to be applied to ladder at the floor level to prevent slipping. **(S – 2018)**

5. A body of weight 50 N is pulled along a rough horizontal plane by a force of 18 N acting at an angle of 14° with the horizontal. Find the coefficient of friction. **(S – 2019)**

6. A ladder 5 meters long rests on a horizontal ground and leans against a smooth vertical wall at an angle 70° with the horizontal. The weight of the ladder is 900 N and acts at its middle. The ladder is at the point of sliding, when a man weighing 750N stands on a rung 1.5 metre from the bottom of the ladder. Calculate the coefficient of friction between the ladder and the floor. **(S – 2019)**

CHAPTER NO. – 04

CENTROID & CENTRE OF GRAVITY

Centre of gravity:

- Centre of gravity of a body may be defined as the point through which the whole weight of a body may be assumed to act.

Centroid:

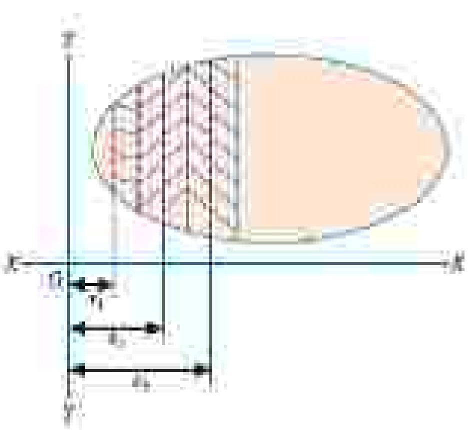
- Centroid or Centre of area is defined as the point where the whole area of the figure is assumed to be concentrated.

Comparison between Center of Gravity and Centroid

Parameter of Comparison	Center of Gravity	Centroid
Perception	Center of gravity is the point where total mass of the object acts.	Centroid is the geometric center of the object where whole area is assume to be concentrate.
Object density	Centre of Gravity is applicable to objects with any density.	Centroid is the central point of objects with uniform density.
Dealing with structure	Generally, deals with 3D structure.	Generally, deals with 2D structure.
Subject association	Centre of Gravity a term often found in Physics.	Centroid a term often used in Mathematics, in relation to any figure.
Examples	Cube, Cone, Cylinder, Sphere, Hemi sphere... etc.	Square, Rectangle, Triangle, Circle, Semi-circle, Quarter circle ... etc.

Moment of an area about an axis:

Consider a body of area A whose centroid is required to be found out. Divide the body into small masses, Let a_1, a_2, a_3 .... etc. be the masses of the particles and (x_1, y_1), (x_2, y_2), (x_3, y_3) .... be the co-ordinates of the centroid from a fixed-point O as shown in Fig. below.



Let $\bar{x}$ and $\bar{y}$ be the co-ordinates of the centroid of the body. From the principle of moments, we know that

$$A\bar{x} = a_1x_1 + a_2x_2 + a_3x_3 + \dots\dots\dots$$

$$\Rightarrow \bar{x} = \frac{\sum ax}{A}$$

Similarly,

$$\bar{y} = \frac{\sum ay}{A}$$

Where,

$$A = a_1 + a_2 + a_3 + \ldots\ldots\ldots$$

Centroid by geometrical consideration:

The centroid of simple figures may be found out from the geometry of the figure as given below.

1. The centroid of uniform rod is at its middle point.
2. The centroid of a rectangle (or a parallelogram) is at the point, where its diagonals meet each other. It is also a middle point of the length as well as the breadth of the rectangle as shown in Fig. 6.1

3. The centroid of a triangle is at the point, where the three medians (a median is a line connecting the vertex and middle point of the opposite side) of the triangle meet as shown in Fig. 6.2.

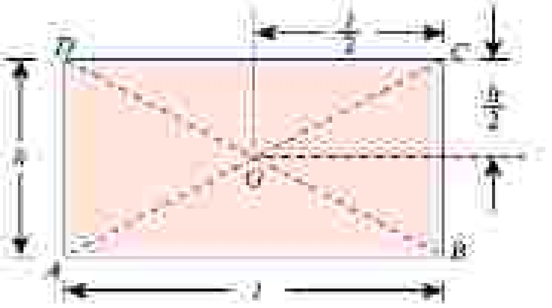


Fig. 6.1. Rectangle

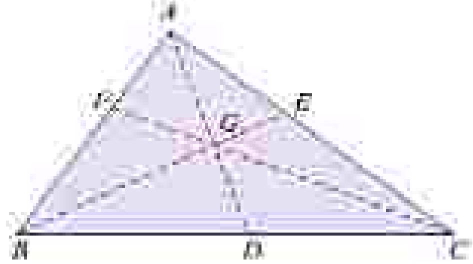


Fig. 6.2. Triangle

4. The centroid of a trapezium with parallel sides a and b is at a distance of $\frac{h}{3} \times \left(\frac{b+2a}{b+a}\right)$ measured form the side b as shown in Fig. 6.3.

5. The centroid of a semicircle is at a distance of $\frac{4r}{3\pi}$ from its base measured along the vertical radius as shown in Fig. 6.4.

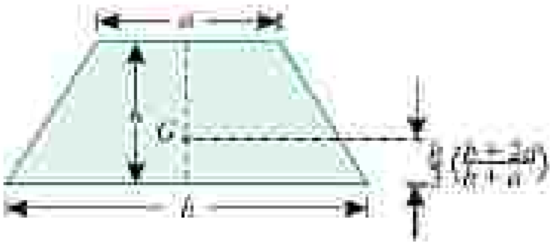


Fig. 6.3. Trapezium

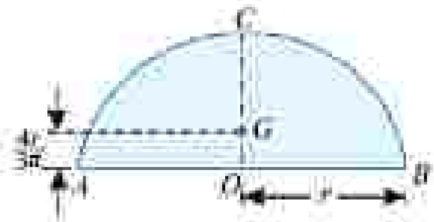


Fig. 6.4. Semicircle

6. The centroid of a circular sector making semi-vertical angle α is at a distance of $\left(\frac{2r}{3}\frac{\sin\alpha}{\alpha}\right)$ from the centre of the sector measured along the central axis as shown in Fig. 6.5.

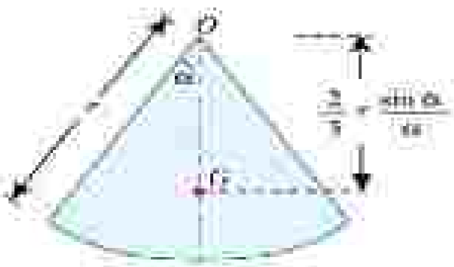


Fig. 6.5. Circular sector

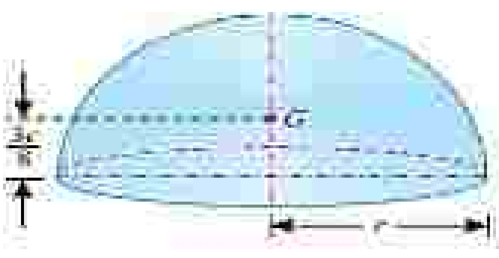


Fig. 6.6. Hemisphere

7. The centroid of a cube is at a distance of $l/2$ from every face (where l is the length of each side).

8. The centroid of a sphere is at a distance of $d/2$ from every point (where d is the diameter of the sphere).

9. The centroid of a hemisphere is at a distance of 3r/8 from its base, measured along the vertical radius as shown in Fig. 6.6

10. The centroid of right circular solid cone is at a distance of h/4 from its base, measured along the vertical axis as shown in Fig. 6.7.

11. The centroid of a segment of sphere of a height h is at a distance of $\frac{3}{4}\frac{(2r-h)^2}{(3r-h)}$ from the centre of the sphere measured along the height. as shown in Fig. 6.8.

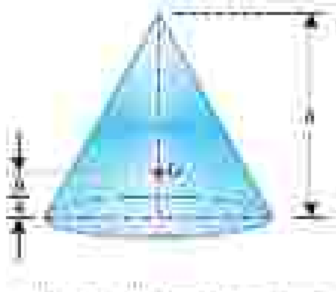


Fig. 6.7. Right circular solid cone

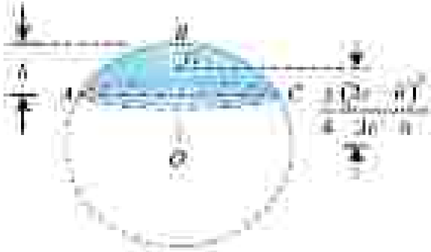
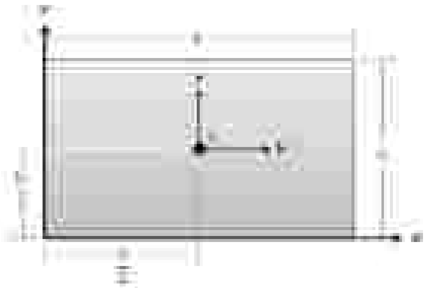


Fig 6.8. Segment of a sphere

Centroid of geometrical figures:

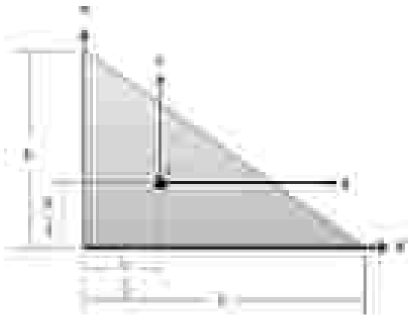
1. Rectangle:



Area = bh

$$\bar{x} = \frac{b}{2} \text{ and } \bar{y} = \frac{h}{2}$$

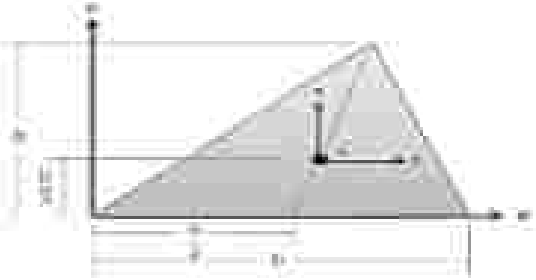
2. Right triangle:



$Area = \frac{1}{2}bh$

$$\bar{x} = \frac{b}{3} \text{ and } \bar{y} = \frac{h}{3}$$

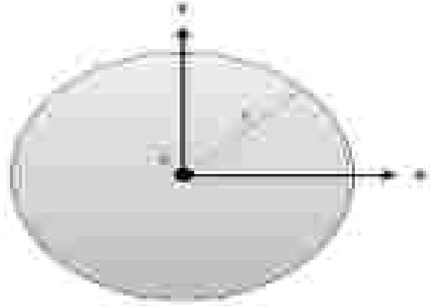
3. Triangle:



$$Area = \frac{1}{2}bh$$

$$\bar{x} = \frac{b}{2} \; and \; \bar{y} = \frac{h}{3}$$

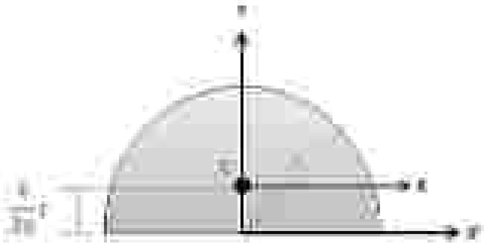
4. Circle:



$$Area = \pi r^2$$

$$\bar{x} = r \; and \; \bar{y} = r$$

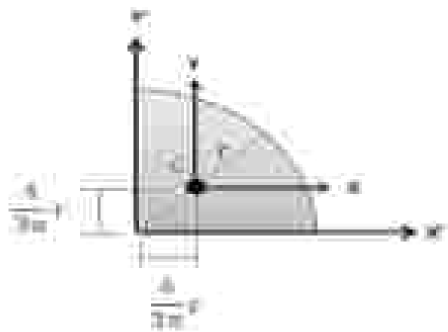
5. Semicircle:



$$Area = \frac{\pi}{2}r^2$$

$$\bar{x} = r \; and \; \bar{y} = \frac{4r}{3\pi}$$

6. Quadrant:



$$Area = \frac{\pi}{4}r^2$$

$$\bar{x} = \frac{4r}{3\pi} \; and \; \bar{y} = \frac{4r}{3\pi}$$

Note: $\bar{x}$ = Horizontal distance between the centroid and the vertical axis.
$\bar{y}$ = Vertical distance between the centroid and the horizontal axis.

Centroid of plane figures:

- The plane geometrical figures (such as T-section, I-section, L-section etc.) have only areas but no mass. The centre of gravity of such figures is found out in the same way as that of solid bodies. The centre of area of such figures is known as centroid, and coincides with the centre of gravity of the figure.
- Let $\bar{x}$ and $\bar{y}$ be the co-ordinates of the centre of gravity with respect to some axis of reference, then

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3 + \cdots}{a_1 + a_2 + a_3 + \cdots}$$

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3 + \cdots}{a_1 + a_2 + a_3 + \cdots}$$

Centre of gravity of symmetrical sections:

Sometimes, the given section, whose centre of gravity is required to be found out, is symmetrical about X-X axis or Y-Y axis. In such cases, the procedure for calculating the centre of gravity of the body is very much simplified; as we have only to calculate either $\bar{x}$ or $\bar{y}$. This is due to the reason that the centre of gravity of the body will lie on the axis of symmetry.

Example – 1: ***Find the centre of gravity of a 100 mm × 150 mm × 30 mm T-section.***

Solution: As the section is symmetrical about Y-Y axis, bisecting the web, therefore its

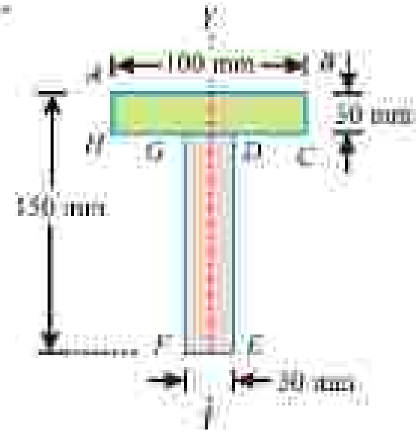
Centre of gravity will lie on this axis. Split up the section into two rectangles ABCH and DEFG as shown in figure.

Let bottom of the web FE be the axis of reference.

(*i*) Rectangle ABCH

$$a_1 = 100 \times 30 = 300 mm^2$$

$$y_1 = \left(150 - \frac{30}{2}\right) = 135 mm$$



(*ii*) Rectangle DEFG

$$a_2 = 120 \times 30 = 3600 mm^2$$

$$y_2 = \frac{120}{2} = 60 mm$$

We know that the distance between the centre of gravity of the section and bottom of the flange FE

$$\bar{y} = \frac{a_1 + y_1}{a_1 + a_2} = \frac{(3000 \times 135) + (3600 \times 60)}{3000 + 3600} = 94.1 mm\ (\boldsymbol{Ans})$$

Example – 2: ***Find the centre of gravity of a channel section 100 mm × 50 mm × 15 mm.***

Solution: As the section is symmetrical about X-X axis, therefore its centre of gravity will lie on this axis. Now split up the whole section into three rectangles ABFJ, EGKJ and CDHK as shown in figure.

Let the face AC be the axis of reference.

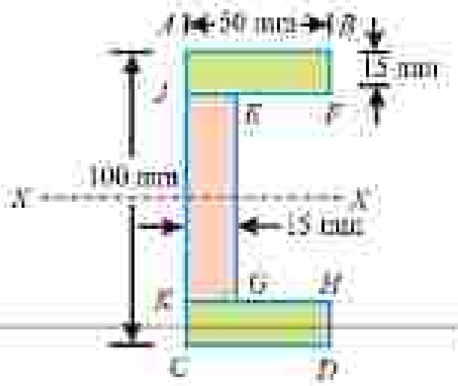
(*i*) Rectangle ABFJ

$$a_1 = 50 \times 15 = 750 mm^2$$

$$x_1 = \frac{50}{2} = 25 mm$$

(*ii*) Rectangle EGKJ

$$a_2 = (100 - 30) \times 15 = 1050 mm^2$$



$$x_2 = \frac{15}{2} = 7.5mm$$

(*iii*) Rectangle CDHK

$$a_3 = 15 \times 50 = 750mm^2$$

$$x_3 = \frac{50}{2} = 25mm$$

We know that the distance between the centres of gravity of the section and left face of the section AC

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3} = \frac{(720 \times 25) + (1050 \times 7.5) + (750 \times 20)}{750 + 1050 + 750} = 17.8mm\ (Ans)$$

Example – 3: ***An I-section has the following dimensions in mm units:***
Bottom flange = 300 × 100, Top flange = 150 × 50, Web = 300 × 50.
Determine mathematically the position of centre of gravity of the section.

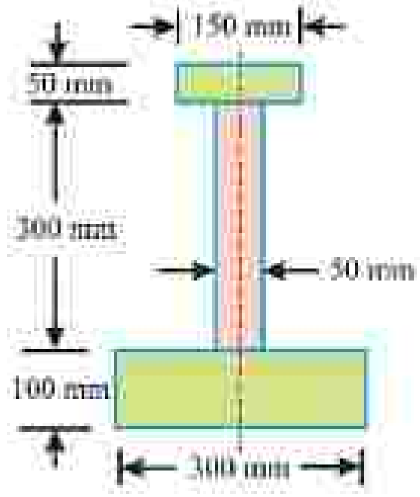
Solution: As the section is symmetrical about Y-Y axis, bisecting the web, therefore its centre of gravity will lie on this axis. Now split up the section into three rectangles as shown in figure.

Let bottom of the bottom flange be the axis of reference.

(*i*) Bottom flange

$$a_1 = 300 \times 100 = 30000mm^2$$

$$y_1 = \frac{100}{2} = 50mm$$



(*ii*) Web

$$a_2 = 300 \times 50 = 15000mm^2$$

$$y_2 = 100 + \frac{300}{2} = 250mm$$

(*iii*) Top flange

$$a_3 = 150 \times 50 = 7500mm^2$$

$$y_3 = 100 + 300 + \frac{50}{2} = 425mm$$

We know that the distance between centre of gravity of the section and bottom of the flange,

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3} = \frac{(30000 \times 50) + (15000 \times 250) + (7500 \times 425)}{30000 + 15000 + 7500} = 160.7mm\ (Ans)$$

Centre of gravity of unsymmetrical sections:

Sometimes, the given section, whose centre of gravity is required to be found out, is not symmetrical either about X-X axis or Y-Y axis. In such cases, we have to find out both the values of $\bar{x}$ and $\bar{y}$.

Example – 4: ***Find the centroid of an unequal angle section 100 mm × 80 mm × 20 mm.***

Solution: As the section is not symmetrical about any axis, therefore we have to find out the values of $\bar{x}$ and $\bar{y}$ for the angle section. Split up the section into two rectangles as shown in figure.
Let left face of the vertical section and bottom face of the horizontal section be axes of reference.

(i) Rectangle – 1

$$a_1 = 100 \times 20 = 2000mm^2$$

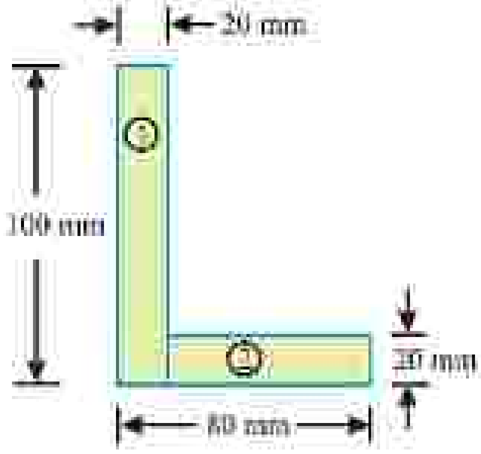
$$x_1 = \frac{20}{2} = 10mm$$

(ii) Rectangle – 2

$$a_2 = (80 - 20) \times 20 = 1200mm^2$$

$$x_2 = 20 + \frac{60}{2} = 50mm$$

$$y_2 = \frac{20}{2} = 10mm$$



And we know that the distance between the centre of gravity of the section and left face

$$\bar{x} = \frac{a_1x_1 + a_2x_2}{a_1 + a_2} = \frac{(2000 \times 10) + (1200 \times 50)}{(2000 + 1200)} = 25mm\ (Ans)$$

Similarly, the distance between centres of gravity of the section and bottom face

$$\bar{y} = \frac{a_1y_1 + a_2y_2}{a_1 + a_2} = \frac{(2000 \times 50) + (1200 \times 10)}{2000 + 1200} = 35mm\ (Ans)$$

Example – 5: ***A uniform lamina shown in Figure consists of a rectangle, a circle and a triangle. Determine the centre of gravity of the lamina. All dimensions are in mm.***

Solution: As the section is not symmetrical about any axis, therefore we have to find out the values of both $\bar{x}$ and $\bar{y}$ for the lamina.

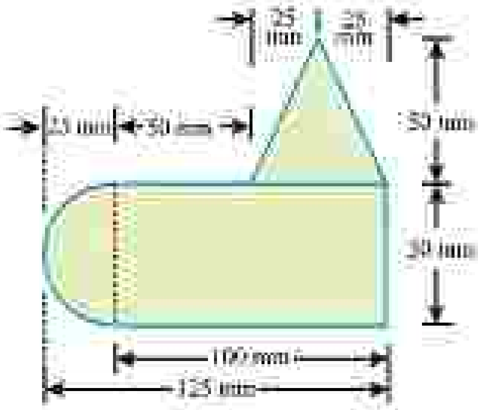
Let left edge of circular portion and bottom face rectangular portion be the axes of refarence.

(i) Rectangular portion

$$a_1 = 100 \times 50 = 5000mm^2$$

$$x_1 = 25 + \frac{100}{2} = 75mm$$

$$y_1 = \frac{50}{2} = 25mm$$



(ii) Semicircular portion

$$a_2 = \frac{\pi}{2} \times r^2 = \frac{\pi}{2}(25)^2 = 982mm^2$$

$$x_2 = 25 - \frac{4r}{3\pi} = 25 - \frac{4 \times 25}{3\pi} = 14.4mm$$

$$y_2 = \frac{50}{2} = 25mm$$

(iii) Triangular portion

$$a_3 = \frac{50 \times 50}{2} = 1250mm^2$$

$$x_3 = 25 + 50 + 25 = 100mm$$

$$y_3 = 50 + \frac{50}{3} = 66.7mm$$

We know that the distance between the center of gravity of the section and the left edge of the circular portion

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3} = \frac{(5000 \times 75) + (982 \times 14.4) + (1250 \times 100)}{5000 + 982 + 1250} = 71.1 \text{ mm } (\boldsymbol{Ans})$$

Similarly, the distance between the centre of gravity of the section and the bottom face of the rectangular portion.

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3} = \frac{(5000 \times 25) + (982 \times 25) + (1250 \times 66.7)}{5000 + 982 + 1250} = 32.2 mm \ (\boldsymbol{Ans})$$

Centre of gravity of sections with cut out holes:

The centre of gravity of such a section is found out by considering the main section, first as a complete one, and then deducting the area of the cut-out hole i.e., by taking the area of the cut-out hole as negative. Now substituting a_2 (i.e., the area of the cut-out hole) as negative, in the general equation for the centre of gravity, we get

$$\bar{x} = \frac{a_1x_1 - a_2x_2}{a_1 - a_2} \quad and \quad \bar{y} = \frac{a_1y_1 - a_2y_2}{a_1 - a_2}$$

Example – 6: ***A square hole is punched out of circular lamina, the diagonal of the square being the radius of the circle as shown in Figure. Find the centre of gravity of the remainder, if r is the radius of the circle.***

Solution: As the section is symmetrical about X-X axis, therefore its centre of gravity will lie on this axis. Let A be the point of reference.

(*i*) Main circle

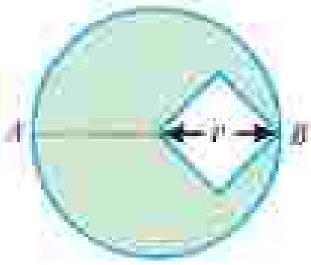
$$a_1 = \pi r^2$$

$$x_1 = r$$

(*ii*) Cut out square

$$a_2 = \frac{r \times r}{2} = 0.5r^2$$

$$x_2 = r + \frac{r}{2} = 1.5r$$

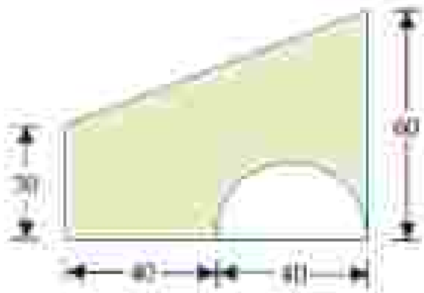


We know that distance between centres of gravity of the section and A,

$$\bar{x} = \frac{a_1x_1 - a_2x_2}{a_1 - a_2} = \frac{(\pi r^2 \times r) - (0.5\,r^2 \times 1.5\,r)}{\pi r^2 - 0.5\,r^2}$$

$$= \frac{r^3(\pi - 0.75)}{r^2(\pi - 0.5)} = \frac{r(\pi - 0.75)}{\pi - 0.5} \ (\boldsymbol{Ans})$$

Example – 7: ***A semi-circular area is removed from a trapezium as shown in Figure (dimensions in mm) Determine the centroid of the remaining area (shown hatched).***



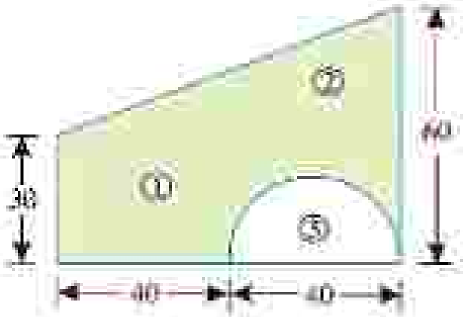
Solution: As the section in not symmetrical about any axis, therefore we have to find out the values of $\bar{x}$ and $\bar{y}$ for the area. Split up the area into three parts as shown in Fig. 6.25. Let left face and base of the trapezium be the axes of reference.

(*i*) Rectangle

$$a_1 = 80 \times 30 = 2400mm^2$$

$$x_1 = \frac{80}{2} = 40mm$$

$$y_1 = \frac{30}{2} = 15mm$$



(*ii*) Triangle

$$a_2 = \frac{80 \times 30}{2} = 1200mm^2$$

$$x_2 = \frac{80 \times 2}{3} = 53.3mm$$

$$y_2 = 30 + \frac{30}{3} = 40mm$$

(*iii*) Semi circle

$$a_3 = \frac{\pi}{2} \times r^2 = \frac{\pi}{2} 20^2 = 628.3mm^2$$

$$x_3 = 40 + \frac{40}{2} = 60mm$$

$$y_3 = \frac{4 \times 20}{3\pi} = \frac{4r}{3\pi} = 8.5mm$$

We know that the distance between center of gravity of the area and left face of trapezium.

$$\bar{x} = \frac{a_1x_1 + a_2x_2 - a_3x_3}{a_1 + a_2 - a_3} = \frac{(2400 \times 40) + (1200 \times 53.3) - (628.3 \times 60)}{2400 + 1200 - 628.3} = 41.1mm\ (\boldsymbol{Ans})$$

Similarly, the distance between center of gravity of the area and the base of the trapezium

$$\bar{y} = \frac{a_1y_1 + a_2y_2 - a_3y_3}{a_1 + a_2 - a_3} = \frac{(2400 \times 15) + (1200 \times 40) - (628.3 \times 8.5)}{2400 + 1200 - 628.3} = 26.5mm\ (\boldsymbol{Ans})$$

CENTRIOD OF COMPOSITE FIGURES

When more than one standard plane figures combine to gather, it forms a composite plane figures. To find the centroid (CG) of composite figure, we have to break up in to the standard plane figures and follow the steps as explain in next sub-topic 4.3.1. We have to study only the composite figure, which are composed of not more than three geometrical figures in this book.

Steps for finding centriod of Composite figures

To find CG of composite figures, we have to follow following steps.

Step-1: Divide the given composite (compound) shape into various standard figures. These standard figures include square, rectangles, circles, semicircles, triangles and many more. In dividing the composite figure, include parts with holes (cut out) are to treat as components with negative values. There is also possibility of rotation (90°, 180°, 270° & 360°) of standard figure to adjust in composite section. Make sure that you break down every part of the compound shape in to various components with designate name (Component-1, Component-2 & so on) before proceeding to the next step.

Step-2: Calculate the area of each component as per standard shape from table 4.1 (B). Make the area negative for designated areas that act as holes (cut out).

Step-3: The given figure should have an X-axis and Y-axis as reference line. Draw the X-axis as the horizontal line passing through bottom most point of the given composite figure, while the Y-axis as the vertical line passing through left most point of the given composite figure.

Step-4: Get the distance of the centroid of each component, as divided into standard figure in step-1, from the X-axis and Y-axis as reference lines.

Step-5: Make a calculation in table as shown below.

Sr. No.	Component Name	Area of Component A in mm²	Distance of CG of component from Reference lines		A·x	A·y
			x	y		
1	Component 1	A_1	x_1	y_1	A_1x_1	A_1y_1
2	Component 2	A_2	x_2	y_2	A_2x_2	A_2y_2
n	Component n	A_n	x_n	y_n	A_nx_n	A_ny_n
	Summation	$\Sigma A =$	—	—	$\Sigma A \cdot x =$	$\Sigma A \cdot y =$

Step-6: Use the equations to find the coordinates (x, y) of centroid (CG) from reference lines.
(a) $\bar{x} = \Sigma \cdot \Sigma A x A$ **and (b)** $\bar{y} = \Sigma \cdot \Sigma A y A$

Let to explain above point, take some examples of composite figure (section), as per syllabus composite figure must be composed of not more than three geometrical figures.

Example 1. Find the centroid (CG) of a 100 mm × 150 mm × 30 mm T-section as shown in the figure.

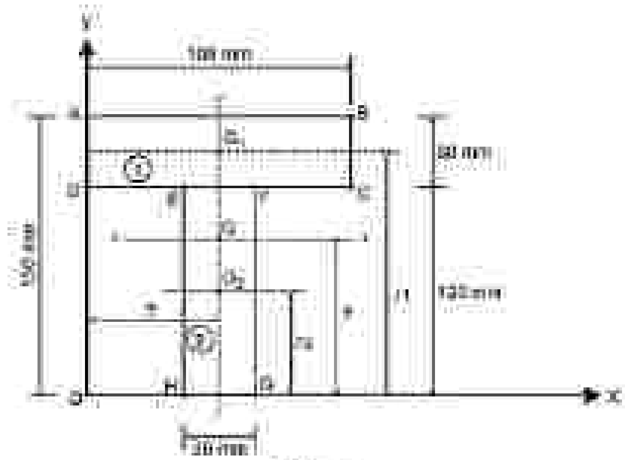


Fig. 4.4

Solution:

Step-1: Divide the given composite (compound) shape into various standard figures. In this case, the T-shape is combination of two rectangles. Name the two components as component-1 for top rectangle (Flange) ABCD & component-2 for vertical rectangle (Web) EFGH as shown in Table below.

Step-2: Calculate the area of each component as per standard shape (Rectangles).

Step-3: The given figure should have an X-axis (OX line) and Y-axis (OY line) as reference line.

Step-4: Get the distance of the centroid (x & y) of each component as per standard figure from reference lines (X-axis & Y-axis). Step-5: Put all the value obtained from step-2 to step-4 in the table as follow.

Sr. No.	Component Name	Area of Component A in mm²	Distance of CG of component from Reference lines		A·x	A·y
			x	y		
1.	Top Rectangle-1 ABCD (100 x 30 mm)	100 x 30 = 3000	$\frac{100}{2} = 50$	$120 + \frac{30}{2} = 135$	150000	405000
2.	Vertical Rectangle-2 EFGH (30 x 120 mm)	30 x 120 = 3600	50 from symmetry	$\frac{120}{2} = 60$	180000	216000
	Summation	$\Sigma A = 6600$	—	—	$\Sigma A \cdot x =$ 330000	$\Sigma A \cdot y =$ 621000

Step-6: Use the equations, to calculate Centroid (CG) of composite plane figure by putting the value from table

(a) $\bar{x} = \Sigma A.x/\Sigma A$

= 776000/11600

=66.90mm (Answer)

(b) $\bar{y} = \Sigma A.y/\Sigma A$

=754000/11600

=65.00mm (Answer)

[As per symmetry of composite figure about yy axis (vertical), we can directly,
Find y = Total width/2 = 130/2=65.00mm, As we obtained by calculations.]

Example 3. Find the centroid of the given composite figure shown in figure

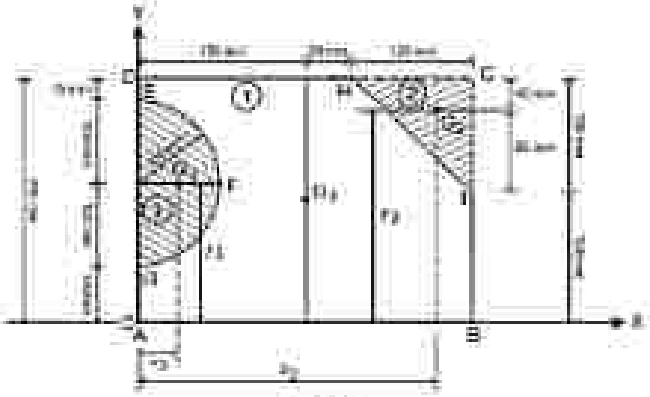


Fig. 4.8

Step-1: Divide the given composite (compound) shape into various stand rd figures. In this case, its combination of three figures. Name the three components as component-1, cut of component-2, and cut of component-3 as shown in Table below. Here both cutouts are oriented from standard shape shown in table 4.1(B), Hence the position of CG shifted accordingly. For semicircle as it oriented to 90°, x coordinator becomes y & vice versa. For right angle triangle as it oriented at 180°, base goes at top, CG distance may be measured accordingly.

Step-2: Calculate the area of each component as per standard shape with negative sign for cuts of component- 2 & 3.

Step-3: The given figure should have an X-axis (AB line) and Y-axis (AD line) as reference line.

Step-4: Get the distance of the centroid (x & y) of each component as per standard figure.

Step-5: Put all the value obtained in step-2 to step-4 in the table.

Sr. No.	Component Name	Area of Component A in mm^2	Distance of CG of component from Reference lines		A.x	A.y
			X	Y		
1	Rectangle-1 [ABCD] (300 x 250 mm)	300 x 250 = 75000	$\frac{300}{2} = 150$	$\frac{250}{2} = 125$	11250000	9375000
2	Cut of Triangle-2 [CHI] (120 mm Base & Height 120mm)	$120 \times \frac{120}{2} = 7200$	300 − 40 = 260	250 − 40 = 210	−1872000	−1512000
3	Cut of Semicircle-3 [EFG] (Radius = 100 mm)	$\frac{\pi \times 100^2}{2} = -15707.96$	$\frac{4 \times 100}{(3 \times \pi)} = 42.44$	35 + 100 = 135	−666643.82	−2120574.60
	Summation	ΣA = 52092.04	—	—	ΣA.x = 8711356.18	ΣA.y = 5742425.40

Step-6: Use the equations, to calculate Centroid (CG) of composite plane figure, put the value from table

(a) $\bar{x} = \Sigma A x / \Sigma A$

= 8711354.18 52092.04

= 167.23 mm (Answer)

(b) $\bar{y} = \Sigma A y / \Sigma A$

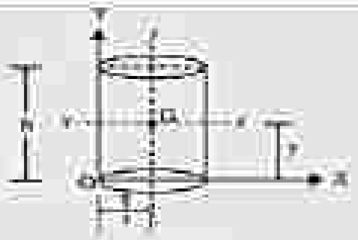
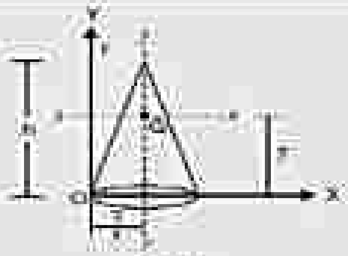
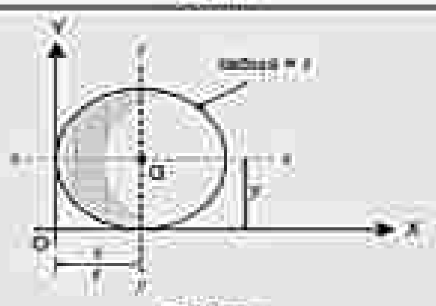
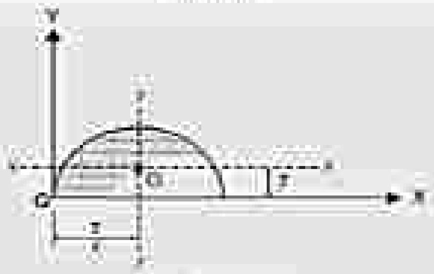
= 5742425.40 52092.04

= 110.23 mm (Answer)

CENTER OF GRAVITY OF SIMPLE SOLIDS [3-D ELEMENTS]

For standard 3 D elements (Simple solids), the center of gravity (CG) are shown in Table 4.2

Table 4.2: Center of Gravity (CG) of Three Dimensional Standard Solid

Sr. No.	Geometrical Shape	Volume	$\bar{x}$	$\bar{y}$
1.	 Cylinder	$V = \pi r^2 h$	r	$\frac{h}{2}$
2.	 Cone	$V = \frac{\pi}{3} r^2 h$	r	$\frac{h}{4}$
3.	 Sphere	$V = \frac{4}{3} \pi r^3$	r	r
4.	 Hemisphere	$V = \frac{2}{3} \pi r^3$	r	$\frac{3r}{8}$

CENTRE OF GRAVITY (CG) OF COMPOSITE SOLIDS

In this, we have to consider the volume of solids (V) instead of area (A) considered in plane figure. All other process remains same, but for conveyance the step list out as follows.

Step-1: Divide the given composite (compound)solidsinto variousstandard solids. These standard solids include Cone, Cylinder, Sphere, Hemi sphere. In dividing the composite solids, include parts with holes (cut out) are to treat as components with negative values. Make sure that you break down every part of the compound solids in to various components with designate name (Component-1, Component-2 & so on) before proceeding to the next step.

Step-2: Calculate the volume of each components as per standard solid from table 4.2. Make the volume negative for designated solid that act as holes (cut out).

Step-3: The given solid should have an X-axis and Y-axis as reference line. Draw the X-axis as the horizontal line passing through bottom most point of the given composite solid, while the Y-axis as the vertical line passing through left most point of the given composite solid.

Step-4: Get the distance of the centroid of each components as divided into standard solid in step-1 from the X-axis and Y-axis as reference lines.

Step-5: Make a calculation in tableas shown below.

Sr. No.	Component Name	Volume of Component V in mm³	Distance of CG of component from Reference lines		V·x	V·y
			x	y		
1	Component 1	V_1	x_1	y_1	V_1x_1	V_1y_1
2	Component 2	V_2	x_2	y_2	V_2x_2	V_2y_2
n	Component n	V_n	x_n	y_n	V_nx_n	V_ny_n
	Summation	ΣV =	—	—	ΣV·x =	ΣV·y =

Step-6: Use the equations to find the coordinates ($\bar{x}$, $\bar{y}$) of centroid (CG) from reference lines.

(a) $\bar{x} = \Sigma\ V \cdot x / \Sigma V$ and

(b) $\bar{y} = \Sigma A \cdot x / \Sigma A$

Let to explain above point, take some examples of composite solid (section), as per syllabus composite solid must be composed of not more than two geometrical solids.

Example 4. Find the centre of gravity (CG) of the composite solid having cylinder of diameter and height as same with 160 mm which supports a cone of base diameter and height as same with 160 mm. Show the position of CG in the figure.

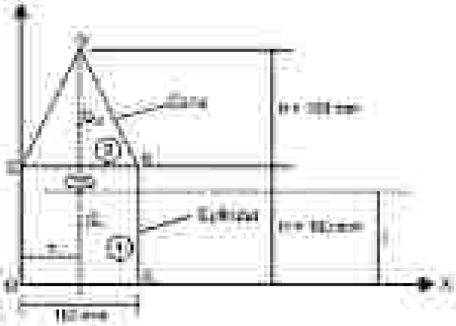


Fig. 4.7

Solution:

Step-1: Divide the given composite (compound) solids into two standard solids. Here bottom part cylinder OABC designate as component-1 and upper part cone BCD designate as component-2 as shown in fig.

Step-2: Calculate the volume of each components as per standard solid from table 4.2.

Step-3: The given solid should have an X-axis and Y-axis as reference line. Draw the X-axis as the horizontal line passing through bottom most point of the given composite solid, while the Y-axis as the vertical line passing through left most point of the given composite solid.

Step-4: Get the distance of the centroid of each components as divided into standard solid in step-1 from the X-axis and Y-axis as reference lines. Step-5: Make a calculation in table as shown below.

Sr. No.	Component Name	Volume of Component V in mm³	Distance of CG of component from Reference lines		V·x	V·y
			x	y		
1	Cylinder OABC (D = H = 160 mm)	$\pi R^2 H$ $= \pi \times 80^2 \times 160$ $= 3216990.88$	$\frac{D}{2} = \frac{160}{2} = 80$	$\frac{H}{2} = \frac{160}{2} = 80$	257359270.40	257359270.40
2	Cone BCD (d = h = 160 mm)	$\frac{\pi r^2 h}{3} = \frac{\pi \times 80^2 \times 160}{3} = 1072330.29$	$\frac{d}{2} = \frac{160}{2} = 80$	$H + \frac{h}{4} = 160 + \frac{160}{4} = 200$	85786423.20	214466058.00
	Summation	ΣV = 4289321.17	—	—	ΣV·x = 343145693.60	ΣV·y = 471825328.40

Step-6: Use the equations to find the coordinates (x, y) of centroid (CG) from referance lines.

(a) $\bar{x} = \Sigma\ V.x / \Sigma V$

= 343145693.60 4289321.17

= 80.00 mm (Answer)

(b) $\bar{y} = \Sigma\ V \cdot y / \Sigma V$

= 471825328.40 4289321.17

= 110.00 mm (Answer)

Example 5. A frustum of cone is having base diameter 100 mm and top diameter 50 mm with height as 100 mm. Find the CG of this frustum.

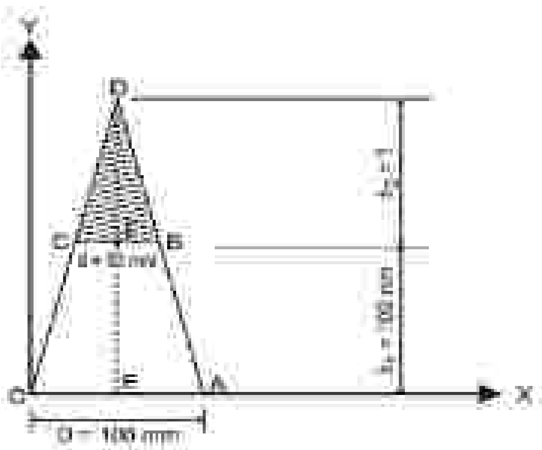


Fig. 4.8

Solution: Here to get frustum, we have to substrate upper cone BCD from full cone OAD as shown in fig.
For full cone OAD, compare triangles DOE & DCF,
we get DE / OE = DF / CF
DE / 50 = (DE - 100)/ 25
25 DE = 50 DE – 5000
25 DE = 5000
DE = 200 mm.
h2 = DF
= DE – EF
= 200 – 100
h2 = 100 mm

Step-1: To get the given solids, we have to substrate upper cone BCD from full cone OAD as shown in fig. Here full cone OAD designate as component- 1 and upper cone BCD designate as component- 2 as shown in fig.
Step-2: Calculate the volume of each components as per standard solid from table 4.2.
Step-3: The given solid should have an X-axis and Y-axis as reference line. Draw the X-axis as the horizontal line passing through bottom most point of the given composite solid, while the Y-axis as the vertical line passing through left most point of the given composite solid.
Step-4: Get the distance of the centroid of each components as divided into standard solid in step-1 from the X-axis and Y-axis as reference lines.
Step-5: Make a calculation in table as shown below.

Sr. No.	Component Name	Volume of Component V in mm^3	Distance of CG of component from Reference lines x	y	V·x	V·y
1.	Full cone OAD (D = 100 mm & H = 200 mm)	$\pi r^2 H$ = π x 50² x 200 = 1570796.33	$\frac{D}{2} = \frac{100}{2}$ = 50	$\frac{H}{4} = \frac{200}{4}$ = 50	78539816.50	78539816.50
2.	Cut of Upper Cone BCD (d = 50 mm & h_2 = 100 mm)	$-\pi r^2 h$ = π x 25² x 100 = −196349.54	50 From Symmetry	$h_1 + \frac{h_2}{4} = 100 + \frac{100}{4}$ = 200	-9817477.04	-24543692.50
	Summation	ΣV = 1374446.79	—	—	ΣV·x = 68722339.46	ΣV·y = 53996124.00

Step-6: Use the equations to find the coordinates (x, y) of centroid (CG) from reference lines.

(a) $\bar{x}$ = Σ V · x/ Σ V
= 343145693.60 / 4289321.17
= 50.00 mm (Answer)
[As per symmetry of composite figure about YY axis (vertical), we can directly,
Find x = Total width / 2
= 100 / 2
= 50.00 mm; As we obtained by calculations.]
(b) $\bar{y}$ = Σ V· y Σ V
= 53996124.00 / 4289321.17
= 39.29 mm (Answer)

POSSIBLE SHORT TYPE QUESTIONS WITH ANSWER

1. Define centre of gravity. (Possible)
Ans: Centre of gravity of a body may be defined as the point through which the whole weight of a body may be assumed to act.

2. Define centroid. (W – 2016 & S – 2019)
Ans: Centroid or Centre of area is defined as the point where the whole area of the figure is assumed to be concentrated.

3. State perpendicular axis theorem. (S – 2019)
Ans: It states, If I_{XX} and I_{YY} be the moments of inertia of a plane section about two perpendicular axis meeting at O, the moment of inertia I_{ZZ} about the axis Z-Z, perpendicular to the plane and passing through the intersection of X-X and Y-Y is given by:

$$I_{ZZ} = I_{XX} + I_{YY}$$

4. State parallel axis theorem. (Possible)
Ans: It states, If the moment of inertia of a plane area about an axis through its centre of gravity is denoted by IG, then moment of inertia of the area about any other axis AB, parallel to the first, and at a distance h from the centre of gravity is given by:

$$I_{AB} = I_G + ah^2$$

5. What is the distance of centroid of a semi-circular area from the base? (W – 2017 & S – 2018)
Ans:

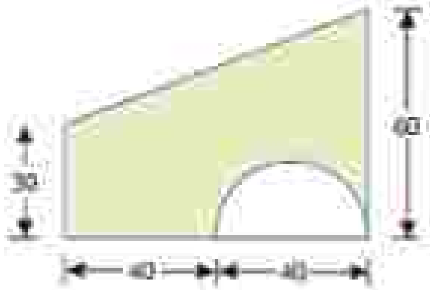
$$\bar{y} = \frac{4r}{3\pi} = 0.424\,r$$

POSSIBLE LONG TYPE QUESTIONS

1. State and prove perpendicular axis theorem. **(Possible)**

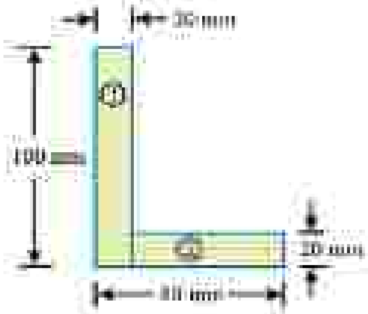
2. State and prove parallel axis theorem. **(S – 2019 Old)**

3. A semi-circular area is removed from a trapezium as shown in Figure (dimensions in mm) Determine the centroid of the remaining area (shown hatched). **(W – 2016)**



4. Find the position of the centroid of an angle section (L – section) having dimension of 150 mm × 200 mm × 20 mm. **(S – 2018)**

5. Find the position of centroid of a L section as shown in the figure below. **(S – 2019)**

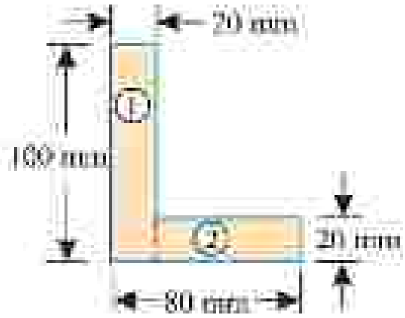


6. Find the moment of inertia of an I – section having following dimensions about centroidal X-X axis and Y-Y axis.

Top flange: 150 mm × 20 mm
Web: 150 mm × 20 mm
Bottom flange: 300 mm × 30 mm

(S – 2018)

7. Find the moment of inertia of the given section about the centroidal X-X and Y-Y axes. **(S – 2019 old)**



CHAPTER NO. – 05

SIMPLE LIFTING MACHINE

Introduction:

- Man invented various types of machines for his easy work. Sometimes, one person cannot do heavy work, but with the help of machine, the same work can be easily done.
- To change the tyre of a car, number of persons will be required. But with the help of a "Jack", the same work can be done by a single man. Therefore, jack acts as a machine by which the load of a car can be lifted by applying very small force as compared to the load of car.

Simple lifting machine:

- It is a device, which enables us to lift a heavy load (W) by applying a comparatively smaller effort (P).

Compound lifting machine:

- A compound lifting machine may be defined as a device, consisting of a number of simple machines, which enables us to do some useful work at a faster speed or with a much less effort as compared to a simple machine.

Effort:

- It may be defined as, the force which is applied so as to overcome the resistance or to lift the load. It is denoted by 'P'.

Load:

- The weight to be lifted or the resistive force to be overcome with the help of a machine is called as load (W).

Mechanical Advantage:

- The mechanical advantage (briefly written as M.A.) is the ratio of weight lifted (W) to the effort applied (P) and is always expressed in pure number.
- Mathematically, mechanical advantage,

$$M.A. = \frac{W}{P}$$

Velocity Ratio:

- The velocity ratio (briefly written as V.R.) is the ratio of distance moved by the effort (y) to the distance moved by the load (x) and is always expressed in pure number.
- Mathematically, velocity ratio,

$$V.R. = \frac{y}{x}$$

Input of a Machine:

- The input of a machine is the work done on the machine. In a lifting machine, it is measured by the product of effort (P) and the distance (y) through which it has moved.
- Mathematically,

$$Input\ of\ a\ machine = P \times y$$

Output of a Machine:

- The output of a machine is the actual work done by the machine. In a lifting machine, it is measured by the product of the weight lifted (W) and the distance (x) through which it has been lifted.
- Mathematically,

$$Output\ of\ a\ machine = W \times x$$

Efficiency of a Machine:

- It is the ratio of output to the input of a machine and is generally expressed as a percentage.
- Mathematically, efficiency,

$$\eta = \frac{Output}{Input} \times 100$$

Ideal Machine:

- If the efficiency of a machine is 100% i.e., if the output is equal to the input, the machine is called as a perfect or an ideal machine.
- In practical cases no machine is 100% efficient. All are real machines whose efficiencies are less than unity.

Relation between efficiency, mechanical advantage and velocity ratio of a lifting machine:

- It is an important relation of a lifting machine, which throws light on its mechanism. Now consider a lifting machine, whose efficiency is required to be found out.

- Let, W = Load lifted by the machine,
  P = Effort required to lift the load,
  y = Distance moved by the effort, in lifting the load, and
  x = Distance moved by the load.

- We know that,

$$M.A. = \frac{W}{P} \text{ and } V.R. = \frac{y}{x}$$

- We also know that input of a machine = Effort applied × Distance through which the effort has moved = $P \times y$_______ (i)
- output of a machine = Load lifted × Distance through which the load has been lifted = $W \times x$ _____(ii)
- ∴ Efficiency,

$$\eta = \frac{\textbf{Output}}{\textbf{Input}} = \frac{W \times x}{P \times y} = \frac{W/P}{y/x} = \frac{M.A.}{V.R.}$$

Example – 1: ***In a certain weight lifting machine, a weight of 1 kN is lifted by an effort of 25 N. While the weight moves up by 100 mm, the point of application of effort moves by 8 m. Find mechanical advantage, velocity ratio and efficiency of the machine.***

Solution. Given: Weight (W) = 1 kN = 1000 N; Effort (P) = 25 N; Distance through which the weight is moved (x) = 100 mm = 0.1 m and distance through which effort is moved (y) = 8 m.

Mechanical advantage of the machine.

We know that mechanical advantage of the machine

$$M.A. = \frac{W}{P} = \frac{1000}{25} = \mathbf{40}$$

Velocity ratio of the machine

We know that velocity ratio of the machine

$$V.R. = \frac{y}{x} = \frac{8}{0.1} = \mathbf{80}$$

Efficiency of the machine

We also know that efficiency of the machine

$$\eta = \frac{M.A.}{V.R.} = \frac{40}{80} = 0.5 = \mathbf{50\%}$$

Reversibility of a Machine:

- When a machine is capable of doing some work in the reverse direction even on removal of effort, it is called as reversible machine and its action is known as reversibility of the machine.

Condition For the Reversibility of a Machine:

- Consider a reversible machine, whose condition for the reversibility is required to be found out.

- Let, W = Load lifted by the machine,
  P = Effort required to lift the load,
  y = Distance moved by the effort, and

x = Distance moved by the load.

- We know that input of the machine = $P \times y$ and Output of the machine = $W \times x$
- We also know that machine friction = Input – Output = $(P \times y) - (W \times x)$
- A little consideration will show that in a reversible machine, the output of the machine should be more than the machine friction, when the effort (P) is zero. i.e.

$$(W \times x) > (P \times y) - (W \times x)$$

$$\Rightarrow 2(W \times x) > (P \times y)$$

$$\Rightarrow \frac{W \times x}{P \times y} > \frac{1}{2}$$

$$\Rightarrow \frac{W/P}{y/x} > \frac{1}{2}$$

$$\Rightarrow \frac{M.A.}{V.R.} > \frac{1}{2}$$

$$\Rightarrow \eta > \frac{1}{2} = 0.5 = 50\%$$

- Hence the condition for a machine, to be reversible, is that its **efficiency should be more than 50%.**

Self-Locking Machine:

- When a machine is not capable of doing some work in the reverse direction even on removal of effort, it is called as irreversible machine or non-reversible machine or self-locking machine.
- The condition for a machine to be non – reversible or self – locking is that its efficiency should not be more than 50% i.e., **η < 50%.**

Example – 2: ***A certain weight lifting machine of velocity ratio 30 can lift a load of 1500 N with the help of 125 N effort. Determine if the machine is reversible.***

Solution. Given: Velocity ratio (V.R.) = 30; Load (W) = 1500 N and effort (P) = 125 N.

We know that

$$M.A. = \frac{W}{P} = \frac{1500}{125} = 12$$

and efficiency

$$\eta = \frac{M.A.}{V.R.} = \frac{12}{30} = 0.4 = 40\%$$

Since efficiency of the machine is less than 50%, therefore the machine is non-reversible.

Example – 3: ***In a lifting machine, whose velocity ratio is 50, an effort of 100 N is required to lift a load of 4 kN. Is the machine reversible? If so, what effort should be applied, so that the machine is at the point of reversing?***

Solution. Given: Velocity ratio (V.R.) = 50; Effort (P) = 100 N and load (W) = 4 kN = 4000 N.

Reversibility of the machine

We know that

$$M.A. = \frac{W}{P} = \frac{4000}{100} = 40$$

and efficiency

$$\eta = \frac{M.A.}{V.R.} = \frac{40}{50} = 0.8 = 80\%$$

Since efficiency of the machine is **more than 50%,** therefore the machine is reversible.

Effort to be applied

A little consideration will show that the machine will be at the point of reversing, when its efficiency is 50% or 0.5.
Let P_1 = Effort required to lift a load of 4000 N when the machine is at the point of reversing.

We know that

$$M.A. = \frac{W}{P_1} = \frac{4000}{P_1}$$

and efficiency

$$0.5 = \frac{M.A.}{V.R.} = \frac{4000/P_1}{50} = \frac{80}{P_1}$$

$$\Rightarrow P_1 = \frac{80}{0.5} = 160\ N$$

Ideal machine :

A machine having 100% efficiency is called an ideal machine. In an ideal machine friction is zero.
For ideal machine, Output = Input or MA = VR

Effort lost in friction (Pf) :

In a simple machine, effort required to overcome the friction between various parts of a machine is called effort lost in friction.
Let P = Effort, Po = Effort for ideal machine, Pf = Effort lost in friction
Effort lost in friction, $P_f = P - P_o$
For Ideal machine MA = VR
W/P_o = VR
P_o = W/ VR = Ideal effort
Due to friction, Actual P > Ideal Effort P_o
$P_f = P - P_o$
$P_f = P - (W/VR)$

Friction load (Wf) :

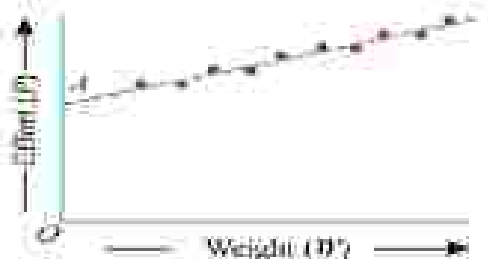
Total friction force produced, when machine is in motion, is called friction load.
Let W = Load (Actual), Wo = Load for Ideal machine and P = Effort
For ideal machine, MA = VR
$W_o = P \times VR$ = Ideal load
Now, friction load $W_f = W_o - W$
$W_f = (P \times VR) - W$

Law of Machine:

- The equation which gives the relation between load lifted and effort applied in the form of a slope and intercept of a straight line is called as Law of a machine.
- Mathematically, the law of a lifting machine is given by the relation:

$$P = mW + C$$

- Where, P = Effort applied to lift the load,
  m = A constant (called coefficient of friction) which is equal to the slope of the line AB,
  W = Load lifted, and
  C = another constant, which represents the machine friction, (i.e. OA)



Maximum Mechanical Advantage of a Lifting Machine:

- We know that mechanical advantage of a lifting machine,

$$M.A. = \frac{W}{P}$$

- For maximum mechanical advantage, substituting the value of $P = mW + C$ in the above equation,

$$Max.\,M.A. = \frac{W}{mW + C} = \frac{1}{m + \frac{C}{W}} = \frac{1}{m}$$

$$\ldots\ldots\left(Neglecting\ \frac{C}{W}\right)$$

Maximum Efficiency of a Lifting Machine:

- We know that efficiency of a lifting machine,

$$\eta = \frac{M.A.}{V.R.}$$

- A little consideration will show that the efficiency will be maximum, when the mechanical advantage will be maximum.

$$Max.\,\eta = \frac{Max.\ M.A.}{V.R.} = \frac{1}{m \times V.R.}$$

Example – 4: ***What load can be lifted by an effort of 120 N, if the velocity ratio is 18 and efficiency of the machine at this load is 60%? Determine the law of the machine, if it is observed that an effort of 200 N is required to lift a load of 2600 N and find the effort required to run the machine at a load of 3.5 kN.***

Solution. Given: Effort (P) = 120 N; Velocity ratio (V.R.) = 18 and efficiency (η) = 60% = 0.6.

Load lifted by the machine

Let W = Load lifted by the machine.

We know that

$$M.A. = \frac{W}{P} = \frac{W}{120}$$

and efficiency,

$$0.6 = \frac{M.A.}{V.R.} = \frac{W/120}{18} = \frac{W}{2160}$$

$$\therefore W = 0.6 \times 2160 = 1296\,N$$

Law of the machine

In the second case, P = 200 N and W = 2600 N

Substituting the two values of P and W in the law of the machine, i.e., $P = mW + C$,

$$120 = m \times 1296 + C \ldots\ldots\ldots(i)$$

$$200 = m \times 2600 + C \ldots\ldots\ldots(ii)$$

Subtracting equation (*i*) from (*ii*),

$$80 = 1304\,m$$

$$\Rightarrow m = \frac{80}{1304} = 0.06$$

and now substituting the value of *m* in equation (*ii*)

$$200 = (0.06 \times 2600) + C = 156 + C$$

$$\Rightarrow C = 200 - 156 = 44$$

Now substituting the value of m = 0.06 and C = 44 in the law of the machine,

$$P = 0.06\,W + 44$$

Effort required to run the machine at a load of 3.5 kN.

Substituting the value of W = 3.5 kN or 3500 N in the law of machine,

$$P = (0.06 \times 3500) + 44 = 254\,N$$

Example – 5: ***In a lifting machine, an effort of 40 N raised a load of 1 kN. If efficiency of the machine is 0.5, what is its velocity ratio? If on this machine, an effort of 74 N raised a load of 2 kN, what is now the efficiency? What will be the effort required to raise a load of 5 kN?***
Solution. Given: When Effort (P) = 40 N; Load (W) = 1 kN = 1000 N; Efficiency (η) = 0.5; When effort (P) = 74 N and load (W) = 2 kN = 2000 N.
Velocity ratio when efficiency is 0.5.
We know that

$$M.A. = \frac{W}{P} = \frac{1000}{40} = 25$$

and efficiency

$$0.5 = \frac{M.A.}{V.R.} = \frac{25}{V.R.}$$
$$\Rightarrow V.R. = \frac{25}{0.5} = 50$$

Efficiency when P is 74 N and W is 2000 N

We know that

$$M.A. = \frac{W}{P} = \frac{2000}{74} = 27$$

and efficiency

$$\eta = \frac{M.A.}{V.R.} = \frac{27}{50} = 0.54 = 54\%$$

Effort required to raise a load of 5 kN or 5000 N

Substituting the two values of P and W in the law of the machine, i.e., $P = mW + C$

$$40 = m \times 1000 + C \ldots\ldots\ldots\ldots (i)$$
$$74 = m \times 2000 + C \ldots\ldots\ldots\ldots (ii)$$

Subtracting equation (i) from (ii),

$$34 = 1000\,m$$
$$\Rightarrow m = \frac{34}{1000} = 0.034$$

and now substituting this value of m in equation (i),

$$40 = (0.034 \times 1000) + C = 34 + C$$
$$\Rightarrow C = 40 - 34 = 6$$

Substituting these values of m = 0.034 and C = 6 in the law of machine,

$$P = 0.034\,W + 6 \ldots\ldots\ldots\ldots (iii)$$

Effort required to raise a load of 5000 N,

$$P = (0.034 \times 5000) + 6 = 176\,N$$

N.B: <u>Friction in a machine</u>

- $F_{effort} = P - \frac{W}{V.R.}$

- $F_{load} = (P \times V.R.) - W$

Simple Wheel and Axle:

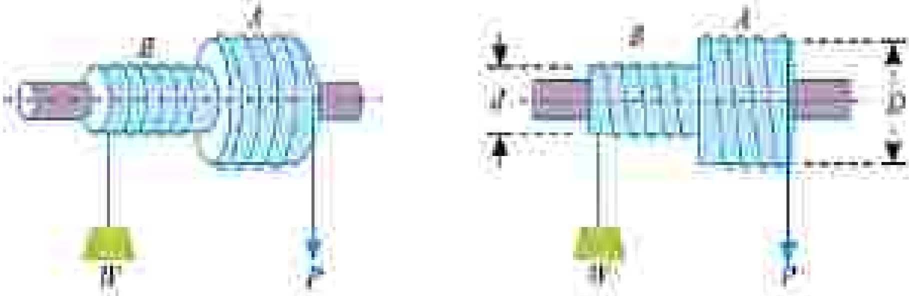


Fig. Simple wheel and axle

- The above figure shows a simple wheel and axle, in which the wheel A and axle B are keyed to the same shaft. The shaft is mounted on ball bearings, order to reduce the frictional resistance to a minimum. A string is wound round the axle B, which carries the load to be lifted. A second string is wound round the wheel A in the opposite direction to that of the string on B.
- Let, D = Diameter of effort wheel,
  d = Diameter of the load axle,
  W = Load lifted, and
  P = Effort applied to lift the load.
- One end of the string is fixed to the wheel, while the other is free and the effort is applied to this end. Since the two strings are wound in opposite directions, therefore a downward motion of the effort (P) will raise the load (W).
- Since the wheel as well as the axle is keyed to the same shaft, therefore when the wheel rotates through one revolution, the axle will also rotate through one revolution.
- We know that displacement of the effort in one revolution of effort wheel A = πD .... (i)
  And displacement of the load in one revolution = πd .... (ii)

$$\therefore V.R. = \frac{Distance\ moved\ by\ the\ effort}{Distance\ moved\ by\ the\ load} = \frac{\pi D}{\pi d} = \frac{D}{d}$$

$$M.A. = \frac{Load\ lifted}{Effort\ applied} = \frac{W}{P}$$

$$\eta = \frac{M.A.}{V.R.}$$

Example – 6: ***A simple wheel and axle has wheel and axle of diameters of 300 mm and 30 mm respectively. What is the efficiency of the machine, if it can lift a load of 900 N by an effort of 100 N?***

Solution. Given: Diameter of wheel (D) = 300 mm; Diameter of axle (d) = 30 mm; Load lifted by the machine (W) = 900 N and effort applied to lift the load (P) = 100 N

We know that velocity ratio of the simple wheel and axle,

$$V.R. = \frac{D}{d} = \frac{300}{30} = 10$$

and mechanical advantage

$$M.A. = \frac{W}{P} = \frac{900}{100} = 9$$

$\therefore$ Efficiency,

$$\eta = \frac{M.A.}{V.R.} = \frac{9}{10} = 0.9 = 90\%$$

Example 5.8: A drum weighing 60 N and holding 420N of water is to be raised from a well by means of wheel and axle. The axle is 100 mm diameter and the wheel is 500 mm diameter. If a force of 120 N has to be applied to the wheel, find (i) mechanical advantage, (ii) velocity ratio and (iii) efficiency of the machine.

Solution. Given: Total load to be lifted (W) = 60 + 420 = 480 N; Diameter of the load axle (d) = 100 mm; Diameter of effort wheel (D) = 500 mm and effort (P) = 120 N.

Mechanical advantage

We know that mechanical advantage

$$M.A. = \frac{W}{P} = \frac{480}{120} = 4$$

Velocity ratio

We know that velocity ratio

$$V.R. = \frac{D}{d} = \frac{500}{100} = 5$$

Efficiency of the machine

We also know that efficiency of the machine,

$$\eta = \frac{M.A.}{V.R.} = \frac{4}{5} = 0.8 = 80\%$$

Single Purchase Crab Winch:

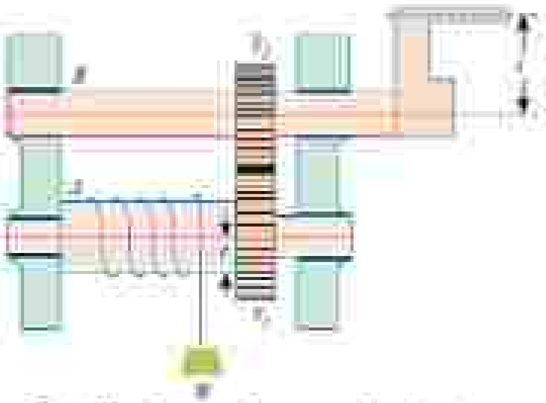


Fig. Single purchase crab winch

- In single purchase crab winch, a rope is fixed to the drum and is wound a few turns round it. The free end of the rope carries the load W. A toothed wheel A is rigidly mounted on the load drum. Another toothed wheel B, called pinion, is geared with the toothed wheel A as shown in the Figure. The effort is applied at the end of the handle to rotate it.

- Let, T_1 = No. of teeth on the main gear (or spur wheel) A,
  T_2 = No. of teeth on the pinion B,
  l = Length of the handle,
  r = Radius of the load drum,
  W = Load lifted, and
  P = Effort applied to lift the load

- $We\ know\ that\ distance\ moved\ by\ the\ effort\ in\ one\ revolution\ of\ the\ handle = 2\pi l$

$\therefore No.\ of\ revolutions\ made\ by\ the\ pinion\ B = 1$

$And\ no.\ of\ revolutions\ made\ by\ the\ wheel\ A = \frac{T_2}{T_1}$

$\therefore$ *No. of revolutions made by the load drum* $= \frac{T_2}{T_1}$

And distance moved by the load $= 2\pi r \times \frac{T_2}{T_1}$

$$\therefore V.R. = \frac{\textbf{Distance moved by the effort}}{\textbf{Distance moved by the load}} = \frac{2\pi l}{2\pi r \times \frac{T_2}{T_1}} = \frac{l}{r} \times \frac{T_1}{T_2}$$

$$M.A. = \frac{Load\ lifted}{Effort\ applied} = \frac{W}{P}$$

$$\eta = \frac{M.A.}{V.R.}$$

Double Purchase Crab Winch:

- A double purchase crab winch is an improved form of a single purchase crab winch, in which the velocity ratio is intensified with the help of one more spur wheel and a pinion. In a double purchase crab winch, there are two spur wheels of teeth T_1 and T_2 and T_3 as well as two pinions of teeth T_2 and T_4.
- The arrangement of spur wheels and pinions are such that the spur wheel with T_1 gears with the pinion of teeth T_2. Similarly, the spur wheel with teeth T_3 gears with the pinion of the teeth T_4, the effort is applied to a handle as shown in the figure below.

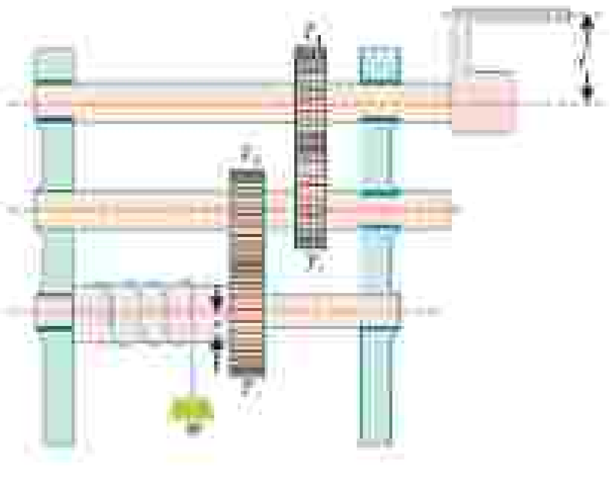


Fig. Double purchase crab winch

- Let, T_1 and T_3 = No. of teeth of spur wheels,
  T_2 and T_4 = No. of teeth of the pinions,
  l = Length of the handle,
  r = Radius of the load drum,
  W = Load lifted, and
  P = Effort applied to lift the load, at the end of the handle.
- *We know that distance moved by the effort in one revolution of the handle* $= 2\pi l$

$\therefore$ *No. of revolutions made by the pinion 4* $= 1$

and no. of revolutions made by the wheel 3 $= \frac{T_4}{T_3}$

$\therefore$ *No. of revolutions made by the pinion 2* $= \frac{T_4}{T_3}$

and no. of revolutions made by the wheel 1 $= \frac{T_2}{T_1} \times \frac{T_4}{T_3}$

$$\therefore \text{Distance moved by the load} = 2\pi r \times \frac{T_2}{T_1} \times \frac{T_4}{T_3}$$

$$\therefore V.R. = \frac{\textbf{Distance moved by the effort}}{\textbf{Distance moved by the load}}$$

$$\Rightarrow V.R. = \frac{2\pi l}{2\pi r \times \frac{T_2}{T_1} \times \frac{T_4}{T_3}} = \frac{l}{r}\left(\frac{T_1}{T_2} \times \frac{T_3}{T_4}\right)$$

$$M.A. = \frac{\text{Load lifted}}{\text{Effort applied}} = \frac{W}{P}$$

$$\eta = \frac{M.A.}{V.R.}$$

Worm And Worm Wheel:

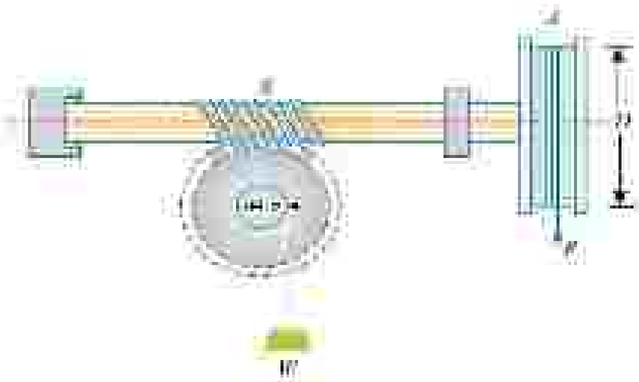


Fig. Worm and worm wheel

- It consists of a square threaded screw, S (known as worm) and a toothed wheel (known as worm wheel) geared with each other, as shown in the above figure. A wheel 'A' is attached to the worm, over which passes a rope as shown in the figure. Sometimes, a handle is also fixed to the worm (instead of the wheel). A load drum is securely mounted on the worm wheel.

- Let, D = Diameter of the effort wheel,
  r = Radius of the load drum
  W = Load lifted,
  P = Effort applied to lift the load, and
  T = No. of teeth on the worm wheel.

- We know that distance moved by the effort in one revolution of the wheel (or handle) $= \pi D$

- If the worm is single-threaded (i.e., for one revolution of the wheel A, the screw S pushes the worm wheel through one teeth), then the load drum will move through

$$= \frac{1}{T}$$

and distance, through which the load will move

$$= \frac{2\pi r}{T}$$

$$\therefore V.R. = \frac{\textbf{Distance moved by the effort}}{\textbf{Distance moved by the load}} = \frac{\pi D}{\frac{2\pi r}{T}} = \frac{DT}{2r}$$

$$M.A. = \frac{Load\ lifted}{Effort\ applied} = \frac{W}{P}$$

$$\eta = \frac{M.A.}{V.R.}$$

Notes:

- If the worm is double-threaded i.e., for one revolution of wheel A, the screw S pushes the worm wheel through two teeth, then

$$V.R. = \frac{DT}{2 \times 2r} = \frac{DT}{4r}$$

- In general, if the worm is n threaded, then

$$V.R. = \frac{DT}{2nr}$$

Simple Screw Jack:

- It consists of a screw, fitted in a nut, which forms the body of the jack. The principle, on which a screw jack works, is similar to that of an inclined plane.

- The figure shows a simple screw jack, which is rotated by the application of an effort at the end of the lever, for lifting the load. Now consider a single threaded simple screw jack.
- Let, l = Length of the effort arm,
  p = Pitch of the screw,
  W = Load lifted, and
  P = Effort applied to lift the load at the end of the lever

- We know that distance moved by the effort in one revolution of screw = $2\pi l$

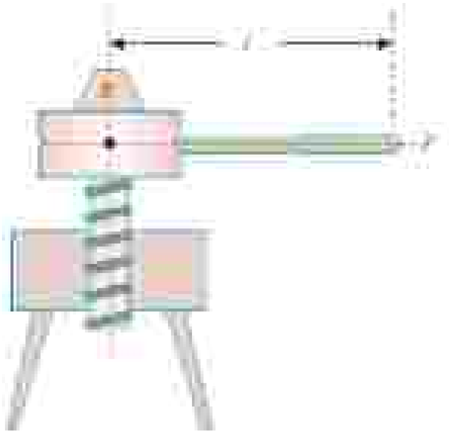


Fig. Simple screw Jack

- Distance moved by the load = p

$$\therefore V.R. = \frac{Distance\ moved\ by\ the\ effort}{Distance\ moved\ by\ the\ load} = \frac{2\pi l}{p}$$

$$M.A. = \frac{Load\ lifted}{Effort\ applied} = \frac{W}{P}$$

$$\eta = \frac{M.A.}{V.R.}$$

Differential axle and wheel

In fig. 5.3 is shown a differential axle and wheel. In this case, the load axle BC is made of two parts of different diameters & effort wheel A are key to same shaft.

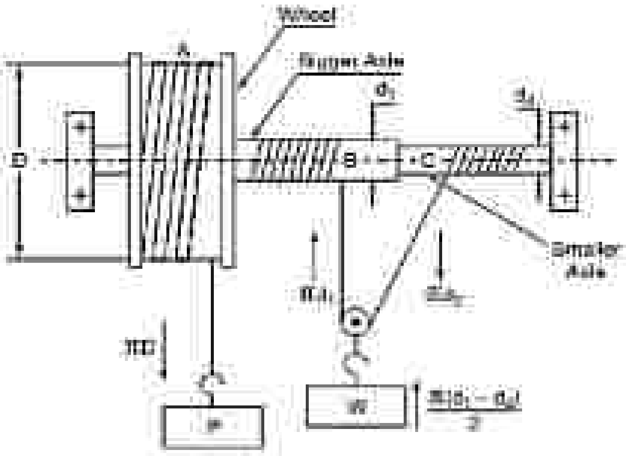


Fig. 5.3: Differential axle and wheel

The effort string is wound round the wheel A and another string is wound round the axle B which after passing round the pulley (to which the weight to be lift is attached) is wound round the axle C in opposite direction to that of axle B. So unwinds string from wheel A, other string also unwinds from axle C. But it winds on axle B to lift the load W.

Let D = Diameter of wheel, d1 = Diameter of bigger axle & d2 = Diameter of smaller axle,

then

$$VR = 2D/\ (d_1\text{-}d_2)$$

Weston's differential pulley block

It consists of two pulley blocks A and B. The upper block A has two pulleys (P1 & P2), one having its diameter a little larger than that of the other. i.e. both of pulley behaves as one pulley with two grooves. The lower block B also carries a pulley, to which the load W is attach to lift up. A continuous chain passes around the pulley P1 then around the lower block pulley and then finally round the pulley P2. The effort P is apply to the chain passing over the pulley P1, so that load W can be lift up as shown in fig 5.8.

Let D = Diameter of bigger pulley and d = Diameter of smaller pulley,

then

$$VR = 2D/(\ D - d\ )$$

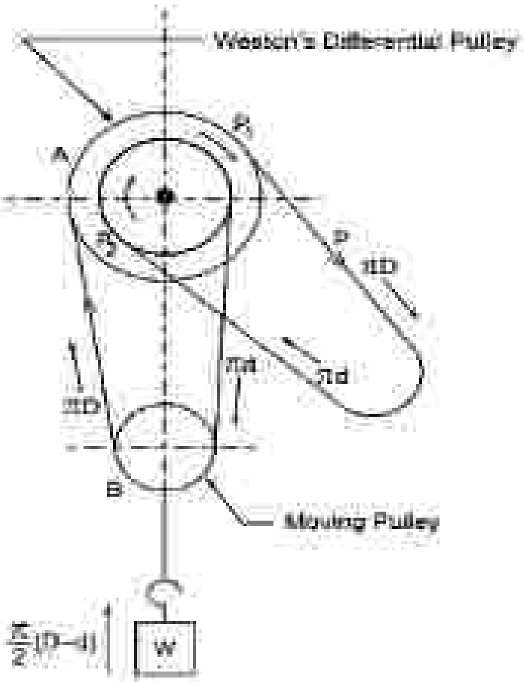


Fig. 5.8: Weston's differential pulley block

Geared pulley block

In consists of a cog wheel A, around which is passed an endless chain. A small gear wheel B known as pinion is key to the same shaft as that of A. The wheel axle B is gear with another bigger wheel C called the spur wheel. A cogwheel D is key to the same shaft as that of spur wheel C. The load W is attach to a chain that passes over the cogwheel D and the effort P is applied to the endless chain, which passes over the wheel A as shown in fig. 5.9.

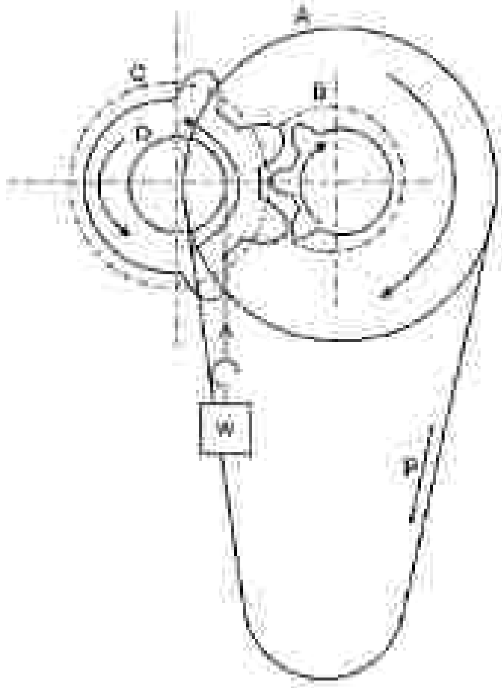


Fig. 5.9: Geared pulley block

Let T1 = No. of cogs on effort wheel A, T2 = No. of teeth on pinion wheel B, T3 = No. of teeth on spur wheel C, T4 = No. of cogs on load wheel D,
then

$$\mathbf{VR = (T_1/T_2) \times (T_3/T_4)}$$

POSSIBLE SHORT TYPE QUESTIONS WITH ANSWER:

1. Define simple lifting machine. (W – 2016)

Ans: It is a device, which enables us to lift a heavy load (*W*) by applying a comparatively smaller effort (*P*). A simple machine may be defined as a device, which enables us to do some useful work at some point or to overcome some resistance, when an effort or force is applied to it, at some other convenient point.

2. Define mechanical advantage of a machine. (Possible)

- Ans: The mechanical advantage (briefly written as M.A.) is the ratio of weight lifted (*W*) to the effort applied (*P*) and is always expressed in pure number.
- Mathematically, mechanical advantage,

$$M.A. = \frac{W}{P}$$

3. Define velocity ratio of a machine. (Possible)

- Ans: The velocity ratio (briefly written as V.R.) is the ratio of distance moved by the effort (*y*) to the distance moved by the load (*x*) and is always expressed in pure number.
- Mathematically, velocity ratio,

$$V.R. = \frac{y}{x}$$

4. What is reversible machine. (S – 2018)
Ans: When a machine is capable of doing some work in the reverse direction even on removal of effort, it is called as reversible machine and its action is known as reversibility of the machine.

5. What is the condition of reversibility of a lifting machine? (W – 2017)
Ans: The condition for a machine, to be reversible, is that its **efficiency should be more than 50%.**

$$i.e.\ \eta > 50\%$$

6. Define self-locking machine. (Possible)
Ans: When a machine is not capable of doing some work in the reverse direction even on removal of effort, it is called as irreversible machine or non-reversible machine or self-locking machine.

7. Write the expression for velocity ratio of a simple wheel and axle. (S – 2018)
Ans:

$$V.R. = \frac{Distance\ moved\ by\ the\ effort}{Distance\ moved\ by\ the\ load} = \frac{\pi D}{\pi d} = \frac{D}{d}$$

8. What is law of machine? (S – 2019)
- Ans: The equation which gives the relation between load lifted and effort applied in the form of a slope and intercept of a straight line is called as Law of a machine.
- Mathematically, the law of a lifting machine is given by the relation:

$$P = mW + C$$

9. State the relation between M.A., V.R. and efficiency of a simple lifting machine. (S – 2019)
Ans:

$$\eta = \frac{Output}{Input} = \frac{W \times x}{P \times y} = \frac{W/P}{y/x} = \frac{M.A.}{V.R.}$$

POSSIBLE LONG TYPE QUESTIONS:

1. Derive the velocity ratio of a compound gear train. **(W – 2016, 2017 & S – 2018, 2019)**

2. Define mechanical advantage, velocity ratio and efficiency of a lifting machine and derive their relationship. **(S – 2018)**

3. Derive the condition for reversibility of a simple lifting machine. **(S – 2019)**

4. In a weight lifting machine, an effort of 40 N can lift a load of 1000 N and an effort of 55 N can lift a load of 1500 N. Find the law of the machine. Also find maximum mechanical advantage and maximum efficiency of the machine. Take velocity ratio of the machine as 48. **(W – 2017)**

[Ans. P = 0.03 W + 10; 33.3; 69.4%]

5. In a certain weight lifting machine, an effort of 15 N can lift a load of 300 N and an effort of 20 N can lift a load of 500 N. Find the law of the machine. Also find the effort required to lift a load of 880 N.
(S – 2018, 2019)

[Ans. P = 0.025 W + 7.5; 29.5 N]

6. In a simple wheel and axle, radii of effort wheel and axle is 240 mm and 40 mm respectively. Find the efficiency of the machine, if a load of 600 N can be lifted by an effort of 120 N. **(Possible)**

[Ans. 83.3%]