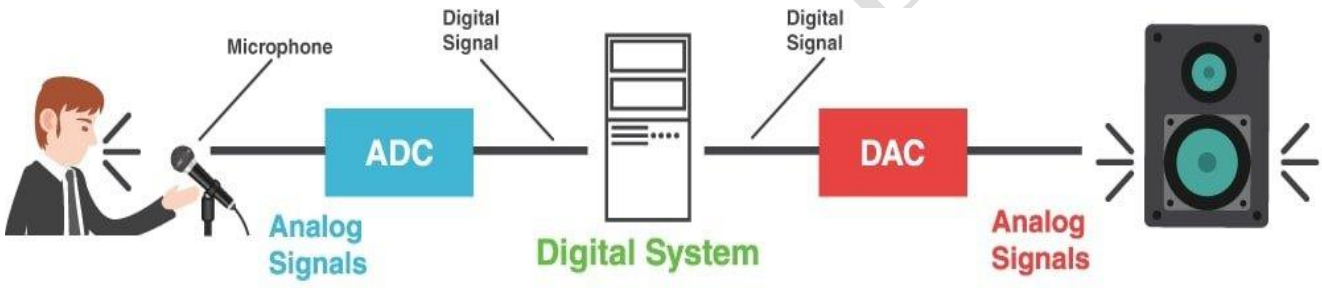
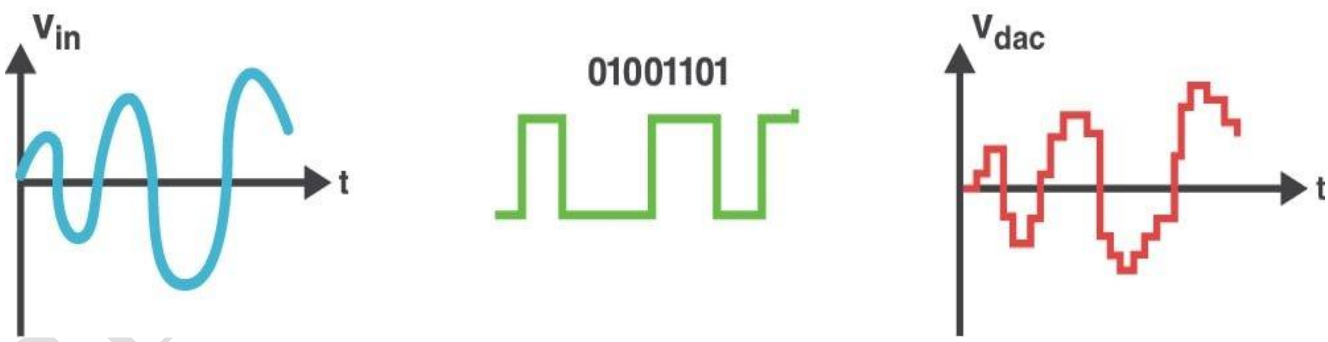
BHADRAK ENGINEERING SCHOOL & TECHNOLOGY (BEST), ASURALI, BHADRAK

Digital Signal Processing
(Th- 03)

(As per the 2020-21 syllabus of the SCTE&VT, Bhubaneswar, Odisha)





Sixth Semester

Electronics & Telecommunication Engg.

Prepared By: *Er A K Patra/ Er. K M Jena*

DIGITAL SIGNAL PROCESSING (Th- 03)

CHAPTER-WISE DISTRIBUTION OF PERIODS & MARKS

Chapter No.	Name of the Chapter	No. of Periods as per Syllabus	Expected Marks
1	INTRODUCTION OF SIGNALS, SYSTEMS, AND SIGNAL PROCESSING	10	15
2	DISCRETE TIME SIGNALS & SYSTEMS	14	25
3	THE Z -TRANSFORM & ITS APPLICATION TO THE ANALYSIS OF LTI SYSTEM	14	30
4	DISCUSS FOURIER TRANSFORM: ITS APPLICATION & PROPERTIES	12	20
5	FAST FOURIER TRANSFORM ALGORITHM & DIGITAL FILTERS	10	10
Total		60	100

CHAPTER NO.- 01

INTRODUCTION OF SIGNALS, SYSTEMS AND SIGNAL PROCESSING

Learning Objectives:

1.1. Basics of signal, system & signal processing: - basic elements of a digital signal processing system-compare the advantages of digital signal processing over analog signal processing.

1.2. Classify signals-Multi channel & multi-dimensional signals-Continuous time verses Discrete-times signal. continuous valued verses Discrete-valued signals.

1.3. Concept of frequency in continuous time & discrete time signals- Continuous time sinusoidal signals-Discrete-time sinusoidal signals.

1.4. Analog to Digital & Digital to Analog conversion & explain the following.

a) Sampling of Analog signal

b) The sampling theorem.

c) Quantization of continuous amplitude signals.

d) Coding of quantized sample.

e) Digital to analog conversion

f) Analysis of digital systems signals vs discrete time signals systems.

1.1 INTRODUCTION TO DIGITAL SIGNAL PROCESSING: -

- Digital Signal Processing is the mathematical manipulation of an information signal, such as audio,temperature, voice, and video and modify or improve them in some manner.

➢ BASICS OF SIGNAL, SYSTEM & SIGNAL PROCESSING: -

➢ **Signals: -**

- In electrical engineering, the fundamental quantity of representing some information is called a signal. It does not matter what the information is i-e: Analog or digital information. In mathematics, a signal is a function that conveys some information. In fact, any quantity measurable through time over space or any higher dimension can be taken as a signal. A signal could be of any dimension and could be of anyform.

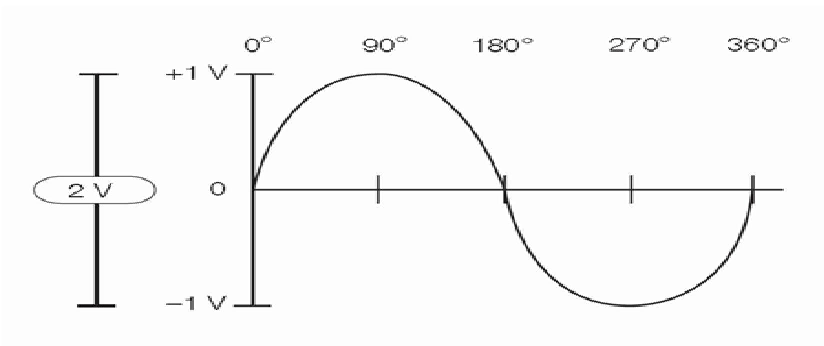
➢ **Analog signals: -**

- A signal could be an analog quantity that means it is defined with respect to the time. It is a continuoussignal.
- **For example:**

I. Human voice

Human voice is an example of analog signals. When you speak, the voice that is produced travel through air in the form of pressure waves and thus belongs to a mathematical function, having independent variables of space and time and a value corresponding to air pressure.

Another example is of sin wave which is shown in the figure below. Y = sin(x) where x is independent



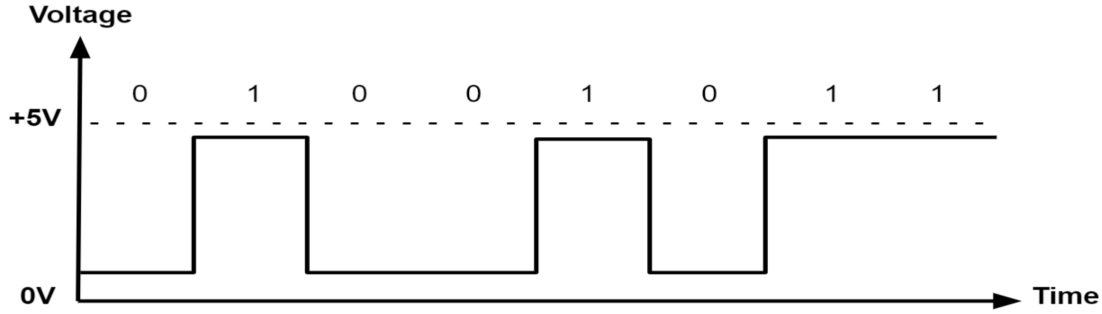
- **Digital signals**

- The word digital stands for discrete values and hence it means that they use specific values to represent any information. In digital signal, only two values are used to represent something i-e: 1 and 0 (binary values). Digital signals are denoted by square waves. They are discontinuous signals.

  For example:

  i. Computer keyboard

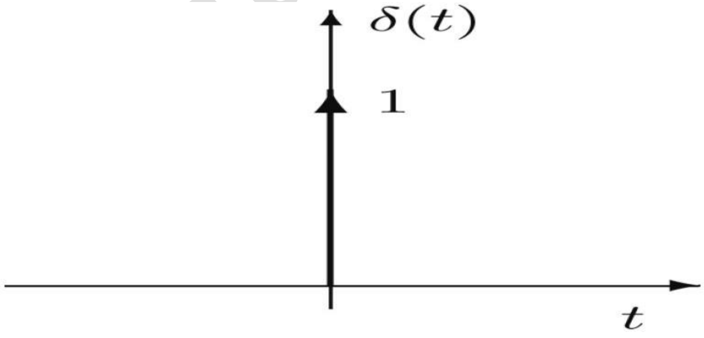
  Whenever a key is pressed from the keyboard, the appropriate electrical signal is sent to keyboard controller containing the ASCII value that particular key. For example, the electrical signal that is generated when keyboard key a is pressed, carry information of digit 97 in the form of 0 and 1, which is the ASCII value of character a.



Some other important signals are there. Such as:

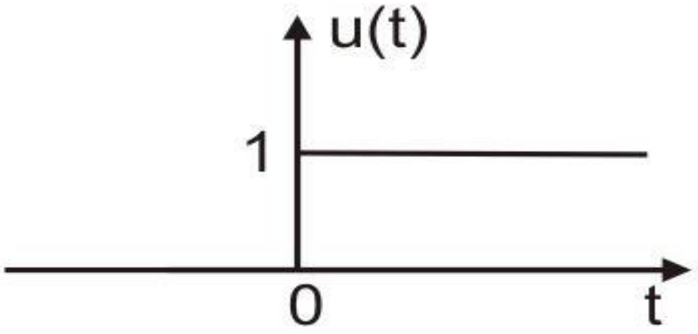
- **Unit Impulse or Delta Function**

A signal, which satisfies the condition, $\delta(t)=\lim_{e\to\infty} x(t)x$ is known as unit impulse signal. This signal tends to infinity when t = 0 and tends to zero when $t \neq 0$ such that the area under its curve is always equals to one. The delta function has zero amplitude everywhere at t = 0.



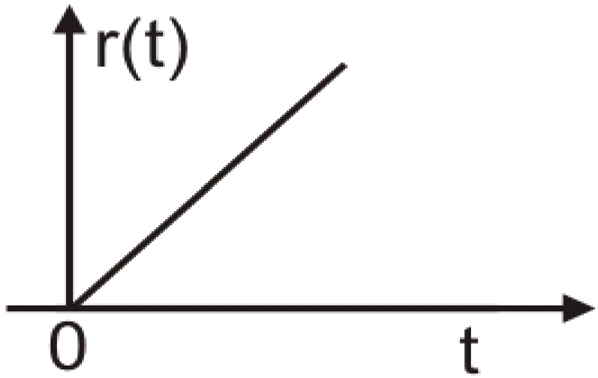
- **Unit Step Signal**

A signal, which satisfies the following two conditions-
U(t)=1(whent≥0) and (t)=0 (when<0) is known as a unit step signal.



- **Ramp Signal**

Integration of step signal results in a Ramp signal. It is represented by rt. Ramp signal also satisfies the condition $r(t) = \int_0^\infty U(t)\,dt = tU(t)$. It is neither energy nor power type signal.



➢ **<u>Systems</u>**

- A system is a defined by the type of input and output it deals with. Since we are dealing with signals, so in our case, our system would be a mathematical model, a piece of code/software, or a physicaldevice, or a black box whose input is a signal and it performs some processing on that signal, and the output is a signal. The input is known as excitation and the output is known as response.

- In the above figure a system has been shown whose input and output both are signals but the input is an analog signal. And the output is a digital signal. It means our system is actually a conversionsystem that converts analog signals to digital signals.

➢ **<u>Continuous systems vs discrete systems</u>**

1. Continuous systems

- The type of systems whose input and output both are continuous signals or analog signals are called continuous systems.

2. Discrete systems

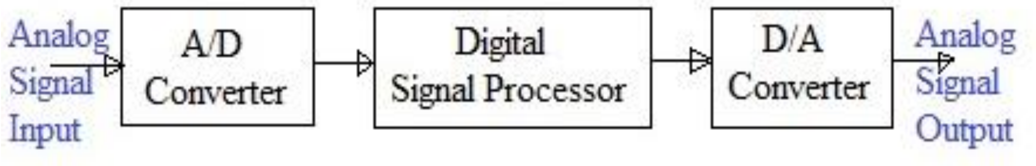
- The type of systems whose input and output both are discrete signals or digital signals are called digital systems.

➢ **Signal processing**

- Signal processing is an electrical engineering subfield that focuses on analyzing, modifying, andsynthesizing signals such as sound, images, and scientific measurements.
- Signal processing techniques can be used to improve transmission, storage efficiency and subjective quality and to also emphasize or detect components of interest in a measured signal.

➢ BASIC ELEMENTS OF A DIGITAL SIGNAL PROCESSING SYSTEM: -

- In its most general form, a DSP system will consist of three main components, as illustrated inFigure.
- The analog-to-digital (A/D) converter transforms the analog signal $x_a(t)$ at the system input into adigital signal x_d [n]. An A/D converter can be thought of as consisting of a sampler (creating a discrete timesignal), followed by a quantizer (creating discrete levels).
- The digital system performs the desired operations on the digital signal x_d[n] and produces acorresponding output y_d [n] also in digital form.
- The digital-to-analog (D/A) converter transforms the digital output y_d[n] into an analog signal y_a(t)suitable for interfacing with the outside world.
- In some applications, the A/D or D/A converters may not be required; we extend the meaning of DSPsystems to include such cases.



ANALOG I/P SIGNAL- $xa(t)$
A/D O/P- $xd[n]$
DIGITAL SYSTEM- $yd[n]$
D/A O/P- $ya(t)$

- Discrete-time signals are typically written as a function of an index n (for example, x(n)) represent a discretization of x(t) sampled every T second). In contrast to Continuous signalsystems,
- where the behavior of a system is often described by a set of linear differential equations,discrete-time systems are described in terms of difference equations.
- Transform-domain analysis of discrete-time systems often makesuse of the Z transform.

ADVANTAGES OF DSP OVER ASP: -

1. Physical size of analog systems is quite large while digital processors are more compact and lighter in weight.
2. Analog systems are less accurate because of component tolerance ex R, L, C and active components. Digital components are less sensitive to the environmental changes, noise and disturbances.
3. Digital system is most flexible as software programs & control programs can be easily modified.
4. Digital signal can be stores on digital hard disk, floppy disk or magnetic tapes. Hence becomes transportable. Thus, easy and lasting storage capacity.
5. Digital processing can be done offline.
6. Mathematical signal processing algorithm can be routinely implemented on digital signal processing systems. Digital controllers are capable of performing complex computation with constantaccuracy at high speed.

7. Digital signal processing systems are upgradeable since that are software controlled.
8. Possibility of sharing DSP processor between several tasks.
9. The cost of microprocessors, controllers and DSP processors are continuously going down. Forsome complex control functions, it is not practically feasible to construct analog controllers.
   Single chip microprocessors, controllers and DSP processors are more versatile and powerful.

1.2- CLASSIFY SIGNALS-MULTI CHANNEL & MULTI-DIMENTIONAL SIGNALS: -

CLASSIFICATION OF SIGNALS: -

1. Single channel and multi-channel signals
2. Single dimensional and multi-dimensional signals
3. Continuous time and Discrete time signals.
4. Continuous valued and discrete valued signal
5. Analog and digital signals.

1. Single channel and Multi-channel signals: -

- If signal is generated from single sensor or source, it is called as single channel signal. If the signals are generated from multiple sensors or multiple sources or multiple signals are generated from same source called as multi-channel signal. Example ECG signals. Multi- channel signal will be the vector sum of signals generated from multiple sources.

2. Single Dimensional (1-D) and Multi-Dimensional signals (M-D): -

- If signal is a function of one independent variable it is called as single dimensional signal like speech signal and if signal is function of M independent variables called as Multi - dimensional signals. Gray scale level of image or Intensity at particular pixel on black and white TV is examples of M-D signals

3. Continuous time and Discrete time signals: -

- **Continuous Time (CTS)**

  1.This signal can be defined at any time instance & they can take all values in the continuous interval (a, b) where a can be -∞ & b can be ∞

  2.These are described by differential equations.

  3.This signal is denoted by x(t).

  4.The speed control of a dc motor using a tacho generator feedback or Sine or exponential waveforms.

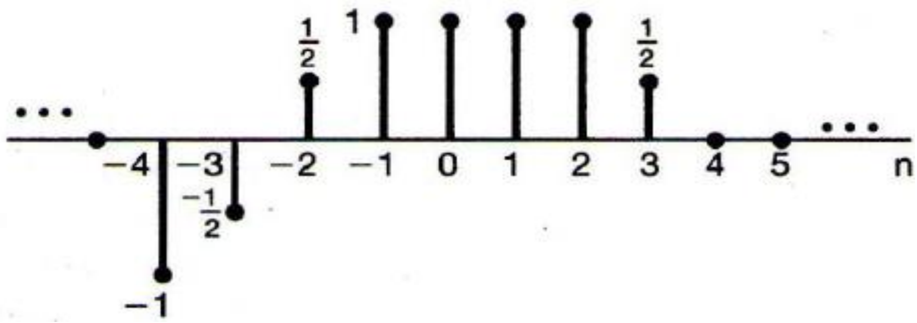
- **Discrete time (DTS)**

  1.This signal can be defined only at certain specific values of time. These time instance need not be equidistant but in practice they are usually takes at equally spaced intervals.

  2.These are described by difference equation.

  3.These signals are denoted by x(n) or notation x(nT) can also be used.

  4.Microprocessors and computer-based systems uses discrete time signals.



4. Continuous valued and Discrete Valued signals: -

- **Continuous Valued**

1.If a signal takes on all possible values on a finite or infinite range, it is said to be continuousvalued signal.

2.Continuous Valued and continuous time signals are basically analog signals.

- **Discrete Valued**

1.If signal takes values from a finite set of possible values, it is said to be discrete valued signal.

2.Discrete time signal with set of discrete amplitude are called digital signal.

5. Analog and digital signal: -

- **Analog signal**

1.These are basically continuous time & continuous amplitude signals.

2.ECG signals, Speech signal, Television signal etc. All the signals generated from varioussources in nature are analog.

- **Digital signal**

1.These are basically discrete time signals & discrete amplitude signals. These signals arebasically obtained by sampling & quantization process.

2.All signal representation in computers and digital signal processors are digital.

Note: Digital signals (discrete time & discrete amplitude) are obtained by sampling the analog signal at discrete instants of time, obtaining discrete time signals and then by quantizing its values to a set of discrete values & thus generating discrete amplitude signals.

1.3-DISCUSS THE CONCEPT OF FREQUENCY IN CONTINUOUS TIME & DISCRETE TIME SIGNALS: -

- **Continuous time and discrete time signal: -**

- The independent variable(s) for a signal may be continuous or discrete. A signal is consideredto be
  a continuous time signal if it is defined over a continuum of the independent variable.
- A signal is considered to be discrete time if the independent variable only has discrete values.

- **Frequency of a signal**

- **Frequency** is the rate at which current changes direction per second. It is measured in hertz (Hz),an international unit of measure where 1 hertz is equal to 1 cycle per second. Hertz (Hz) = One hertz is equal to one cycle per second. Cycle = One complete wave of alternating current or voltage.

CONCEPT OF FREQUENCY IN CONTINUOUS TIME & DISCRETE TIME SIGNALS:

- Let us find out the representation of frequency for discrete time signals and also the relationship between sampling frequency, continuous time frequency and discrete time frequency. Let us consider the following analog signal:

$$x(t) = A\cos(\omega t + \phi), -\infty < t < \infty \ldots\ldots\ldots (1)$$

- The above represented analog signal is a continuous cosine wave having amplitude, frequency and phase A, ω and Φ respectively where ω = 2 π f. The above expression can also be written as

$$= \frac{A}{2} e^{-j(\omega t+\varphi)} + \frac{A}{2} e^{-j(\omega t+\varphi)}$$

$$x(t) = A\cos(2\pi ft + \phi), -\infty < t < \infty \; .... (2)$$

Above Equation shows that the cosine wave can be expressed in terms of two equal signal.

➢ <u>CONTINEOUS TIME SINUSOIDAL SIGNAL: -</u>

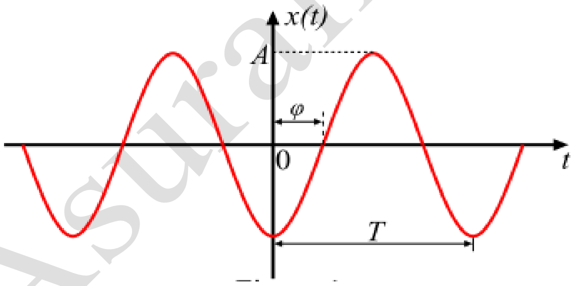
- A sinusoidal signal which is defined for every instant of time is called continuous-time sinusoidal signal. The continuous time sinusoidal signal is given as follows -

$$x(t) = A\sin(\omega t + \varphi) = A\sin(2\pi ft + \varphi)$$

Where,
A is the amplitude of the signal. That is the peak deviation of the signal from zero.
$\omega = 2\pi f$ is the angular frequency in radians per seconds.
f is the frequency of the signal in Hz.
φ is the phase angle in radians.
All the continuous-time sinusoidal signals are periodic signal. The time period (T) of a continuous-time sinusoidal signal is given by,

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

The graphical representation or waveform of a continuous time sinusoidal signal x(t) is shown in Figure.



➢ DISCRETE TIME SINUSOIDAL SIGNAL

- The discrete-time sinusoidal sequence is given by

$$X(n) = A\ \sin(\omega n + \varphi)$$

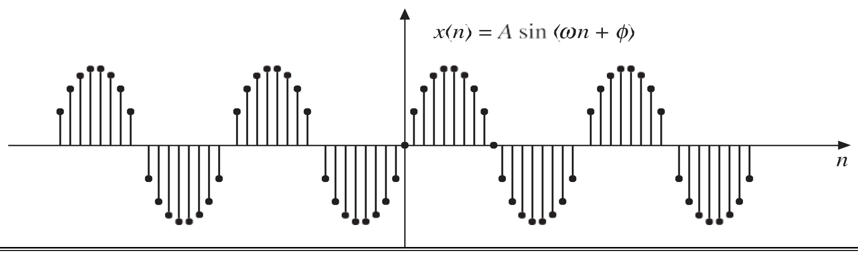
Where A is the amplitude, is angular frequency, is phase angle in radians and n is an integer.

The period of the discrete-time sinusoidal sequence is:

$$N = \frac{2\pi}{\omega} m$$

Where N and m are integers.
All continuous-time sinusoidal signals are periodic, but discrete-time sinusoidal sequences may or may not be periodic depending on the value of ω.
For a discrete-time signal to be periodic, the angular frequency must be a rational multiple of 2π.

- The graphical representation of a discrete-time sinusoidal signal is shown in Figure .



➢ HARMONICALY RELATED COMPLEX EXPONENTIAL: -

Set of periodic exponentials with fundamental frequencies that are multiples of a single positive frequency ω_0

$$x_k(t) = e^{jk\omega_0 t} \text{ for } k = 0, \mp 1, \mp 2, \ldots$$
$$k = 0 \Rightarrow x_k(t) \text{ is a constant}$$
$$k \neq 0 \Rightarrow x_k(t) \text{ is periodic with fundamental frequency}$$
$$\text{and fundamental period } \frac{2\pi}{|k|\omega_0} = \frac{T_0}{|k|}, \text{ where } T_0 = \frac{2\pi}{\omega_0}$$

k^{th} harmonic $x_k(t)$ is still periodic with T_0 as well
Harmonic (from music): tones resulting from variations in acoustic pressures that are integer multiples of a fundamental frequency.

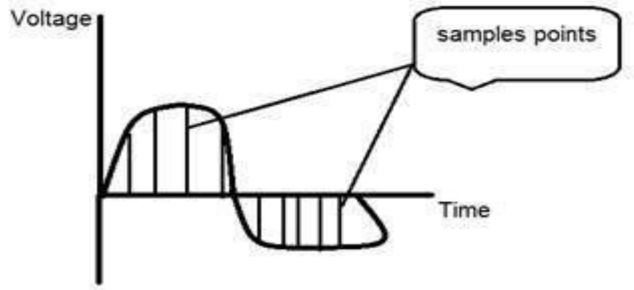
1.4 CONVERSION OF ANALOG TO DIGITAL SIGNALS: -

Since there are lot of concepts related to this analog to digital conversion and vice-versa. We will only discuss those which are related to digital image processing. There are two main concepts that are involvedin the conversion.

- Sampling
- Quantization

➢ Sampling

Sampling as its name suggests can be defined as take samples. Take samples of a digital signal over x- axis. Sampling is done on an independent variable.

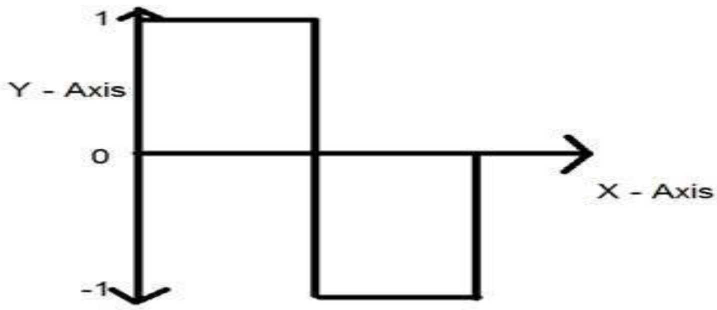


- Sampling is done on the x variable. We can also say that the conversion of x axis (infinite values) to digital is done under sampling.
- Sampling is further divide into up sampling and down sampling. If the range of values on x-axis are less then we will increase the sample of values. This is known as up sampling and its vice versais known as down sampling.

➢ Quantization

- Quantization as its name suggest can be defined as dividing into quanta (partitions). Quantization is done on dependent variable. It is opposite to sampling.
- In case of this mathematical equation y = sin(x)
- Quantization is done on the Y variable. It is done on the y axis. The conversion of y axis infinite values to 1, 0, -1 (or any other level) is known as Quantization.
- These are the two basics steps that are involved while converting an analog signal to a digitalsignal.

- The quantization of a signal has been shown in the figure below.



➢ Need to convert an analog signal to digital signal: -

- The first and obvious reason is that digital image processing deals with digital images, that are digital signals. So, whenever the image is captured, it is converted into digital format and then it is processed.
- The second and important reason is, that in order to perform operations on an analog signal with a digital computer, you have to store that analog signal in the computer. And in order to store an analog signal, infinite memory is required to store it. And since that's not possible, so that's why we convert that signal into digital format and then store it in digital computer and then performs operations on it.

1.4.a. SAMPLING OF ANALOG SIGNAL: -

- Sampling is defined as, "The process of measuring the instantaneous values of continuous-timesignal in a discrete form." . When a source generates an analog signal and if that has to be digitized, having 1s and 0s i.e., High or Low, the signal has to be discretized in time.

1.4.b. SAMPLING THEOREM: -

- The sampling theorem specifies the minimum-sampling rate at which a continuous-time signal needs to be uniformly sampled so that the original signal can be completely recovered or reconstructed by these samples alone. This is usually referred to as Shannon's sampling theorem inthe literature.

➢ **Sampling theorem:**

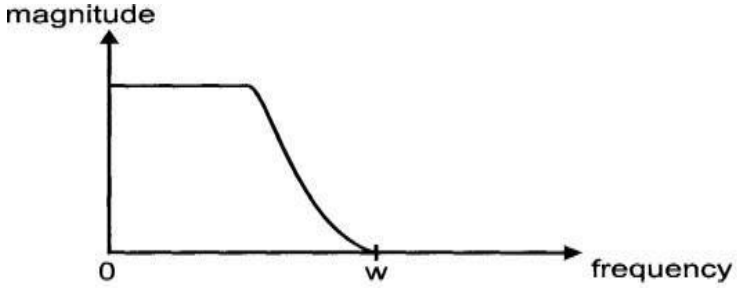
- If a continuous time signal contains no frequency components higher than W_{hz}, then it can becompletely determined by uniform samples taken at a rate f_s samples per second where

$$f_s \geq 2W$$

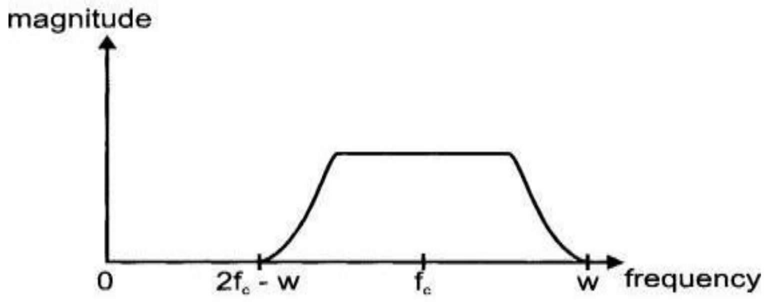
or, in term of the sampling period

$$T \leq 1/2W$$

- A signal with no frequency component above a certain maximum frequency is known as a band limited signal. Figure 2.4 shows two typical band limited signal spectra: one low-pass and oneband-pass.



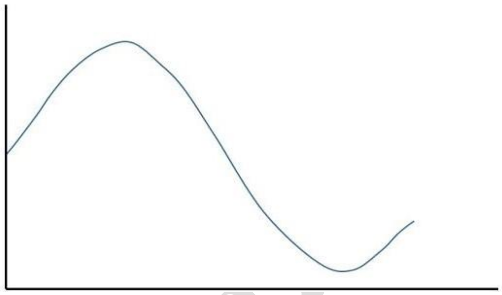
(a) a low-pass spectrum



(b) a band-pass spectrum

1.4.c. QUANTIZATION OF CONTINUOUS AMPLITUDE SIGNALS: -

- **Quantization: -**
- The digitization of analog signals involves the rounding off of the values which are approximately equal to the analog values. The method of sampling chooses a few points on the analog signal and then these points are joined to round off the value to a near stabilized value. Such a process is calledas Quantization.
    - **Quantizing an Analog Signal**
- The analog-to-digital converters perform this type of function to create a series of digital values outof the given analog signal. The following figure represents an analog signal. This signal to get converted into digital has to undergo sampling and quantizing.



- The quantizing of an analog signal is done by discretizing the signal with a number of quantizationlevels. Quantization is representing the sampled values of the amplitude by a finite set of levels, which means converting a continuous-amplitude sample into a discrete-time signal.
- The following figure shows how an analog signal gets quantized. The blue line represents analogsignal while the brown one represents the quantized signal.

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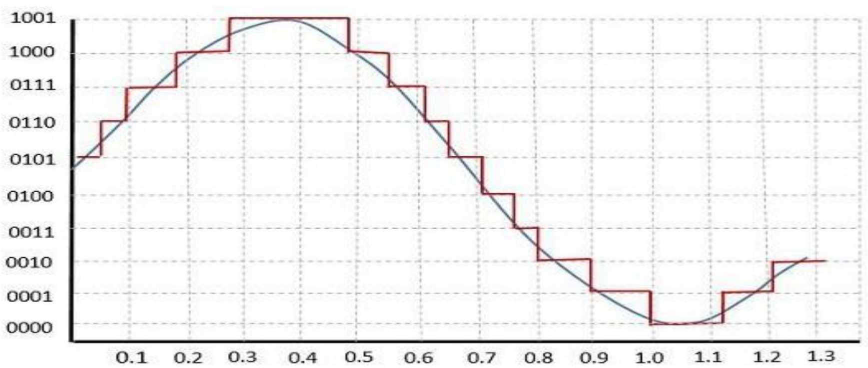
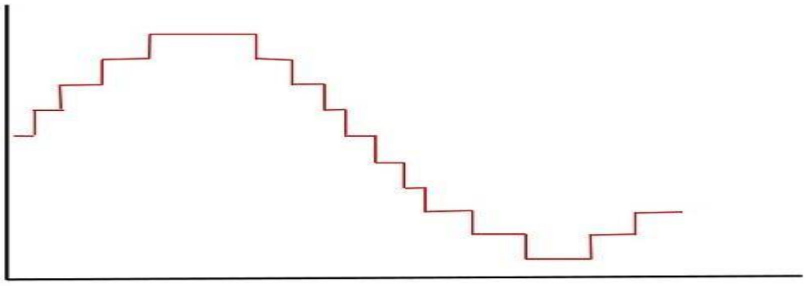


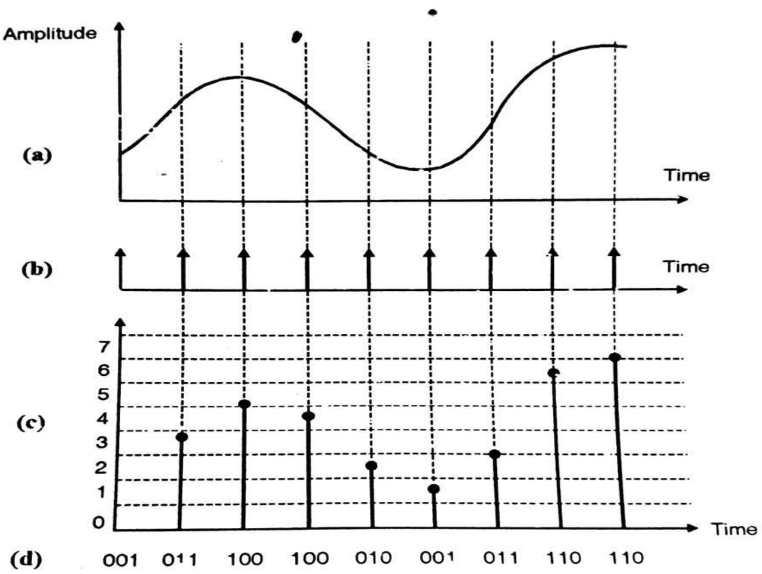
- Both sampling and quantization result in the loss of information. The quality of a Quantizer output depends upon the number of quantization levels used. The discrete amplitudes of thequantized output are called as representation levels or reconstruction levels. The spacing between the two adjacent representation levels is called a quantum or step-size.

- The following figure shows the resultant quantized signal which is the digital form for thegiven analog signal.



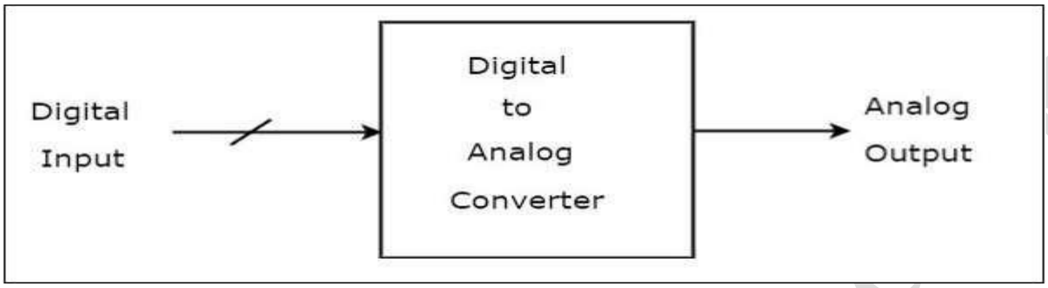
1.4.d CODING OF QUANTIZED SAMPLE: -

- **Quantization:**
- The process of converting continuous sample values into discrete values is called quantization.
- In this process we divide the signal range into a fixed number of intervals.
- Each interval is of same size and is assigned a number. These intervals are numbered between 0 to 7.
- Each sample falls in one of the intervals and is assigned that interval's number.
- **Coding:**
- The process of representing quantized values digitally is called coding
- In our example, eight quantizing levels are used. These levels can be coded using 3 bits if the binary system is used, so each sample is represented by 3 bits.
- The analog signal is represented digitally by the following series of binary numbers: 001, 011, 100, 100,010, 001, 011, 110, and 110.



1.4.e DIGITAL TO ANALOG CONVERSION: -

- A **Digital to Analog Converter (DAC)** converts a digital input signal into an analog output signal. The digital signal is represented with a binary code, which is a combination of bits 0 and 1. This chapter deals with Digital to Analog Converters in detail.
- The **block diagram** of DAC is shown in the following figure −



- A Digital to Analog Converter (DAC) consists of a number of binary inputs and a single output. Ingeneral, the **number of binary inputs** of a DAC will be a power of two.

➢ Digital-to-Analog Conversion: -

- A digital-to-analog converter (DAC), as the name implies, is a data converter which generates an analog output from a digital input. A DAC converts a limited number of discrete digital codes to acorresponding number of discrete analog output values.
- Because of the finite precision of any digitized value, the finite word length is a source of error in the analog output. This is referred to as quantization error. Any digital value is really only an approximation of the real-world analog signal. The more digital bits represented by the DAC, the more accurate the analog output signal.
- Basically, one LSB of the converter will represent the height of one step in the successive analog output. You can think of a DAC as a digital potentiometer that produces an analog output that is afraction of the full-scale analog voltage determined by the value of the digital code applied to the converter.
- Similar to ADCs, the performance of a DAC is determined by the number of samples it can process and the number of bits used in the conversion process.
- For example, a three-bit converter as shown in fast fig. will have less performance than the four-bitconverter shown in 2nd Fig

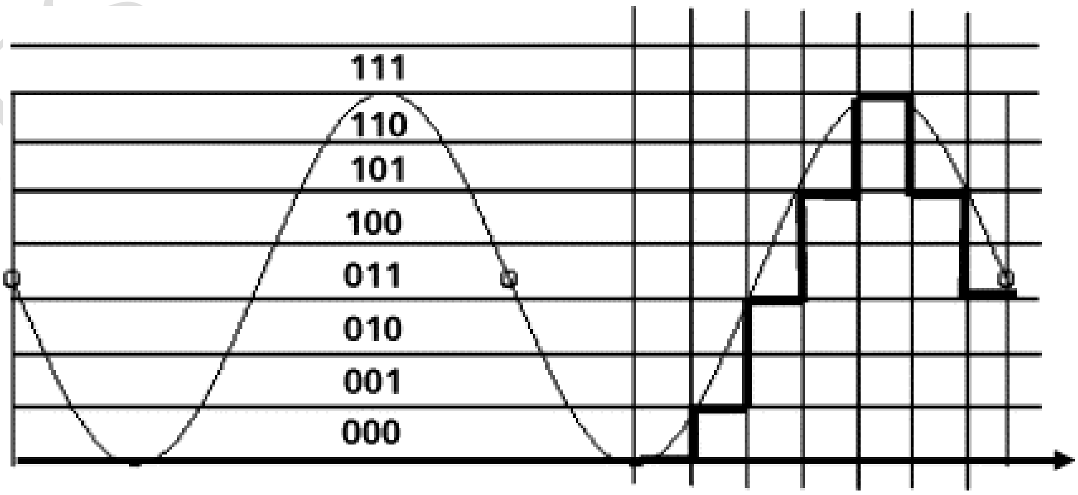


Fig.1

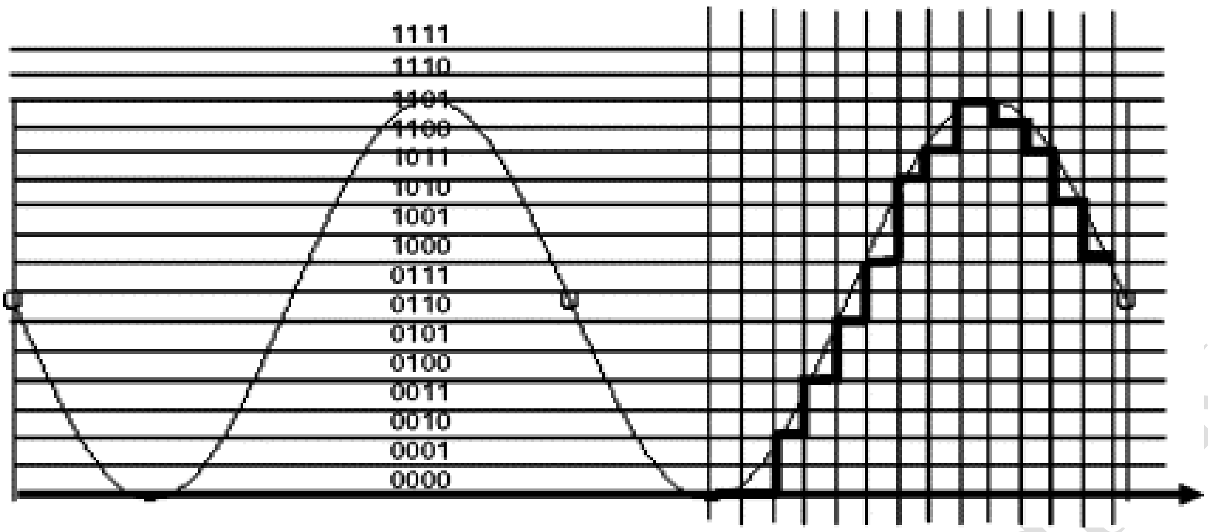
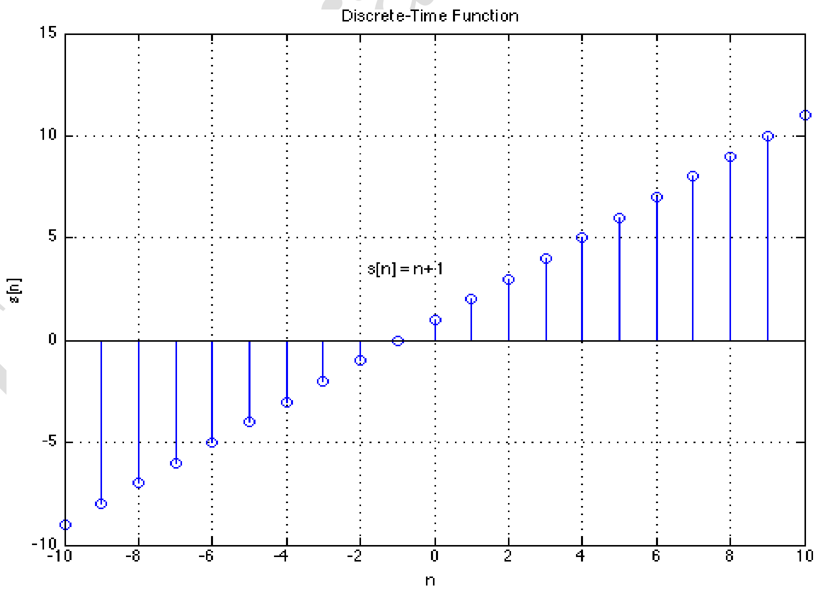


Fig.2

1.4.f. DIGITAL SYSTEM SIGNAL VS DISCRETE TIME SIGNAL SYSTEM: -

➢ **Discrete-time (DT) Signal**

- A discrete-time signal is a bounded, continuous-valued sequence s[n]. Alternately, it may be viewed as a continuous-valued function of a discrete index n. We often refer to the index n as time,since discrete-time signals are frequently obtained by taking snapshots of a continuous-time signalas shown below. More correctly, though, n is merely an index that represents sequentially of the numbers in s[n].



If they DT signals are snapshots of real-world signals realness and finiteness apply.

Below are several characterizations of size for a DT signal

1. Energy signal: A signal is said to be an energy signal if and only if its total energy E is finite, i.e., $0 < E < \infty$.

$$E = \sum_n s^2[n].$$

2. Power signal: A signal is said to be a power signal if its average power P is finite, i.e., $0 < P < \infty$.

$$P = \lim_{n\to\infty} \frac{1}{2N+1}\sum_{n=-N}^{N} s^2[n].$$

3. Amplitude= max|s[n]| Smoothness is not applicable.

➢ Digital Signal

- We will work with digital signals but develop theory mainly around discrete-time signals.
- Digital computers deal with digital signals, rather than discrete-time signals. A *digital* signal is a sequence s[n], where index the values s[n] are not only finite, but can only take a finite set of values. For instance, in a digital signal, where the individual numbers s[n] are stored using 16 bits integers, s[n] can take one of only 2^{16} values.
- In the digital valued series s[n] the values *s* can only take a fixed set of values.
- Digital signals are discrete-time signals obtained after "digitalization." Digital signals too are usually obtained by taking measurements from real-world phenomena. However, unlike the accepted norm for analog signals, digital signals may take complex values presented above are some criteria for real-world signals. Theoretical signals are not constrained.
- Real- this is often violated; we work with complex numbers.
- Finite/bounded.
- Energy – violated all the time.
- Signals that have infinite temporal extent, *i.e.,* which extend from −∞ to ∞, can have infiniteenergy.
- Power - almost never: nearly all the signals we will encounter have bounded power
- Smoothness-- this is often violated by many of the continuous time signals.

POSSIBLE SHORT TYPE QUESTIONS WITH ANSWERS:

1.Define DSP?

Ans. Digital Signal Processing is the mathematical manipulation of an information signal, such as audio, temperature, voice, and video and modify or improve them insome manner.

2. Define signal?

Ans. In electrical engineering, the fundamental quantity of representing someinformation is called a signal.

3.Define system?

Ans. A system is a defined by the type of input and output it deals with. Since we are dealing with signals, so in our case, our system would be a mathematical model,a piece of code/software, or a physical device, or a black box whose input is a signaland it performs some processing on that signal, and the output is a signal.

4.Define discrete time signal?

Ans. The word digital stands for discrete values and hence it means that they use specific values to represent any information. In digital signal, only two values are used to represent something i-e: 1 and 0 (binary values). Digital signals are denotedby square waves. They are discontinuous signals.

5.Define Digital to Analog Converter (DAC)? [2018(S)]

Ans. A Digital to Analog Converter (DAC) converts a digital input signal into an analog output signal. The digital signal is represented with a binary code, which is acombination of bits 0 and 1. This chapter deals with Digital to Analog Converters indetail.

POSSIBLE LONG TYPE QUESTIONS:

1.Explain basic elements of digital signal processing system.

2.Compare DSP over ASP. [2019(S-NEW)]

3.Explain analog to digital conversion?.

4.Explain digital to analog conversion.

5.State and explain sampling theorem.(S-24)

CHAPTER NO.-02:
DISCRETE TIME SIGNALS & SYSTEMS

Learning Objectives:

2.1 Concepts of discrete time signals.
2.1.1 Elementary discrete time signals.
2.1.2 Classification of discrete time signal.
2.1.3 Simple manipulation of discrete time signal
2.2 Discrete time system.
2.2.1 Input-output of system.
2.2.2 Block diagram of discrete time system.
2.2.3 Classify discrete time system.
2.2.4 Inter connection of discrete time system.
2.3 Discrete time time-invariant system.
2.3.1 Different technique for the analysis of linear system.
2.3.2 Resolution of a discrete time signal in to impulse.
2.3.3 Response of LTI system to arbitrary inputs using convolution sum.
2.3.4 Convolution & interconnection of LTI system-properties.
2.3.5 Study systems with finite duration and infinite duration impulse response.
2.4 Discrete time system described by difference equation.
2.4.1 Recursive & non-recursive discrete time system.
2.4.2 Determine the impulse response of linear time invariant recursive system.
2.4.3 Correlation of Discrete Time signals.

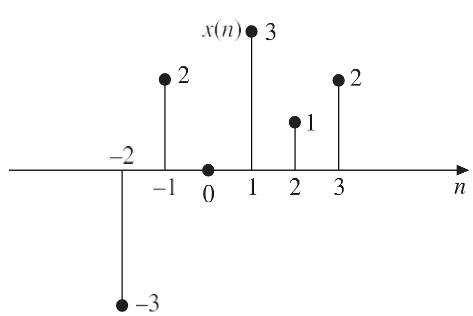
2.1CONCEPT OF DISCRETE TIME SIGNALS:

A discrete time signal S(n) is a function of an independent variable 'n'.Discrete Time signal can be represented in the following 4 ways,

1. Graphical representation
2. Functional representation
3. Tabular representation
4. Sequential representation

1. Graphical representation:

Consider a single x (n) with values
$X(-2) = -3, x(-1) = 2, x(0) = 0, x(1) = 3, x(2) = 1$ and $x(3) = 2$
This discrete-time single can be represented graphically as shown below figure.



Graphical representation of discrete-time signal

2. **Functional representation:**

$$x(n) = \begin{cases} -3, & n = -2 \\ 2, & n = -1 \\ 0, & n = 0 \\ 3, & n = 1 \\ 1 & n = 2 \\ 2 & n = 3 \end{cases}$$

3. **Tabular representation:**

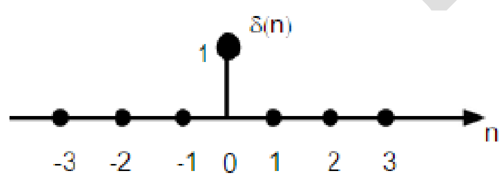
n	-2	-1	0	1	2	3
X(n)	-3	2	0	3	1	2

4. Sequential representation:

$$x(n) = \{-3,2,0,3,1,2\}$$

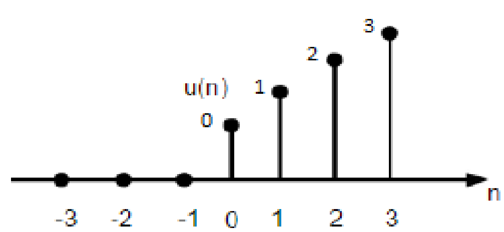
2.1.1 ELEMENTARY DISCRETE TIME SIGNALS:

1. Unit Impulse Signal:



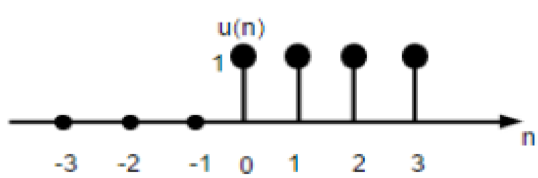
$$\delta(n) = \begin{cases} 1, n = 0 \\ 0, n \neq 0 \end{cases}$$

2. Unit Ramp Signal:



$$R(n) = \begin{cases} n, n \geq 0 \\ 0, n < 0 \end{cases}$$

3. Unit Step Signal:



$$u(n) = \begin{cases} 1, n \geq 0 \\ 0, n < 0 \end{cases}$$

2.1.2. CLASSIFICATION DISCRETE TIME SIGNAL:

1.Energy Signal and Power Signal:

- The energy of a discrete time signal x(n) is defined as.

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

A signal x(n) is called as energy signal if the energy is finite (0<E<infinity) and powerequal to zero.

The average power of a discrete time signal is defined as

$$P = \lim_{N=\infty} \frac{1}{2N+1} \sum_{n=-\infty}^{\infty} |x(n)|^2 \quad \text{where} \begin{cases} 0 < P < \infty \\ E = \infty \end{cases}$$

The signal x(n) is called as power signal if and only if the power is finite and energy is infinite.

- Infinite summation formula:

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}, \quad |a| < 1$$

$$\sum_{n=0}^{\infty} na^n = \frac{a}{(1-a)^2}, \quad |a| < 1$$

$$\sum_{n=0}^{\infty} n^2 2^2 = \frac{a^2+a}{(1-a)^3}$$

- Finite summation formula:

$$\sum_{n=0}^{N} a^n = \frac{1-a^n+1}{1-a}, a \neq 1$$

$$\sum_{n=0}^{N} 1 = N+1$$

$$\sum_{n=N}^{N} 1 = N_2 - N_1 + 1$$

$$\sum_{n=0}^{N} n = \frac{N(N+1)}{2}$$

$$\sum_{n=0}^{N} n^2 = \frac{N(N+1)(2N+1)}{6}$$

Q.1.Cheak whether the unit step signal is energy or power signal.

Sol^n: $x(n) = u(n)$

This signal is periodic (since u(n) repeats after every sample) and of finite duration hence it is an power signal

Let us calculate power directly

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$\Rightarrow \sum_{n=0}^{\infty} u(n)^2 = \sum_{n=0}^{\infty} (1)^2 = \sum_{n=0}^{\infty} 1 = \infty$$

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |u(n)|^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-\infty}^{N} 1$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \; N+1$$

$$= \lim_{N \to \infty} \frac{N+1}{2N+1}$$

$$= \lim_{N \to \infty} \frac{1+\frac{1}{N}}{2+\frac{1}{N}} = \frac{1}{2}$$

Q.2.Find the energy and power of a signal $x(n) = \left(\frac{1}{2}\right)^n u(n)$

Hence signal is periodic, we calculate energy directly;

Sol^n:

$$\sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$\Rightarrow \sum_{n \to -\infty}^{\infty} \left[\left(\frac{1}{2}\right)^n\right]^2 u(n) = \sum_{n \to 0}^{\infty} \left(\frac{1}{2}\right)^{n*}$$

$$\Rightarrow \sum_{n=0}^{\infty} a^n = \frac{1}{1-a} = \frac{1}{1-a} \quad |a| < 1 \quad = \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$$

2. Periodic Signal and Aperiodic Signal:

A signal which has a definite pattern and repeats itself at regular intervals of time is called a periodic signal, and a signal which does not repeat at regular intervals of time is called a non-periodic or aperiodic signal.

A discrete-time signal $x(n)$ is said to be periodic if it satisfies the condition $x(n) = x(n + N)$ for all integers n.

The smallest value of N which satisfies the above condition is known as fundamental period.

If the above condition is not satisfied even for one value of n, then the discrete-time signalis aperiodic. Sometimes aperiodic signals are said to have a period equal to infinity.

Q. Determine whether the following signals are periodic or not and also find fundamental period?

$$x(t) = sin\ 10\pi t$$

$$x(t) = sin\ \pi t\ u(t)$$

- Given signal is,

$$x(t) = sin\ 10\pi t$$

Since the signal $x(t)$ is a sinusoidal signal, hence, it is a periodic signal.

Now, comparing $x(t)$ with standard signal, i.e., sin ωt, we get,

$$\omega = 10\pi$$

Fundamental period $T = \frac{2\pi}{\omega} = \frac{2\pi}{10\omega} = \frac{1}{5} sec$

- Given signal is,

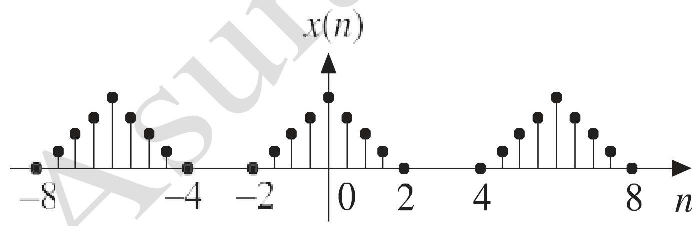
$$x(t) = sin\ \pi t\ u(t)$$

Here, $x(t)$ is the product of sinusoidal signal $(sin\ \pi t)$ and unit step signal$(u(t))$. As we know, the signal $sin\pi t$ is periodic with time period $T = \frac{2\pi}{\omega}$ while the signal $u(t)$ exists only for $0 < t < \infty$. Thus, (t) is not a periodic signal.

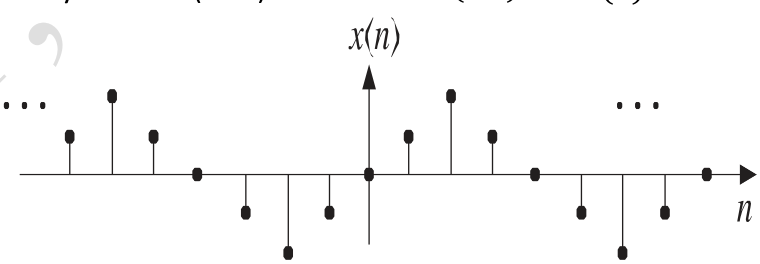
Therefore, the signal $x(t)$ is an aperiodic or non-periodic signal.

3. Symmetric Signal and Asymmetric Signal:

A discrete time signal is said to be symmetric (Even) if it satisfies $x(-n) = x(n)$ for all values of n



A discrete time signal is said to be Asymmetric (Odd) if it satisfies $x(-n) = -x(n)$ for all values of n.



2.1.3 SIMPLE MANIPULATION OF DISCRETE TIME SIGNAL:

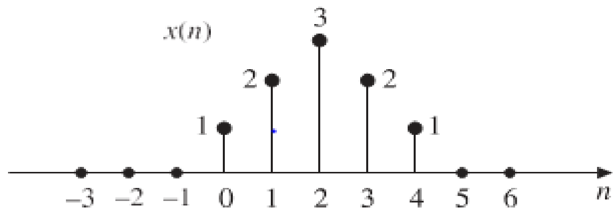
1. Shifting:

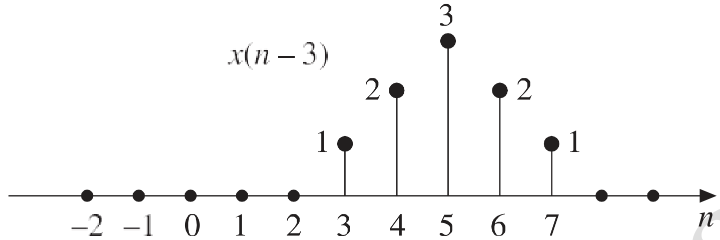
The time shifting of a signal may result in time delay or time advance. The time shifting operationof a discrete-time signal $x(n)$ can be represented by

$$Y(n) = x(n - k)$$

This shows that the signal $y(n)$ can be obtained by time shifting the signal $x(n)$ by k units. If k is positive, it is delay and the shift is to the right, and if k is negative, it is advance and the shift is to the left.

An arbitrary signal $x(n)$ is shown in below figure. $x(n-3)$ which is obtained by shifting $x(n)$ to the right by 3 units (i.e. delay $x(n)$ by 3 units) is shown in below figure.





2. <u>**Time Reversal (Folding) :**</u>

The time reversal also called time folding of a discrete-time signal $x(n)$ can be obtained by folding the sequence about $n = 0$. The time reversed signal is the reflection of the original signal. It is obtained by replacing the independent variable n by $-n$. Figure (a) shows an arbitrary discrete-time signal $x(n)$, and its time reversed version $x(-n)$ is shown in Figure (b).

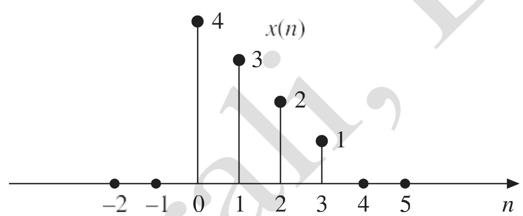


Fig .a

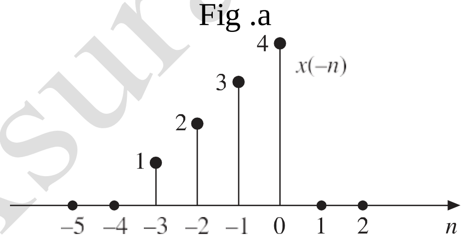


Fig .b

<u>**3. Amplitude Scaling:**</u>

The amplitude scaling of a discrete-time signal can be represented by

$$y(n) = ax(n)$$

where a is a constant.

The amplitude of $y(n)$ at any instant is equal to a times the amplitude of $x(n)$ at that instant. If $a > 1$, it is amplification and if $a < 1$, it is attenuation. Hence the amplitude is rescaled. Hence the name amplitude scaling.

Figure (1) shows a signal $x(n)$ and Figure (2) shows a scaled signal $y(n) = 2x(n)$.

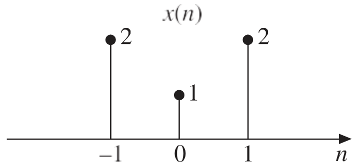


Fig.1

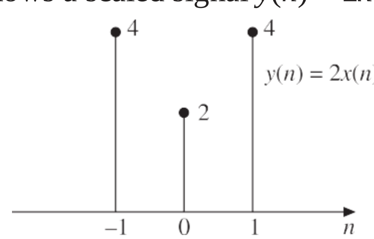


Fig.2

4,Time scaling

Time scaling may be time expansion or time compression. The time scaling of a discrete- time signal $x(n)$ can be accomplished by replacing n by an in it. Mathematically, it can be expressed as:

$$y(n) = x(an)$$

When $a > 1$, it is time compression and when $a < 1$, it is time expansion.
Let $x(n)$ be a sequence as shown in Figure (1). If $a = 2$, $y(n) = x(2n)$. Then

$$y(0) = x(0) = 1$$
$$y(-1) = x(-2) = 3$$
$$y(-2) = x(-4) = 0$$
$$y(1) = x(2) = 3$$
$$y(2) = x(4) = 0$$

and so on.

So to plot $x(2n)$ we have to skip odd numbered samples in $x(n)$.
We can plot the time scaled signal $y(n) = x(2n)$ as shown in Figure (2). Here the signal is compressed by 2
If $a = (1/2)$, $y(n) = x(n/2)$, then

$$y(0) = x(0) = 1$$
$$y(2) = x(1) = 2$$
$$y(4) = x(2) = 3$$
$$y(6) = x(3) = 4$$
$$y(8) = x(4) = 0$$
$$y(-2) = x(-1) = 2$$
$$y(-4) = x(-2) = 3$$
$$y(-6) = x(-3) = 4$$
$$y(-8) = x(-4) = 0$$

We can plot $y(n) = x(n/2)$ as shown in Figure(3). Here the signal is expanded by 2. All odd components in $x(n/2)$ are zero because $x(n)$ does not have any value in between the sampling instants.

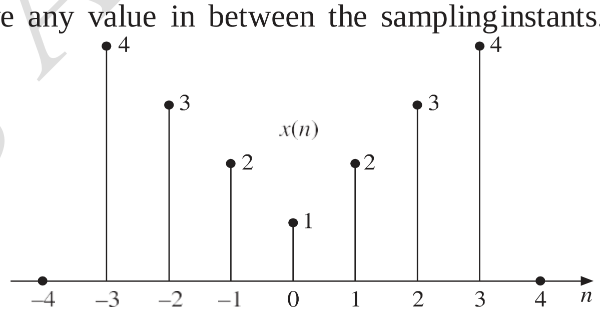


Fig.1

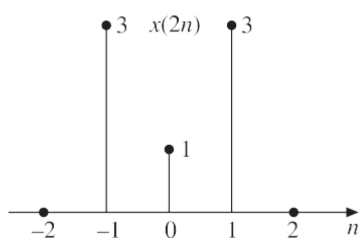


Fig.2

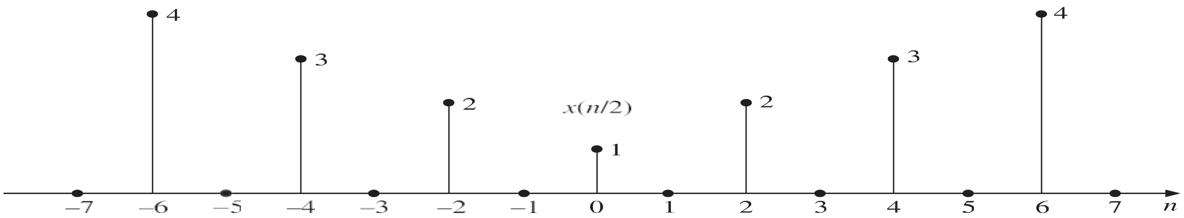


Fig.3

2.2 DISCRETE TIME SYSTEM.

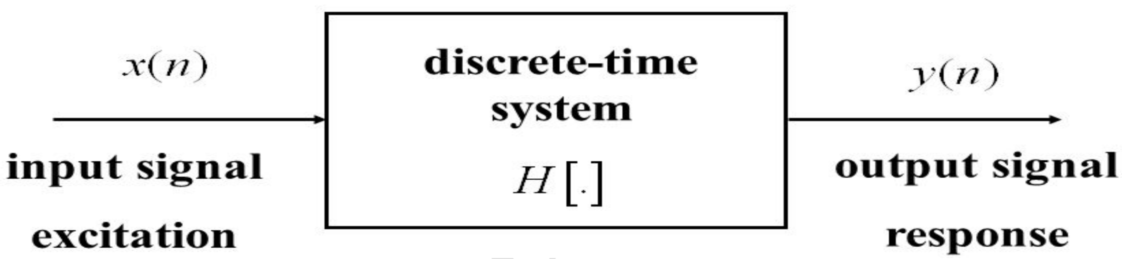
A system is defined as an entity that acts on an input signal and transforms it into an output signal. A system may also be defined as a set of elements or functional blocks which are connected together and produces an output in response to an input signal.

A discrete-time system is represented by a block diagram as shown in below figure. An arrow entering the box is the input signal (also called excitation, source or driving function)and an arrow leaving the box is an output signal (also called response). Generally, the inputis denoted by $x(n)$ and the output is denoted by $y(n)$.

The relation between the input $x(n)$ and the output $y(n)$ of a system has the form:

$$y(n) = \text{Operation on } x(n)$$

which represents that $x(n)$ is transformed to $y(n)$. In other words, $y(n)$ is the transformedversion of $x(n)$.



2.2.1 INPUT OUTPUT OF SYSTEM:

- Symbol H denotes transformations
- There is a mathematical relation between the input and output of discrete time system
- In order to give the input to the input terminal is use and in order to get the output to the output terminals used
- The input and output must be discreate in nature
- We can express as:

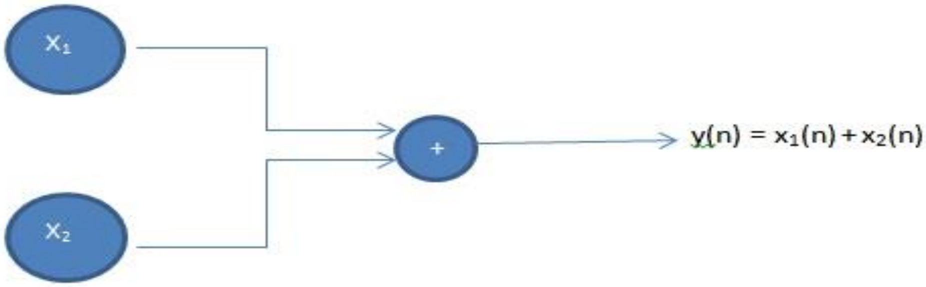
$$y(n) = H[x(n)]$$

2.2.2. BLOCK DIAGRAM OF DISCRETE- TIME SYSTEMS:

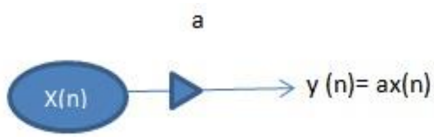
Digital Systems are represented with blocks of different elements or entities connected with arrows which also fulfils the purpose of showing the direction of signal flow,

Some common elements of Discrete-time systems are: -

Adder: It performs the addition or summation of two signals or excitation to have a response. An adder is represented as,



Constant Multiplier: This entity multiplies the signal with a constant integer or fraction. And is represented as, in this example the signal x(n) is multiplied with a constant "a" to have the response of the system as y(n).



2.2.3CLASSIFY DISCRETE TIME SYSTEM:

The discrete time systems can be classified as follows:

- Static/Dynamic
- Causal/Non-Causal
- Time invariant/Time variant
- Linear/Non-Linear
- Stable/Unstable

Static/Dynamic:

A system is said to be static or memoryless if the response is due to present input alone, i.e., for a static or memoryless system, the output at any instant *n* depends only on the input applied at that instant *n* but not on the past or future values of input or past values of output.

For example, the systems defined below are static or memoryless systems.

$$y(n) = x(n)$$
$$y(n) = 2x^2(n)$$

In contrast, a system is said to be dynamic or memory system if the response depends upon past or future inputs or past outputs. A summer or accumulator, a delay element is a discrete- time system with memory.

For example, the systems defined below are dynamic or memory systems.

$$y(n) = x(2n)$$
$$y(n) = x(n) + x(n – 2)$$
$$y(n) + 4y(n – 1) + 4y(n – 2) = x(n)$$

Any discrete-time system described by a difference equation is a dynamic system.
A purely resistive electrical circuit is a static system, whereas an electric circuit having inductor and/or capacitors is a dynamic system.

Causal/Non-Causal:

A system is said to be causal (or non-anticipative) if the output of the system at any instant *n* depends only on the present and past values of the input but not on future inputs, i.e., for a causal system, the impulse response or output does not begin before the input function is applied, i.e., a causal system is non anticipatory.

Causal systems are real time systems. They are physically realizable.

The impulse response of a causal system is zero for $n < 0$, since (n) exists only at $n = 0$,

i.e. $h(n) = 0 \quad \text{for} \quad n < 0$

The examples for causal systems are:

$$y(n) = nx(n)$$

$$y(n) = x(n-2) + x(n-1) + x(n)$$

A system is said to be non-causal (anticipative) if the output of the system at any instant *n* depends on future inputs. They are anticipatory systems. They produce an output even before the input is given. They do not exist in real time. They are not physically realizable.

A delay element is a causal system, whereas an image processing system is a non-causal system.

The examples for non-causal systems are:

$$y(n) = x(n) + x(2n)$$

$$y(n)=x^2(n)+2x(n+2)$$

Q. Test whether system is Causal or Non − causal?

$$y(n) = x(2n)$$

For n = –2	y(2)	x(4)
For n = 0	y(0)	x(0)
For n = 2	y(2)	x(4)

For positive values of *n*, the output depends on the future values of input. Therefore, the system is non-causal.

Q. Test whether system is Causal or Non − causal?

$$y(n) = x(n^2)$$

Ans:

Taking n=0

$$y(0) = x(0^2) = x(0)$$

Taking n=1

$$y(1) = x(1^2) = x(1)$$

Taking n=2

$$y(2) = x(2^2) = x(4)$$

Taking n=-1

$$y(-1) = x(-1^2) = x(1)$$

Here the output depends on present and future value to the system is non-causal.

Time invariant/Time variant:

Time-invariance is the property of a system which makes the behavior of the system independent of time. This means that the behavior of the system does not depend on the time at which the input is applied. For discrete-time systems, the time invariance property is called shift invariance.

A system is said to be shift-invariant if its input/output characteristics do not change with time, i.e., if a time shift in the input results in a corresponding time shift in the output , i.e.

If $T[x(n)] = y(n)$

Then $T[x(n-k)] = y(n-k)$

A system not satisfying the above requirements is called a time-varying system (or shift-varying system). A time-invariant system is also called a fixed system.

The time-invariance property of the given discrete-time system can be tested asfollows:

Let $x(n)$ be the input and let $x(n - k)$ be the input delayed by k units.
$y(n) = T[x(n)]$ be the output for the input $x(n)$.

Q. Test whether system is Time variant or Time invariant?

$$y(n) = x(n) + x(n-1)$$

Ans:

$$y(n,k) = x(n-k) + x(n-k-1)$$

$$y(n,k) = x(n-k) + x(n-k-1)$$

$$\Rightarrow y(n,k) = y(n-k)$$

So, the system is time invariant

Q. Test whether system is Time variant or Time invariant?

$$y(n) = x(n)$$

$$y(n,k) = x(-n-k)$$

$$y(n-k) = x(-n(n-k))$$

$$= x(-n+k)$$

$$y(n,k) \neq y(n-k)$$

So, the system is time variant.

Stable/Unstable:

- LDI system is said to be stable if it produces bounded output for AB for bounded.
- The necessary condition for stability is, $\sum_{n=-\infty}^{\infty} h(n) < \infty$

Q. Test stability of system whose impulse system

$$h(n) = \left(\frac{1}{2}\right)^n u(n)$$

$$h(n) = \left(\frac{1}{2}\right)^n u(n)$$

$$= \sum_{n=-\infty}^{\infty} h(n)$$

$$= \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^n u(n)$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n .1$$

$$= \frac{1}{1-\frac{1}{2}} = 2$$

As

$$\sum_{n=-\infty}^{\infty} h(n) < \infty = 2, \text{so the system is stable}$$

Linear/Non-Linear:

- A system that satisfies the principle of superposition is said to be LinearSystem.
- Super position principle said that the response of the system to weighted sum of signal is equal to the corresponding weighted sum of output signal toeach of individual output.

Example of linear system

$$a_1x_1(n) \rightarrow a_1y_1(n)$$

$$T[a_1x_1(n) + a_2x_2(n)] \rightarrow T[a_1x_2(n)] + T[a_2x_2(n)]$$

Example of non-linear system

$$T[a_1x_1(n) + a_2x_2(n)] = a_1T[x_1(n)] + a_2T[x_2(n)]$$

The system which does not follow the principle of superposition is known as Non-linear system

Q. Test whether the following system is linear or non-linear

$$y(n) = x^2(n)$$

$$y_1(n) = x_1^2(n)$$

$$a_1y_1(n) = a_1x_1^2(n)$$

$$y_2(n) = x_2^2(n)$$

$$a_2y_2(n) = a_2x_2^2(n)$$

$$y(n) = a_1x_1^2(n) + a_2x_2^2(n) \ldots \ldots .. (1)$$

$$y(n) = [a_1x_1(n) + a_2x_2(n)]^2 \ldots \ldots (2)$$

Equation (1) and (2) are non-linear

Q. Test whether the following system is linear or non-linear

$$Y(n) = nx(n)$$

$$y_1(n) = nx_1(n)$$

$$y_2(n) = nx_2(n)$$

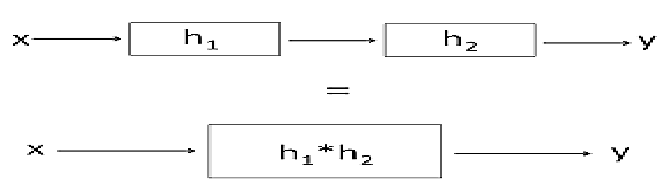
$$y_1(n) = a_1nx_1(n) + a_2nx_2(n) \ldots \ldots \ldots .(1)$$

$$T[a_1x_1(n) + a_2x_2(n)] = a_1nx_1(n) + a_2nx_2(n) \ldots \ldots \ldots .(2)$$

Equation (1) and (2) are equal so it is linear

2,2,3INTERCONNECTION OF DISCRETE -TIME SYSTEM:

I.SERIES INTERCONNECTION



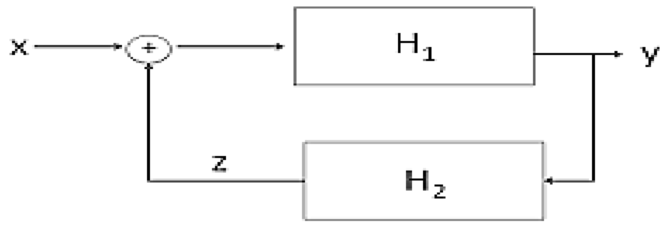
$$y_2(n) = x_1(n) * h(n)$$

$$H(n) = \sum_{n=0}^{\infty} h_1(k) * h_2(n-k)$$

$$= h_1(n) * h_2(n) \qquad [* \rightarrow \text{convolutions}]$$

Here the impulse response of two LTI system connected in cascade is the convolution of individual impulse response.

II. PARALLEL INTERCONNECTION



$$y_1(n) = x(n) * h_1(n)$$

$$y_2(n) = x(n) * h_2(n)$$

$$y(n) = y_1(n) + y_2(n)$$

$$y(n) = \sum_{n=-\infty}^{\infty} x(k)h_1(n-k) + \sum_{n=-\infty}^{\infty} x(k)h_2(n-k)$$

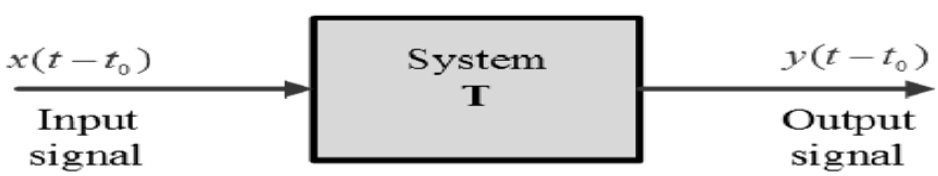
$$\Rightarrow \sum_{n=-\infty}^{\infty} x(k)h(n-k)$$

$$y(n) = x(n) * h(n)$$

When system is connected in parallel the overall impulse response is the summation of individual impulse response.

2.2DISCRETE TIME TIME-INVARIANT SYSTEM:

- A system is called time-invariant if a time shift in the input signal $x(t - t_0)$ causes the same time shift in the output signal $y(t - t_0)$.it is shown in figure.



2.3.1 DIFFERENT TECHNIQUES FOR THE ANALYSIS OF LINEAR SYSTEM:

Time-varying impulse response:

The time-varying impulse response $h(t_2, t_1)$ of a linear system is defined as the response of the system at time $t = t_2$ to a single impulse applied at time $t = t_1$.In other words, if the input x(t) to a linear system.

$$z(t) = \phi(t - t_1)$$

Where $\delta(t)$ represents the Dirac delta function and the corresponding response $y(t)$of the system is

$$y(t)|_{t-t_2} = h(t_2, t_1)$$

Then the function $h(t_2, t_1)$ is the time varying impulse response of the system. since the system cannot respond before the input is applied the following causality condition must be satisfied

$$h(t_2, t_1) = 0, t_2 < t_1$$

The convolution integral:

The output of any general continuous time linear system is related to the input by an integral which may be written over a doubly infinite range because of the causality condition

$$y(t) = \int_{-\infty}^{l} h(t, t^*)x(t^*)\mathrm{d}t^* = \int_{-\infty}^{\infty} h(t, t^*)x(t^*)\mathrm{d}t^*$$

$$y(t) = \int_{-\infty}^{t} h(t - t^*)x(t^*)dt^* = \int_{-\infty}^{\infty} h\Big(t - t^*) \mid x(t^*)dt^* = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)dr = \int_{0}^{x} h(\tau)z(t-\tau)dr$$

Linear time invariant system are most commonly characteristics by the LaPlace transform of the impulse response function is called transfer function which is

$$\mathrm{H(s)} = \int_0^\infty \mathrm{e}^{-\mathrm{st}}$$

In this application this is usually a rational algebraic function of s because $h(t)$ are zero for negative t the integral may equally be written over the doubly infinite range and putting $s = j\omega$ follow the formula of frequency response function.

$$h(\mathrm{j}\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t}dt$$

Discrete time system:

The output of any discrete time linear system is related to the input by the time-varying convolution sum

$$\mathrm{Y[n]} = \sum_{\mathrm{m}=-\infty}^{\mathrm{n}} \mathrm{h[n,m]x[m]} = \sum_{\mathrm{m}=-\infty}^{\infty} \mathrm{h[n,m]x[m]}$$

Or equivalently for a time invariant system on re-finding h(n)

$$\mathrm{Y[n]} = \sum_{\mathrm{k}=0}^{\infty} \mathrm{h[k]x[n-k]} = \sum_{\mathrm{k}=-\infty}^{\infty} \mathrm{h[k]x[n-k]}$$

Where $k = n - m$ represent the lag time between the stimulus at the time m and the response of time n.

2.3.2RESOLUTION OF A DISCRETE TIME SIGNAL IN TO IMPULSE:

$$x(n) = \mathrm{x(n)} = \sum_{\mathrm{k}} \mathrm{c_k\, x_k(n)}$$

Suppose

$$\mathrm{x_k(n)} = \delta(\mathrm{n-k})$$

Then

$$\mathrm{x(n)}\delta(\mathrm{n-k}) = \mathrm{x(k)}\delta(\mathrm{n-k})$$

Is zero everywhere except at $n = k$

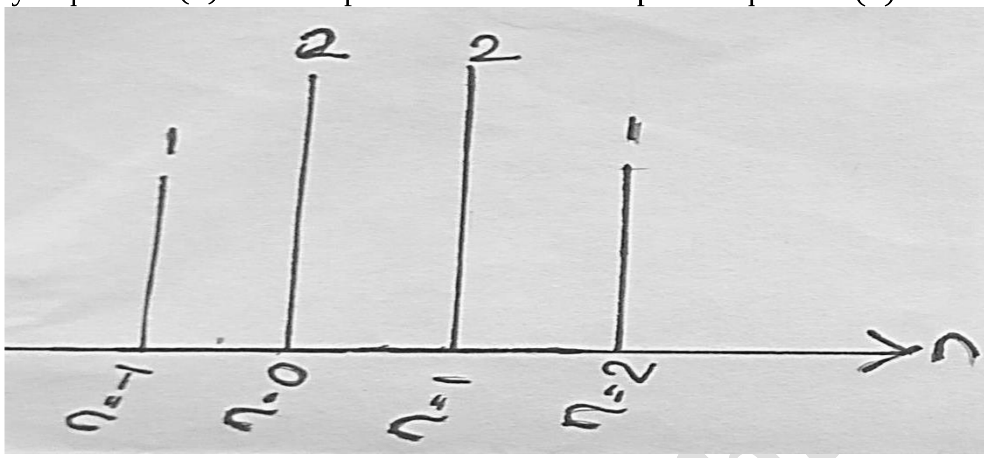
This means we can write $x(n)$ as

$$X(n) = \sum_{\mathrm{k}=-\infty}^{\infty} \mathrm{x(k)}\delta(\mathrm{n-k})$$

2.3.3 RESPONSE OF LTI SYSTEM TO ARBITRARY INPUTS USING CONVOLUTION SYSTEM.:

Convolution sum:

Any arbitrary sequence x(n) can be represents in terms of impulse sequence s(n).



$$x(n) = \begin{matrix} 2, & n = 0,1 \\ 1, & n = -1,2 \end{matrix}$$

$$2\delta(n) + 1\delta(n+1) + 2\delta(n-1) + 1\delta(n-2)$$

- A discrete time system performs an operation on the input signal based on some pre-defined criteria to produce a modified output.
- If the input of the system id unit impulse, then the output of the system is denoted by$h(n)$

$$\mathrm{h(n) = T[s(n)]} \ldots\ldots\ldots\ldots\ldots..(1)$$

- We can write the arbitrary sequence as a weighted sum of discrete impulse

$$\mathrm{y(n)} = \sum_{\mathrm{n}=-\infty}^{\infty} \mathrm{x(k)\delta(n-k)} \ldots\ldots(2)$$

- For a linear system, we can write

$$\mathrm{y(n)} = \sum_{\mathrm{n}=-\infty}^{\infty} \mathrm{x(n) + T[\delta(n-k)]}$$

- For time invariant system

$$\mathrm{h(n,k) = h(n-k)}$$

- The convolution sum can be represented as

$$\mathrm{y(n) = x(n) * h(n)}$$

Steps included in convolution:

The process of convolution between the sequence x(k) and h(k) involves the following steps:

i)Folding:

h(k) is folded and we obtain h(−k)

ii)Shifting:

Shift h(-k) by n

iii)Multiplication:

Multiply x(k) by $h(n-k)$

Q. Determine the convolution sum of two sequence (Graphical method)

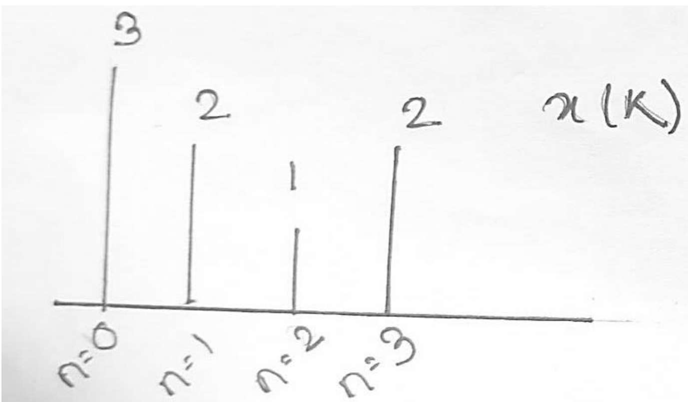
$$x(n) = \{3,2,1,2\}$$
$$h(n) = \{1,2,1,2\}$$
$$\uparrow$$

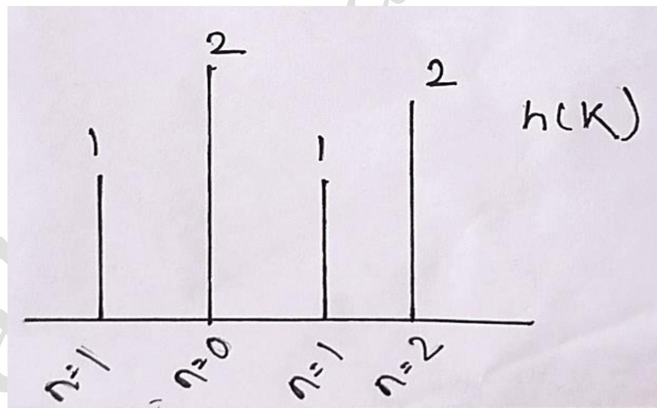
Step-1

The sequence $x(n)$ starts at $n = 0$ and $h(n)$ starts at $n2 = -1$.Therefore the starting time for evaluating the output sequence $y(n)$ is $n = n_1 + n_2 = 0 + (-1) = -1$

Step-2

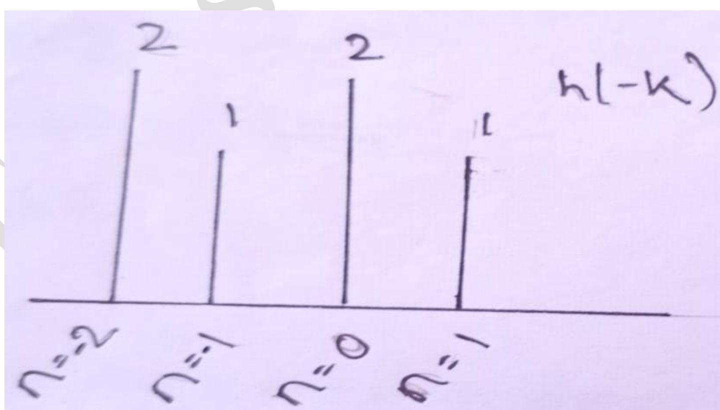
Express both sequences in terms of the index k



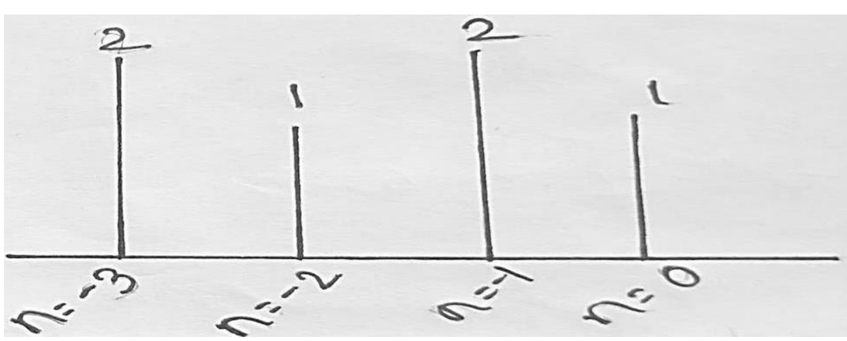


Step-3

Fold $h(k)$ about $k = 0$ to obtain $h(-k)$



As starting time to evaluate $y(n)$is -1,shift $h(k)$ by one unit to obtain $h(-1-k)$



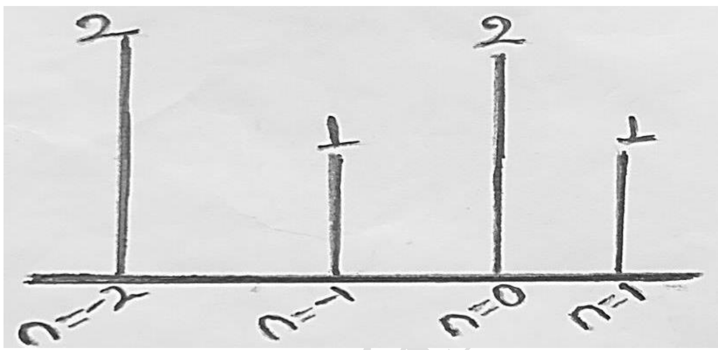
$$y(-1) = \sum_{k=-\infty}^{\infty} x(k)h(-1-k)$$

$$y(-1) = 0(2) + 0(1) + 0(2) + 3(1) + 2(0) + 1(0) + 2(0) = 3$$

Increment the index by 1,shift the sequence to right to obtain h(-k) and multiply the two sequence $x(k)$ And $h(-k)$ element and sum the product.

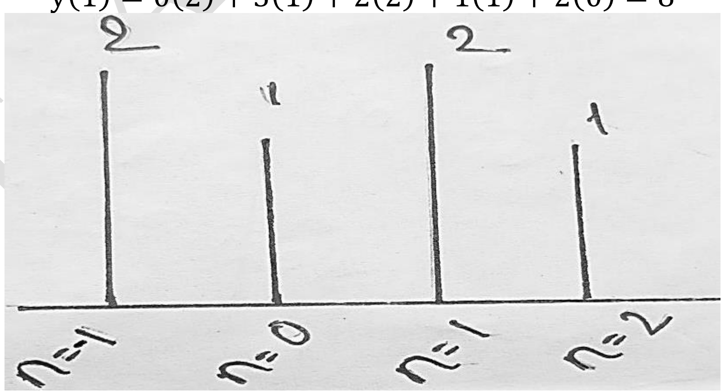
$$y(0) = \sum_{k=-\infty}^{\infty} x(k)h(-k)$$

$$y(0) = 0(2) + 0(1) + 3(2) + 2(1) + 1(0) + 1(0) + 2(0) = 8$$



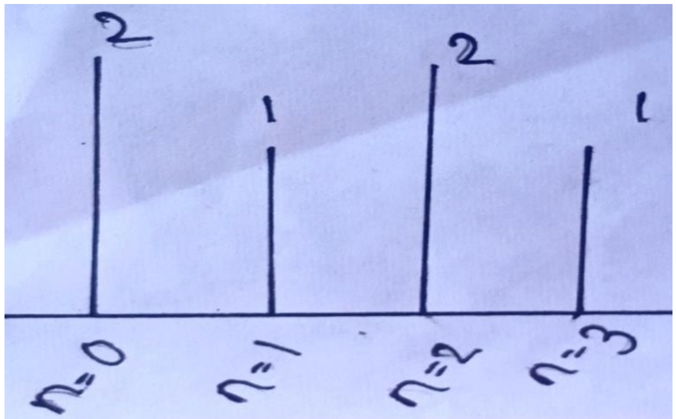
$$y(1) = \sum_{k=-\infty}^{\infty} x(k)h(1-k)$$

$$y(1) = 0(2) + 3(1) + 2(2) + 1(1) + 2(0) = 8$$



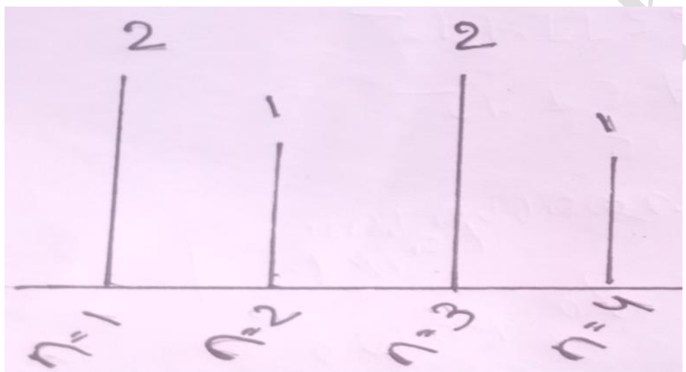
$$y(2) = \sum_{k=-\infty}^{\infty} x(k)h(2-k)$$

$$y(2) = 3(2) + 2(1) + 1(2) + 2(1) = 12$$



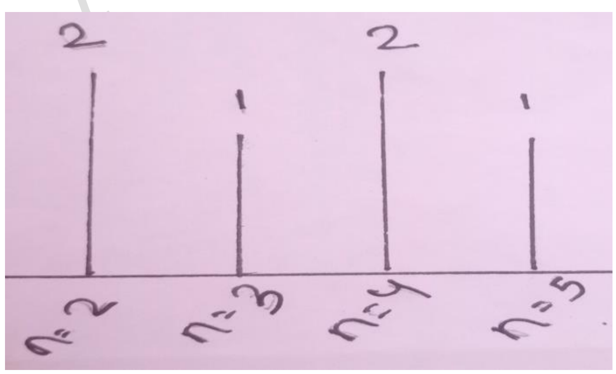
$$y(3) = \sum_{k=-\infty}^{\infty} x(k)h(3-k)$$

$$y(3) = 3(0) + 2(2) + 1(1) + 2(2) = 9$$



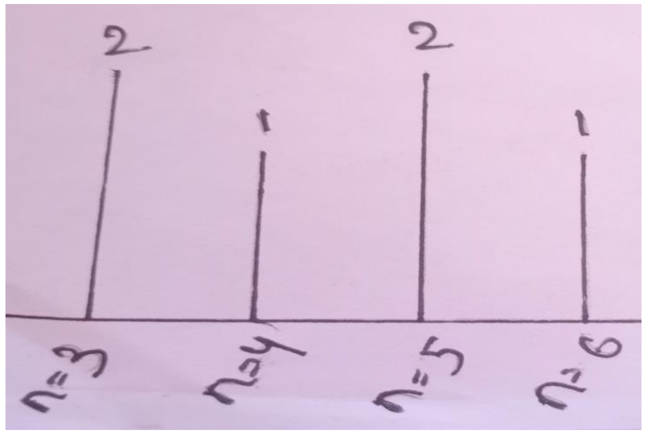
$$y(4) = \sum_{k=-\infty}^{\infty} x(k)h(4-k)$$

$$y(4) = 3(0) + 2(0) + 1(2) + 2(1) + 0(2) = 4$$



$$y(5) = \sum_{k=-\infty}^{\infty} x(k)h(5-k)$$

$$y(5) = 3(0) + 2(0) + 1(0) + 2(2) + 0(1) + 0(2) + 0(1) = 4$$



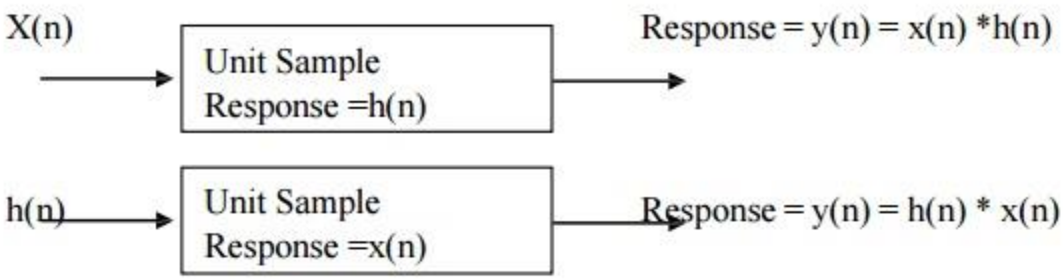
$$y(n) = \{3,8,8,12,9,4,4\}$$
$$\uparrow$$

2.3.4 CONVOLUTION & INTERCONNECTION OF LTI SYSTEM - PROPERTIES:

Properties of linear convolution:

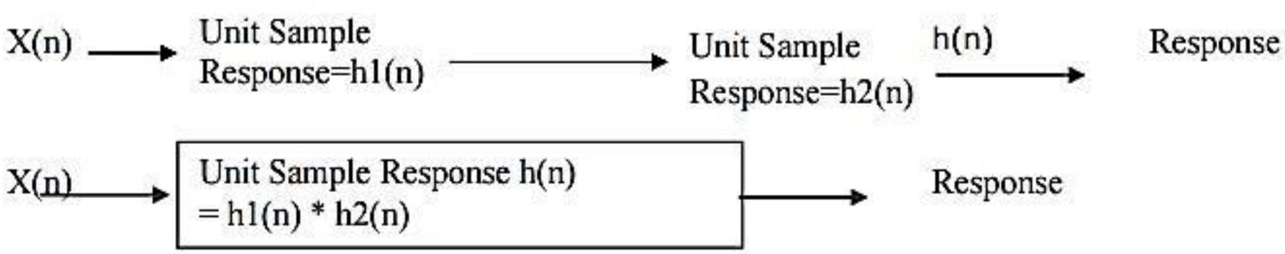
X(n)=Excitation input signal y(n)
=Output Response
h(n)=Unit sample response

1. Commutative Law (commutative property of convolution)



$$x(n) * h(n) = h(n) * x(n)$$

2. Associative Law:(Associative property of convolution)



$$[x(n) * h_1(n)] * h_2(n) = x(n) * [h_1(n) * h_2(n)]$$

3. Distributive Law :(Distributive property of convolution)

$$x(n) * [h_1(n) + h_2(n)] = x(n) * h_1(n) + x(n) * h_2(n)$$

Interconnection of LTI system:

1. Cascade Interconnection
2. Parallel Interconnection
3. Series- parallel Interconnection

2.3.5 STUDY SYSTEMS WITH FINITE DURATION AND INFINITE DURATION IMPULSE RESPONSE:

- According to the duration of impulse response LTI system can be classified into two types.

1) Finite impulse response system
   If the impulse response sequence is of finite duration.
   Example:

$$h(n) = \begin{matrix} 1, & n = 0 \\ 2, & n = 3 \end{matrix}$$

   For causal FIR system $h(n) = 0$ for $n < 0$

2) Finite impulse response system
   The system whose impulse response is infinite known as infinite impulse system.
   Example:

$$h(n) = a^n u(n)$$

2.4DISCRETE TIME SYSTEM DESCRIBED BY DIFFERENCE EQUATION:

2.4.1 RECURSIVE DISCRETE TIME SYSTEM

This system requires two multiplication, one addition, and one memory location. This is a recursive system which means the output at time n depends on any number of a past output values. So, a recursive system has feedback output of the system into the input.

NON-RECURSIVE DISCRETE TIME SYSTEM

A non-recursive formula is a formula for a sequence that does not itself depend on any other terms in the sequence. In other words, the only variable you will need to plug in is the index of the sequence. For instance, $S(n) = n^2$ is one of the most basic non-recursive formulas.

2.4.2 DETERMINE THE IMPULSE RESPONSE OF LINEAR TIME INVARIANT RECURSIVE SYSTEM:

There are two methods:

i)Direct method

ii)Indirect method

i)Direct Method

In this method the solution of difference equations consists of two parts.

a) Homogeneous solution. Denoted by $y_h(n)$

b) Particular solution. Denoted by $y_p(n)$

Total solution=$y(n) = y_h(n) + y_p(n)$

Homogeneous solutions:

It is obtained by putting $x(n) = 0$,so that

$$\sum_{k=0}^{N} a_k y(n-k) = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots.(1)$$

Assume that the solutions of this equations is in exponential form

$$y_h(n) = \lambda^n \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Putting equation (ii) in equation (i)

$$\sum_{k=0}^{N} a_k \lambda^{(n-k)} = 0$$

$$= a_0\lambda^{(n)} + a_1\lambda^{(n-1)} + a_2\lambda^{(n-2)} \dots\dots.. a_N\lambda^{(n-N)} = 0$$

Assume that $a_0 = 1$

Hence,

$$\lambda^{(n)} + a_1\lambda^{(n-1)} + a_2\lambda^{(n-2)} \dots\dots.. a_N\lambda^{(n-N)} = 0$$

Taking λ^{n-N} common, we get

$$\lambda^{n-N}[\lambda^n.\lambda^{-n+N} + a_1\lambda^{n-1}.\lambda^{-n+N} + a_2\lambda^{n-2}.\lambda^{-n+N} + \cdots a_N] = 0$$

Or,

$$\lambda^{n-N}[\lambda^N + a_1\lambda^{N-1} + a_2\lambda^{N-2} + \cdots + a_N] = 0$$

The polynomial in the bracket is known as characteristic equations

Case-1: Roots are distinct

If roots are $\lambda_1, \lambda_2, \lambda_3 \dots.. \lambda_n$.The general solutions is in the form.

$$y_h(n) = c_1\lambda_1^n + c_2\lambda_2^n + \cdots c_n\lambda_N$$

Where $c_1, c_2, c_3 \dots$....are weighting coefficient

Case-2: Repeated Roots

Let λ_1 is repeated for n times so, the general solutions

$$y_h(n) = \lambda_1[c_1 + c_2 n \dots\dots]$$

Case-3: Complex Roots

If the roots are complex conjugate

$$\lambda_1 = a + ib$$
$$\lambda_2 = a - ib$$

Then the solutions

$$y_n(n) = r^n(\lambda_1^n \cos n\theta + \lambda_2^n \sin n\theta)$$

Here

$$r = \sqrt{a^2 + b^2}$$
$$\theta = \tan^{-1}\frac{b}{a}$$

Particular Solution

This can be obtained by assuming a form that depends on the input $x(n)$.There are some general structures given below:x(n)

$$A \rightarrow K$$
$$AM^n \rightarrow KM^n$$
$$An^M \rightarrow K_0 n^M + K_1 n^{M-1} + \cdots$$
$$A^n n^M \rightarrow A^n[K_0 n^M + K_1 n^{M-1} \dots\dots\dots.]$$
$$A\cos\omega_0 n + a\sin\omega_0 n \rightarrow K_1\cos\omega_0 n + K_2\sin\omega_0 n$$

A, K, M, K, C are constants.

Q. Determine the homogeneous solution of the system described by the first order differential equation is:

$$y(n) + a_1 y(n-1) = x(n)$$

$$y_h(n) = y(n) + a_1 y(n-1) = 0 \dots (1)$$

$$\Rightarrow \lambda^{n-1}(\lambda^n + a_1) = 0$$

$$\Rightarrow \lambda = -a_1$$

$$y_h(n) = c_1 \lambda^n$$

$$y_h(n) = c_1(-a_1)^n \dots\dots (ii)$$

Value of c_1 in n=0 in equation(ii)

$$y(0) = c_1 \dots\dots (iii)$$

Put n=0 in equation (i)

$$y(0) + a_1 y(-1) = 0$$

$$y(0) = -a_1 y(-1)$$

Since, $y(0) = c_1$(from equation iii)

$$c_1 = -a_1 y(-1)$$

$$y_h(n) = c_1(\lambda)^n$$

$$y_h(n) = c_1(-a)^n$$

$$y_h(n) = (-a_n) y(-1)(-a_n)^n$$

$$y_h(n) = (-a_1)^{n+1} y(-1)$$

2.4.3 CORRELATION OF DISCRETE TIME SIGNAL:

The concept of correlation namely:

- Auto correlation
- Cross correlation

Auto Correlation of discrete time signal

The auto correlation is the measure of similarly between a sequence $x(n)$ and its shifted version $x(n-k)$.

Auto Correlation of discrete time energy signals

The auto correlation function between the real values discrete time signal $x(n)$and its time shifted version is

$$R(k) = \sum_{n=-\infty}^{\infty} x(n).x(n-k)$$

Where n is the variable and K is the constant representing the shift.

Auto Correlation of discrete time power signals

The auto correlation function of a real valued discrete time power signal$x(n)$is defind as

$$R(k) = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x(n)x(n-k)$$

$$R(k) = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x(n)x(n+k)$$

Cross correlation of discrete time Energy signals

Let $x_1(n)$ and $x_2(n)$ denote a pair of real valued discrete time energy signal. The cross-correlation function defined as

$$R(k) = \sum_{n=-\infty}^{\infty} x_1(n).x_2(n-k)$$

The second cross correlation function of $x_1(n)$ and $x_2(n)$ defind as

$$R(k) = \sum_{n=-\infty}^{\infty} x_1(n-k).x_2(n)$$

Cross correlation of discrete time power signals

If $x_1(n)$and $x_2(n)$ represent two different discrete time power signals then the Cros correlation is defined as:

$$R(k) = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x_1(n)x_2(n-k)$$

$$R(k) = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x_1(n+k)x_2(n)$$

If $x_1(n)$and $x_2(n)$ signal to be orthogonal then cross correlation is zero

$$R(k) = 0$$

POSSIBLE SHORT TYPES QUESTIONS WITH ANSWER

1.What is a Discrete Signal?

Ans -A discrete time signal S(n) is a function of an independent variable 'n'.

2.How Discrete Time signal can be represented?

Ans-Discrete Time signal can be represented in the following 4 ways,

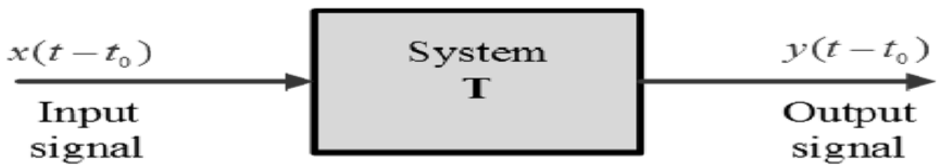
1. Graphical representation
2. Functional representation
3. Tabular representation
4. Sequential representation

3.Define fundamental period?

Ans - A discrete-time signal is periodic if there is a non-zero integer N ∈ discrete time such that for all n ∈ discrete time, $x(n + N) = x(n)$. The smallest value of N is known as the fundamental period.

4.What do you mean by a Discrete Time System?

Ans -The discrete time system is anything that takes a discrete time signal as input and generates discrete time output.



5.Classify Discrete Time System.

Ans-

- Static/Dynamic
- Causal/Non-Causal
- Time invariant/Time variant
- Linear/Non-Linear
- Stable/Unstable
- FIR /IIR

6.What do you mean by Causal and Non-Causal System.(S-24)

Ans:

- A causal system is one whose output depends only on the present and the pastinputs.
- A noncausal system's output depends on the future inputs. In a sense, a noncausalsystem is just the opposite of one that has memory.
- It cannot because real systems cannot react to the future.

7.what is energy and power signal.(S-24)

A: Non periodic signal are energy signal where $0<E<\infty$ & $p=0$

Pratical periodic signal are power signal where $0<p<\infty$ & $E=\infty$

8.Check whether the signal x(n)=u(n)-u(n-1) is causal or non causal.(S-24)

A: It is a causal signal because output depends upon present input and past input u(n-1)

9. Define unit step and unit ramp signal in discrete time domain.(S-24)

In the discrete-time domain, a unit step signal (denoted as u[n]) is a signal that is 0 for all values of n less than 0 and 1 for all values of n greater than or equal to 0,

In a unit ramp signal (denoted as r[n]) is a signal that is 0 for all values of n less than 0 and increases linearly with n (i.e., has a value of n) for all values of n greater than or equal to 0,

POSSIBLE LONG TYPE QUESTIONS

1. Classify Discrete Time system with example.(S-24)
2. Check for i)Linearity ii)Time invariance of given by y(n)=x(n)+nx(n-1) where x(n) is input and y(n) o/p.(S-24)
3. Determine the response of the relaxed system by impulse response $h(n)=(1/2)^n u(n)$ to i/p $x(n)=2^n u(n)$.(S-24)
4. Determine the convolution sum of two sequence (matrix method) [2019(S)]

$$x(n) = \{2,4,-1,2\}$$

$$h(n) = \{-1,4,2,-3\}$$

5. Consider a causal stable system whose input is x(n) and output is y(n) are related by the difference equations. [2018(S)]

$$y(n) - \frac{1}{6}y(n-1) - \frac{1}{6}y(n-2) = x(n)$$

CHAPTER NO.- 03

THE Z-TRANSFORM & ITS APPLICATION TO THE ANALYSIS OF LTI SYSTEM

Learning Objectives:

3.1 Z-transform & its application to LTI system.
3.1.1 Direct Z-transform.
3.1.2 Inverse Z-transform
3.2 Various properties of Z-transform.
3.3 Rational Z-transform.
3.3.1 Poles & zeros.
3.3.2 Pole location time domain behavior for casual signals.
3.3.3 System function of a linear time invariant system.
3.4 Discuss inverse Z-transform.
3.4.1 Inverse Z-transform by partial fraction expansion.
3.4.2 Inverse Z-transform by contour Integration.

3.1. Z-TRANSFORM & ITS APPLICATION TO LTI SYSTEM

- Z-Transform is used to analyze the LTI discrete time signal.
- Z-Transform converts the discrete Time signal into frequency domain.
- It is used for both stable and unstable systems.
- In Z-domain, the convolution of two sequences is equivalent to multiplications of theircorresponding Z-Transform.

3.1.1 DIRECT Z-TRANSFORM:

Z- Transform of a discrete time signal x(n) ca be written as,

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n} \ldots\ldots..(I)$$

Where, z is a complex variable

- Z-Transform of a signal x(n) is denoted by,

$$X(z) = z[x(n)]$$

$$X(z) \overset{z}{\leftrightarrow} x(n)$$

- If $x(n)$ is causal, then $x(n) = 0$ for $n < 0$ and the Z- Transform can be written as,

$$X(z) = \sum_{n=0}^{\infty} x(n)z^{-n} \ldots\ldots..(ii)$$

- If x(n) is causal, then the Z- Transform exists for those values of n for which seriesconverse.

Example: Find the Z-transform of the given sequence

$x(n) = \{1,2,3,4,1,2\}$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$$

$$= \sum_{n=-3}^{2} x(n)z^{-n}$$

$$X(z) = x(-3)z^{-(-3)} + x(-2)z^{-(-2)} + x(-1)z^{-(-1)} + x(0)z^{0} + x(1)z^{-1} + x(2)z^{-2}$$

$$X(z) = 1z^{3} + 2z^{2} + 3z^{1} + 4 + 1z^{-1} + 2z^{-2}$$

ROC (Region Of Convergence)

- The region of convergence of x(z) is the set of all values of Z for which x(z) attains a finite value.

We know that Z-transform is expressed as under

$$Z\{x(n)\} = X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n} \ldots\ldots (1)$$

We may write complex number

$$z = re^{j\omega}$$

Substituting $z = re^{j\omega}$, in equation 1

$$X(z) = X(re^{j\omega}) =$$

$$\sum_{n=-\infty}^{\infty} x(n)r^{-n}e^{-j\omega n} \ldots\ldots (2)$$

The above expression is the discrete time Fourier transform of the modified discrete time signal $\{\mathbf{x(n)r^{-n}}\}$

Now if $\mathbf{r = 1}$ then $|\mathbf{z}| = \mathbf{1}$

Hence the expression (2) will coverage if $\{\mathbf{x(n)r^{-n}}\}$ is absolutely summable

Mathematically, we have

$$\sum_{\mathbf{n}=-\infty}^{\infty} |\mathbf{x(n)r^{-n}}| < \infty$$

Hence $\mathbf{x(n)}$ to be finite the magnitude of its Z-transform $\mathbf{X(z)}$ should be finite.

Therefore, the set of value of Z in the Z-plane for which the magnitude of X(z) is finite, is called the Region of convergence (ROC).

Properties:

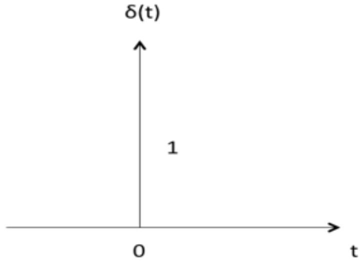
- ROC of z-transform is indicated with circle in z-plane.
- ROC does not contain any poles.
- If x(n) is a finite duration causal sequence or right sided sequence, then the ROC is entire z-plane except at z = 0.
- If x(n) is a finite duration anti-causal sequence or left sided sequence, then the ROC is entire z-plane except at z = ∞.
- If x(n) is a infinite duration causal sequence, ROC is exterior of the circle with radius a. i.e. |z| > a.
- If x(n) is a infinite duration anti-causal sequence, ROC is interior of the circle with radius a. i.e. |z| < a.

- If x(n) is a finite duration two-sided sequence, then the ROC is entire z-plane except at z = 0 & z = ∞.

Z-transform of some common signal

a) Z-transform of unit impulse $\delta(n)$

Unit impulse $\delta(n)$ has been shown in figure



$$\delta(n) = \begin{matrix} 1 & \text{only at n} = 0 \\ 0 & \text{otherwise} \end{matrix}$$

According to definition of Z-transform we have

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

Here $x(n) = \delta(n)$

Therefore,

$$X(z) = \sum_{n=-\infty}^{\infty} \delta(n) z^{-n}$$

Since $\delta(n)$ is present only at n=0 we can directly write under:

$$X(z) = \delta(0) z^0$$

$$\delta(0) = 1 \text{ and } z^0 = 1$$

$$X(z) = 1$$

ROC: In above equation there is no Z term. hence ROC is entire z-plane.

This means that z can have any value

The z-transform pair is given by $\delta(n) \overset{z}{\leftrightarrow} 1$

b) Find the z-transform of discrete time unit-step signal u(n)

We know that z-transform is expressed as

$$z\{x(n)\} = X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \ldots \ldots (1)$$

We know that discrete time unit step function u(n) exists only for positive values i.e. causal

$$X(z) = \sum_{n=0}^{\infty} x(n) z^{-n} = \sum_{n=0}^{\infty} \frac{x(n)}{z^n}$$

$$X(z) = x(0) + \frac{x(1)}{z} + \frac{x(2)}{z} + \frac{x(3)}{z} + \ldots\ldots (2)$$

But given that $x(n) = u(n)$

so $x(0) = u(0) = 1$ and $x(1) = u(1) = 1$

Similarly, $x(2) = x(3) = 1$

Therefore equation 2 becomes:

$$X(z) = 1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \cdots$$

The above expression is a geometric progression, its sum is:

$$X(z) = \frac{1}{1 - \frac{1}{z}} \quad \left|\frac{1}{z}\right| < 1$$

$$X(z) = \frac{z}{z - 1} \quad |z| > 1$$

Hence

$$u(n) \overset{z}{\leftrightarrow} \frac{z}{z - 1} \quad ROC|z| > 1$$

This Z-transform pair for unit-step sequence. The ROC is $|z| > 1$.This means that the ROC is the area outside the circle of radius 1. This type of circle is called unit circle.

3.1.1 INVERSE Z-TRANSFORM:

The process used for transforming the Z-domain signal into time domain signal is known asinverse Z-Transform.

Mathematically, the inverse z-trans form is expressed by:

$$x(n) = z^{-1}[X(z)]$$

There are three methods to perform Z-transform:

- Long division method.
- Partial fraction expansion method.
- Residue method.

3.2VARIOUS PROPERTIES OF Z-TRANSFORM:

1)Linearity Property

The z-transform is linear. this property states that the z-transform of a linear combination of discrete time signal is equal to the same linear combination of their z-transform.

Mathematically if

$$x_1(n) \overset{z}{\leftrightarrow} X_1(z)$$

And

$$x_2(n) \overset{z}{\leftrightarrow} X_2(z)$$

$$x(n) = a_1x_1(n) + a_2x_2(n) \overset{z}{\leftrightarrow} X(z) = a_1X_1(z) + a_2X_2(z)$$

Where a_1, a_2 are two arbitrary constants.

2) Time Reversal

Time Reversal property states that

$$\text{if } x(n) \overset{z}{\leftrightarrow} X(z) \qquad ROC: r_1 < |z| < r_2$$

$$\text{Then } x(-n) \overset{z}{\leftrightarrow} X(z^{-1}) \qquad \text{ROC:} \frac{1}{r_2} < |z| \frac{1}{r_1}$$

3) **Time shifting property**

Time shifting property states that

$$\text{if } x(n) \overset{z}{\leftrightarrow} X(z)$$

$$\text{Then } x(n-n_0) \overset{z}{\leftrightarrow} z^{-n_0}X(z)$$

The ROC of $z^{-n_0}X(z)$ will be same that of $X(z)$ except for $z = 0$ if $n_0 > 0$ and $z = \infty, n_0 < 0$

4) **Scaling property**

The scaling property states that

$$\text{if } x(n) \overset{z}{\leftrightarrow} X(z) \qquad \text{ROC: } r_1 < |z| < r_2$$

$$\text{Then } a^n x(n) \overset{z}{\leftrightarrow} X(a^{-1}z) \quad \text{ROC: } |a|r_1 < |z| < |a|r_2$$

Here, a is any constant which may be real or complex quantity

5) **Differentiation property**

Differentiation property states that

$$\text{if } x(n) \overset{z}{\leftrightarrow} X(z)$$

$$\text{Then } n\, x(n) \overset{z}{\leftrightarrow} -z\frac{dX(z)}{dz}$$

$$\text{Or } n\, x(n) \overset{z}{\leftrightarrow} z^{-1}\frac{dX(z)}{dz^{-1}}$$

6) **Convolution property**

Convolution property sates that

$$\text{if } x_1(n) \overset{z}{\leftrightarrow} X_1(z)$$

$$\text{and } x_2(n) \overset{z}{\leftrightarrow} X_2(z)$$

$$\text{Then } x(n) = x_1(n) * x_2(n) \overset{z}{\leftrightarrow} X_1(z).X_2(z)$$

7) **Correlation property**

Correlation property of z-transform states that

$$\text{if } x_1(n) \overset{z}{\leftrightarrow} X_1(z)$$

$$\text{and } x_2(n) \overset{z}{\leftrightarrow} X_2(z)$$

Then $r_{x_1x_2}(1) =$

$$\sum_{n=-\infty}^{\infty} x_1(n)x_2(n-1) \overset{z}{\leftrightarrow} R_{x_1x_2}(z) = X_1(z).X_2(z^{-1})$$

8) **Conjugation property**

Conjugation property states that

$$\text{if } x(n) \overset{z}{\leftrightarrow} X(z)$$

$$\text{Then} x^{*}(n) \overset{z}{\leftrightarrow} X^{*}(z^{*})$$

$$\text{If, } x(n) \text{ is real then } X(z) = X^{*}(z^{*})$$

9) **Initial value theorem**

Initial value theorem sates that if x(n) is a causal discrete time signal with z-transform X(z), then the initial value may be determined by using expression

$$\mathbf{x(0) = \lim_{n \to 0} x(n)}$$

$$\mathbf{x(0) = \lim_{|z| \to \infty} X(z)}$$

10)**Final value theorem**

Final value theorem states that for a discrete time signal x(n), if $\mathbf{X(z)}$ and poles of X(z) are all inside the unit circle. then final value id discrete time signal,$\boldsymbol{x}(\infty)$,may be determined by:

$$\mathbf{x(\infty) = \lim_{n \to \infty} x(n) = \lim_{|z| \to 1} [(1 - z^{-1})X(z)]}$$

<u>3.3 RATIONAL Z-TRANSFORM.</u>

<u>3.3.1 POLES & ZEROS</u>

An important property of Z-transform is those for which X(z) is a rational function. Rational transfer function is a ratio of two polynomials of Z, i.e.,$\mathbf{X(z) = 0}$.The poles of a z-transform $\mathbf{X(z)}$are the valus of z for which $\mathbf{X(z)} = \infty$.

We can represent X(z) graphically by pole-zero plot in the complete z-plane. Pole is located by X and zero

by 0, Form the definition of ROC, the ROC of a z-transform should not contain any pole.

Example:

Find the pole-zero plot for the signal $x(n) = 2^{n}u(n)$

Solution:

We know that

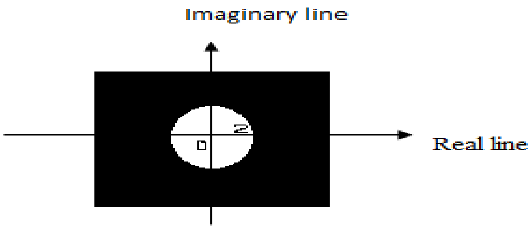
$$X(z) = Z[x(n)] = \sum_{n=-\infty}^{\infty} x(n)z^{-n}$$

$$X(z) = \sum_{n=-\infty}^{\infty} 2^{n}u(n)z^{-n} = \frac{1}{1 - 2z^{-1}}, \qquad \text{ROC: } |z| > |2|$$

Or

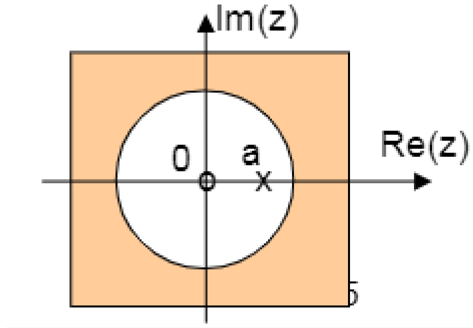
$$X(z) = \frac{z}{z - 2}$$

Thus, X(z) has one zero at $z = 0$ and one pole at $z = 2$.the pole-zero plot has been shown



3.3.2 POLE LOCATION TIME DOMAIN BEHAVIOR FOR CASUAL SIGNALS.

For the signal



$$x(n) = a^n u(n) \quad where\ a > 0$$

The Z-transform is

$$X(z) = \frac{1}{1 - az^{-1}} = \frac{z}{z - a}$$

One pole $z_1 = 0$. One pole $p_1 = a$

The signal is decaying $0 < a < 1$

The signal is fixed if $a = 1$

The signal is growing if $a > 1$

The signal alternates if a is negative

Causal signal with poles outside the unit circle becomes unbounded.

3.3.3 SYSTEM FUNCTION OF A LINEAR TIME INVARIANT SYSTEM.

As a matter of fact, the z-transform plays an important role in analysis and representation of discrete time LTI System.

The convolution property of z-transform states that

$$y(n) = h(n) * x(n) \overset{z}{\leftrightarrow} Y(z) = H(z)X(z)$$

$$Y(z) = H(z).X(z)$$

Where X(z), Y(z) and H(z) are the z-transform of system input, output and impulse response respectably H(z) is known as the transfer function of discrete time LTI systems. sometimes it is also called system function. For z evaluated at unit circle(z=$e^{j\omega}$), the transfer function H(z) reduces to the frequency response of the discrete time LTI system. we assume that the unit circle is in the ROC for H(z).

We also know that if the input to a discrete time LTI system is a complex exponential signal $x(n) = z^n$, the output of the system will be H(z).z^n. in other words, we can say that z^n is an eigen function of the system with eigen value given by H(z).

3.4 DISCUSS INVERSE Z-TRANSFORM.

The process through which x(z) is converted back to x(n) is known as Inverse Z-transform.This can be done by the following methods,

1.Partial fraction Method
2. contour Integration

3.4.1. PARTIAL FRACTION METHOD

In order to determine the inverse Z-transform of $X(z)$ using partial fraction expansion method, the denominator of $X(z)$ must be in factored form. In this method, we obtained the partial fraction expansion of $\frac{X(z)}{z}$ instead of $X(z)$. This is because the Z-transform of time-domain sequences have Z in their numerators.

The partial fraction expansion method is applied only if $\frac{X(z)}{z}$ is a proper rational function, i.e., the order of its denominator is greater than the order of its numerator.

If $\frac{X(z)}{z}$ is not a proper function, then it should be written in the form of a polynomial and a proper function before applying the partial fraction method.

The disadvantage of the partial fraction method is that, the denominator of $X(z)$ must be in factored form. Once the $\frac{X(z)}{z}$ is obtained as a proper function, then using the standard Z-transform pairs and the properties of Z-transform, the inverse Z-transform of each partial fraction can be obtained.

Let a rational function $\frac{X(z)}{z}$ given as -

$$\frac{X(z)}{z} = \frac{N(z)}{D(z)} = \frac{b_0 z^m + b_1 z^{m-1} + b_2 z^{m-2} + \cdots b_m}{z^n + a_1 z^{n-1} + a_2 z^{n-2} + \cdots a_n}$$

When the order of numerator is less than the order of denominator, i.e., $m < n$, then $\frac{X(z)}{z}$ is a proper function.

If $m \geq n$, then $\frac{X(z)}{z}$ is not a proper function, then it is to be written as -

$$\frac{X(z)}{z} = c_0 z^{n-m} + c_1 z^{n-m-1} + \cdots + c_{n-m} + \frac{N_1(z)}{D(z)}$$

Where, $[c_0 z^{n-m} + c_1 z^{n-m-1} + \cdots + c_{n-m}]$ is a polynomial and $\frac{N_1(z)}{D(z)}$ is the proper rational function.

Now, there are two cases for the proper rational function $\frac{X(z)}{z}$ as follows –

Case I - When $\frac{X(z)}{z}$ has all distinct poles -

When all the poles of $\frac{X(z)}{z}$ are distinct, then the function $\frac{X(z)}{z}$ can be expanded in the form given below -

$$\frac{X(z)}{z} = \frac{C_1}{z - K_1} + \frac{C_2}{z - K_2} + \frac{C_3}{z - K_3} + \cdots + \frac{C_n}{z - K_n}$$

Here, the coefficients $C_1, C_2, C_3, \ldots, C_n$ can be determined by using the equation given below -

$$C_i = \left[(z - K_i)\frac{X(z)}{z}\right]_{z=K_i} \; ; \text{ Where }, i = 1,2,3..$$

Case II - When $\frac{X(z)}{z}$ has I-repeated poles and the remaining $(n - I)$ poles are simple Consider p^{th} pole is repeated / times. Then, the function $\frac{X(z)}{z}$ can be expressed as,

$$\frac{X(z)}{z} = \frac{C_1}{z - K_1} + \frac{C_2}{z - K_2} + \cdots + \frac{C_{p1}}{z - K_p} + \frac{C_{p2}}{(z - k_p)^2} + \cdots + \frac{C_{pl}}{(z - k_p)^l}$$

Where

$$C_{pl} = \left[(z - K_p)\frac{lX(z)}{z}\right]_{z=K_p}$$

Also, if the Z-transform $X(z)$ has a complex pole, then the partial fraction can be expressed as -

$$\frac{X(z)}{z} = \frac{C_1}{z - K_1} + \frac{C_1^*}{z - K_1^*}$$

Where, C_1^* is the complex conjugate of C_1 and K_1^* is the complex conjugate of K_1. Therefore, it is clear that the complex poles result in complex conjugate coefficients in the partial fraction expansion.

Q. Find the inverse Z-transform of

$$X(z) = \frac{z^{-1}}{2 - 3z^{-1} + z^{-2}} ; \text{ROC} \rightarrow |z| > 1$$

Solution

Given Z-transform is,

$$X(z) = \frac{z^{-1}}{2 - 3z^{-1} + z^{-2}}$$

$$\Rightarrow X(z) = \frac{z}{2z^2 - 3z + 1} = \frac{z}{2\left[z^2 - \left(\frac{3z}{2}\right) + \left(\frac{1}{2}\right)\right]}$$

$$\Rightarrow X(z) = \frac{1}{2}\left\{\frac{z}{(z-1)\left[z - \left(\frac{1}{2}\right)\right]}\right\}$$

By taking partial fraction, we get,

$$\Rightarrow \frac{X(z)}{z} = \frac{A}{(z-1)} + \frac{B}{\left[z - \left(\frac{1}{2}\right)\right]}$$

Where, A and B are determined as follows -

$$A = \left[(z-1)\frac{X(z)}{z}\right]_{z=1}$$

$$= (z-1)\left[\frac{1}{2}\frac{z}{(z-1)\left[z - \left(\frac{1}{2}\right)\right]}\right]_{z=1}$$

$$= \frac{1}{2}\left[\frac{1}{1 - \left(\frac{1}{2}\right)}\right] = 1$$

Similarly,

$$B = \left[\left(z - \frac{1}{2}\right)\frac{X(z)}{z}\right]_{z=\frac{1}{2}} = \left(z - \frac{1}{2}\right)\left[\frac{1}{2}\frac{z}{(z-1)\left[z - \left(\frac{1}{2}\right)\right]}\right]_{z=\frac{1}{2}} = \frac{1}{2}\left[\frac{1}{\left(\frac{1}{2}\right) - 1}\right] = -1$$

$$\therefore \frac{X(z)}{z} = \frac{1}{(z-1)} - \frac{1}{\left[z - \left(\frac{1}{2}\right)\right]}$$

$$\Rightarrow X(z) = \frac{z}{(z-1)} - \frac{z}{\left[z - \left(\frac{1}{2}\right)\right]}; \text{ROC} \rightarrow |z| > 1$$

Because the region of convergence (ROC) of the given Z-transform is $|z| > 1$, thus both the sequences must be casual. Hence, by taking the inverse Z-transform, we get,

$$Z^{-1}[X(z)] = Z^{-1}\left[\frac{z}{(z-1)} - \frac{z}{\left[z - \left(\frac{1}{2}\right)\right]}\right]$$

$$\therefore x(n) = \left[u(n) - \left(\frac{1}{2}\right)^n u(n)\right]$$

3.4.2 INVERSE Z-TRANSFORM BY CONTOUR INTEGRATION

In this method, we obtain inverse z-transform x(n), by summing residues of $[X(z)z^{n-1}]$at all poles.

Mathematically, this may be express as

$$x(n) = \sum_{\text{all poles}[X(z)]} \text{residues of } [X(z)z^{n-1}]$$

Here the residue for any pole of order m

at $z = \beta$ is

$$\text{Residue} = \frac{1}{(m-n)} \lim_{z \to \beta} \{ \frac{d^{m-1}}{dz^{m-1}} [(z-\beta)^m X(z).\, z^{n-1}] \}$$

Example: Use counter method find inverse z-transform, x(n)for

$$X(z) = \frac{z}{(z-1)(z-2)}$$

X(z) has two poles of order m=1 at z=1 and z=2

We can obtain the corresponding residue as ahead:

For poles at z=1

$$\text{Residue} = \frac{1}{0} \lim_{z \to 1} \{ \frac{d^0}{dz^0} \left[(z-1)^1 . \frac{z.z^{n-1}}{(z-1)(z-2)} \right] \}$$

Residue= -1

similarly for poles at z=2

$$\text{Residue} = \frac{1}{0} \lim_{z \to 2} \{ \frac{d^0}{dz^0} \left[(z-2)^1 . \frac{z.z^{n-1}}{(z-1)(z-2)} \right] \}$$

$$= \lim_{z \to 2} [\frac{z}{z-1} . z^{n-1}]$$

$$= 2.2^{n-1} = 2^n$$

Hence, $x(n) = \{-1 + 2^n\}.\, u[n]$

POSSIBLE SHORT TYPE QUESTIONS WITH ANSWER

1.Define z-transform.

Z-Transform is used to analyze the LTI discrete time signal.

Z-Transform converts the discrete Time signal into frequency domain.

2.Define ROC.Write its two properties [2019(S-NEW)](S-24)

The region of convergence of x(z) is the set of all values of Z for which x(z) attains afinite value.

1) The ROC of the Z-transform is a ring or disc in the z-plane centred at the origin.

2)The ROC of the Z-transform cannot contain any poles.

3.Write Properties of Z-Transform.[2019(S-NEW)]

Properties are:

Differentiation in Z-domain
Parseval's Theorem
Time Shifting
Convolution Property
Initial Value Theorem
Final Value Theorem

4.What are the methods of Inverse Z-transform?(S-24)

Inverse Z-Transform can be done by the following methods,

1)Partial fraction Method
2)contour Integration

5.Define Inverse z-transform.

The process through which x(z) is converted back to x(n) is known as Inverse Z- transform.

6.What are the applications of Z-Transform?

Z-Transform converts the discrete Time signal into frequency domain.

In Z-domain, the convolution of 2 sequences is equivalent to multiplications of theircorresponding Z-Transform.

POSSIBLE LONG TYPE QUESTIONS

1.Find Inverse Z-Transform of

$$x(z) = \frac{1}{1-1.5z^{-1}+0.5z^{-2}} \qquad \begin{array}{ll} ROC & |z| > 1 \\ |z| & < 0.5 \\ 0.5 < & |z| < 1 \end{array}$$

2. State any 5 properties of z-transform.(S-24)

3.Compute poles,zeros and system response of given difference equation y(n)=2y(n-1)+3x(n).(S-24)

4.Find inverse Z-Transform of (S-24)

$$x(z) = \frac{3z}{(z-1)(z-2)} (ROC: [Z] > 2)$$

5.Find inverse Z-Transform of the sequence by using partial fraction method[2019(S-NEW)]

$$X(z) = \frac{Z^2 - 3Z + 8}{(Z-2)(Z+2)(Z+3)}$$

CHAPTER NO.-04
DISCUSS FOURIER TRANSFORM: ITS APPLICATIONS PROPERTIES

Learning Objectives:

4.1 Concept of discrete Fourier transform.
4.2 Frequency domain sampling and reconstruction of discrete time signals.
4.3 Discrete Time Fourier transformation (DTFT).
4.4 Discrete Fourier transformation (DFT).
4.5 Compute DFT as a linear transformation.
4.6 Relate DFT to other transforms. .
4.7 Property of the DFT.
4.8 Multiplication of two DFT & circular convolution.

4.1: CONCEPT OF DISCRETE FOURIER TRANSFORM(DFT): -

It is a finite duration discrete frequency sequence which is obtain by sampling one period of Fourier transform. Sampling is done at "N" equally spaced points over the period points over the period extending from $\boldsymbol{\omega = 0 \text{ to } \omega = 2\pi}$.

Mathematically:
The DFT of discrete sequence x(n) is denoted by X(k).it is given by

$$\mathbf{X(k) = \sum_{n=0}^{N-1} x(n).\, e^{-j2\pi kn/N}}$$

$$\mathbf{Here\ k = 0, 1, 2, \dots N-1}$$

It is called N-point DFT

We can obtain discrete sequence x(n) from its DFT.it is called inverse discrete Fourier transform (IDFT).it is given by

$$\mathbf{x(n) = \frac{1}{N} \sum_{n=0}^{N-1} x(k).\, e^{j2\pi kn/N}}$$

$$\mathbf{Here,\ n = 0, 1, 2 \dots N-1}$$

It is called N-point IDFT.

4.2: DETERMINE FREQUENCY DOMAIN SAMPLING AND RECONSTRUCTION OF DISCRETE TIME SIGNALS: -

Continuous time signal Fourier transform, discrete time Fourier Transform can be used to represent a discrete sequence into its equivalent frequency domain representation and LTI discrete time system and develop various computational algorithms.

X(jω) in continuous Fourier transform, is a continuous function of x(n). However, DFT deals with representing x(n) with samples of its spectrum X(ω). Hence, this mathematical tool carries much importance computationally in convenient representation. Both, periodic and non-periodic sequences can be processed through this tool. The periodic sequences need to be sampled by extending the period to infinity.
From the introduction, it is clear that we need to know how to proceed through frequency domain sampling i.e. sampling $X(\omega)$. Hence, the relationship between sampled Fourier transform and DFT is established in the following manner.

Similarly, periodic sequences can fit to this tool by extending the period N to infinity.

Let a Non periodic sequence be,

$$X(n) = \lim_{n\to\infty} x_N(n)$$

Defining its Fourier transform,

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n} Xk(\delta\omega) \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Here, $X(\omega)$ is sampled periodically, at every $\delta\omega$ radian interval.

As $X(\omega)$ is periodic in 2π radians, we require samples only in fundamental range. The samples are taken after equidistant intervals in the frequency range $0 \le \omega \le 2\pi$. Spacing between equivalent intervals is $\delta\omega = \frac{2\pi}{N}k$ radian.

Now evaluating, $\omega = \frac{2\pi}{N}K$

$$X(\frac{2\pi}{N}K) = \sum_{n\to-\infty}^{\infty} x(n)e^{-\frac{j2\pi nk}{N}} \ldots\ldots\ldots\ldots.. (2)$$

where $k = 0,1,\ldots\ldots N-1$

After subdividing the above, and interchanging the order of summation

$$X\left(\frac{2\pi}{N}k\right) = \sum_{n=0}^{N-1}\left[\sum_{l=-\infty}^{\infty} x(n-Nl)\right]e^{-j2\pi nk/N} \ldots\ldots\ldots\ldots\ldots (3)$$

$$\sum_{l=-\infty}^{\infty} x(n-Nl) = x_p(n)$$

The above is a periodic function of period N and its fourier series

$$= \sum_{k=0}^{n-1} C_k e^{j2\pi nk/N}$$

where, $n = 0,1,\ldots,N-1$; p stands for periodic entity or function. The Fourier coefficients are,

$$C_k = \frac{1}{N}\sum_{n=0}^{N-1} x_p(n)e^{-\frac{j2\pi nk}{N}} \quad k = 0,1,\ldots,N-1 \ \ldots\ldots\ldots\ldots\ldots (4)$$

Comparing equations 3 and 4 , we get;

$$NC_k = X\left(\frac{2\pi}{N}k\right) \text{ where } k = 0,1,\ldots,N-1 \quad \ldots. \quad (5)$$

$$NC_k = X\left(\frac{2\pi}{N}k\right) = X(e^{jw}) = \sum_{n=-\infty}^{\infty} x_p(n)e^{-\frac{j2\pi nk}{N}} \ \ldots\ (6)$$

From Fourier series expansion,

$$x_p(n) = \frac{1}{N}\sum_{k=0}^{N-1} NC_k e^{\frac{j2\pi nk}{N}} = \frac{1}{N}\sum_{k=0}^{N-1} X\left(\frac{2\pi}{N}k\right)e^{\frac{j2\pi nk}{N}} \ \ldots. (7)$$

Where $n = 0,1,\ldots,N-1$

Here, we got the periodic signal from $X\omega$. $x(n)$ can be extracted from $x_p(n)$ only, if

there is no aliasing in the time domain. $N \geq L$
N = period of $x_p(n)$ L = period of $x(n)$

$$x(n) = \begin{cases} x_p(n), & 0 \leq n \leq N-1 \\ 0, & \text{Otherwise} \end{cases}$$

4.3 DISCRETE TIME FOURIER TRANSFORMATION(DTFT): -

The strong similarity between the Fourier analysis and synthesis equation in continuous time, there is a duality between the time domain and frequency domain. However for the discrete time Fourier transform (DTFT) of a-periodic signals, no similar duality exists, since a-periodic signals and their Fourier transforms are very different kinds of functions. A-periodic discrete time signal is, of course, a-periodic sequence, while their Fourier transformation are always periodic functions of a continuous frequency variable.

DTFT is discrete time Fourier transform and is given by:

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$$

The range of ω is form ($-\pi$ to π) or (0 to 2π)

4.4 STATE & EXPLAIN DISCRETE FOURIER TRANSFORMATION(DFT):

The discrete Fourier transform (DFT) is a fundamental transform in digital signal processing, with applications in frequency analysis, fast convolution, image processing, etc. Moreover, fast algorithms exist that make it possible to compute the DFT very efficiently.

The k^{th} DFT coefficient of a length N signal x(n) is defined as

$$X^d(k) = \sum_{n=0}^{N-1} x(n)W_N^{-kn},\ k = 0, \ldots, N-1$$

where

$$W_N = e^{j2\pi/N} = \cos\left(\frac{2\pi}{N}\right) + j\sin\left(\frac{2\pi}{N}\right)$$

It is the principal N-th root of unity. Because W_N^{nk} as a function of k has a period of N, the DFT coefficients $X^d(k)$ are periodic with period N when k is taken outside the range $k = 0,1 \ldots, N-1$. The original sequence $x(n)$ can be retrieved by the inverse discrete Fourier transform (IDFT)-

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X^d(k)W_N^{kn},\ n = 0, \ldots, N-1$$

The inverse DFT can be verified by using a simple observation regarding the principal N-th root of unity W_N.

Namely,

$$\sum_{n=0}^{N-1} W_N^{nk} = N \cdot \delta(k),\ k = 0, \ldots, N-1$$

where $\delta(k)$ is the Kronecker delta function. For example, with $N = 1,2 \ldots 5$ and $k = 0$, the sum gives

$$1 + 1 + 1 + 1 + 1 = 5$$

For $k = 1$, the sum gives

$$1 + W_5 + W_5^2 + W_5^3 + W_5^4 = 0$$

The sums can also be visualized by looking at the illustration of the DFT matrix in . Because W_N^{nk} as a function of k is periodic with period N, we can write

$$\sum_{n=0}^{N-1} W_N^{nk} = N \cdot \delta(\langle k \rangle_N)$$

where k_n denotes the remainder when k is divided by N, i.e., $\langle k \rangle_N$ is k modulo N.
To verify the inversion formula, we can substitute the DFT into the expression for the IDFT:

$$\begin{aligned} x(n) &= \frac{1}{N}\sum_{k=0}^{N-1}\left(\sum_{l=0}^{N-1} x(l)W_N^{-kl}\right)W_N^{kn}, \\ &= \frac{1}{N}\sum_{l=0}^{N-1} x(l)\sum_{k=0}^{N-1} W_N^{k(n-l)}, \\ &= \frac{1}{N}\sum_{l=0}^{N-1} x(l)N\delta(\langle n-l \rangle_N), \\ &= x(n). \end{aligned}$$

4.5 COMPUTE DFT AS A LINEAR TRANSFERMATION: -

Let us understand Linear Transformation -
We know that,

$$DFT(k) = DFT[x(n)] = X\left(\frac{2\pi}{N}k\right) = \sum_{n=0}^{N-1} x(n).W_n^{-nk}$$

$$k = 0,1,\dots,N-1$$

$$x(n) = IDFT[X(k)] = \frac{1}{N}\sum_{k=0}^{N-1} X(k).W_N^{-nk};\ n = 0,1,\dots,N-1$$

Computation of DFT can be performed with N^2 complex multiplication and $N-1$ complex addition.

$$x_N = \begin{bmatrix} x(0) \\ x(1) \\ \cdot \\ . \\ x(N-1) \end{bmatrix} \text{ N point vector of signal } x_N$$

$$X_N = \begin{bmatrix} X(0) \\ X(1) \\ \cdot \\ . \\ X(N-1) \end{bmatrix} \text{ N point vector of signal } X_N$$

$$= \begin{bmatrix} 1 & 1 & 1 & \cdots & \cdots & 1 \\ 1 & W_N & W_N^2 & \cdots & \cdots & W_N^{N-1} \\ \cdot & W_N^2 & W_N^4 & \cdots & \cdots & W_N^{2(N-1)} \\ . & & & & & \\ 1 & W_N^{N-1} & W_N^{2(N-1)} & \cdots & \dots & W_N^{(N-1)(N-1)} \end{bmatrix}$$

N - point DFT in matrix term is given by -

$$X_N = W_N x_N$$

$W_N \longmapsto$ Matrix of linear transformation

$$\text{Now, } x_N = W_N^{-1} X_N$$

IDFT in Matrix form is given by

$$x_N = \frac{1}{N} W_N^* X_N$$

Comparing both the expressions of x_N,

$$W_N^{-1} = \frac{1}{N} W_N^*$$

and

$$W_N \times W_N^* = N[I]_{N \times N}$$

Therefore, W_N is a linear transformation matrix, an orthogonal unitary matrix.
From periodic property of W_N and from its symmetric property, it can be concluded that,

$$W_N^{k+N/2} = -W_N^k$$

4.6 RELATE DFT TO OTHER TRANSFORMS: -

(1) Relationship to the Fourier series co-efficient of a continuous time signal:

Suppose that $x_a(t)$ is a continuous-time periodic signal with fundamental period $T_p = 1/F_0$.
The signal can be expressed in a Fourier Series as under:

$$x_a(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi k t F_0}$$

where, (c_k) are the Fourier co-efficients. If we sample $x_a(t)$ at a uniform rate

$$F_s = \frac{N}{T_p} 1/T$$

we obtain the discrete-time sequence

$$x(n) = x_a(nT) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi k F_0 nT} = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi kn/N}$$

$$= \sum_{k=0}^{N-1} \left[\sum_{l=-\infty}^{\infty} c_{k-lN} \right] e^{j2\pi kn/N}$$

By using IDFT formula, where

$$X(k) = N \sum_{1=-\infty}^{\infty} c_{k-IN} N\bar{c}_k$$

$$\text{and } \bar{c}_k = \sum_{1=-\infty}^{\infty} c_{k-1\,N}$$

Hence, the $(\tilde{c}_k)$ sequence is an realised version of the sequence (c_k).

(2) Relationship Of DFT to the Fourier series co-efficient of A-periodic sequence to the Fourier series co-efficient of a continuous time signal:

Although the Fourier transform of a periodic sequence does not converge in then normal sense, the introduction of impulses permits un to include periodic sequences formally within the framework of Fourier transform analysis. We know that the Fourier series of a periodic sequence $x_p(n)$ with fundamental period N is expressed as:

$$x_p(n) = \sum_{k=0}^{N-1} c_k e^{i2mk/N}, -\infty =< n < \infty$$

where the Fourier series coefficients are expressed by:

$$c_k = \frac{1}{N}\sum_{n=0}^{N-1} x_p(n) - e^{-j2\pi nk/N}, k = 0,1 \ldots N-1$$

Now, by comparing the above two equations with that of DFT pair and defining a sequence which is identical to $x_p(n)$ over a single period, we get

$$X(k) = N \cdot c_k$$

If a periodic sequence $x_p(n)$ is formed by periodically repeating $x(n)$ every N samples i.e.

$$x_p(n) = \sum_{N=-\infty}^{\infty} x(n - 1N)$$

The discrete-frequency domain representation is expressed as:

$$X(k) = \sum_{n=0}^{N-1} x_p(n) e^{-\frac{j2nk}{N}} = Nc_k, k = 1,2 \ldots N-1$$

And the IDFT is $x_p(n)$

$$= \frac{1}{N}\sum_{k=0}^{N-1} X(k) e^{j2\pi nK/N}, -\infty < n < \infty$$

(4) Relationship Of DFT to the Z-transform:

Let X(z) be the z-transform for a sequence x(n) which is expressed as:

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

With a ROC which includes the unit circle. If X(z) is sampled at the N equally spaced points on the unit circle,

$$X(k) = X(z) \ [\text{at } z = e^{\frac{j2\pi k}{N}}, k = 0,1, \ldots . N-1]$$

$$X(k) = \sum_{n=-\infty}^{\infty} x(n) e^{-2\pi nk/N}$$

Now, it may be noted that this is identical to the Fourier transform $X(e^{jm})$ evaluated at the X equally spaced frequencies i.e.,

$$\omega_k = 2\pi k/N, k = 0,1, \ldots . N-1$$

If the sequence $x(n)$ has a finite duration of length N, then the z-transform is given as:

$$X(z) = \sum_{n=0}^{N-1} x(n) \cdot z^{-n}$$

Now, substituting the IDFT relationship for $x(n)$, we obtain

$$X(z) = \sum_{n=0}^{N-1} \left[\frac{1}{N}\sum_{k=0}^{N-1} X(k)e^{j2\pi nk/N}\right] \cdot z^{-n} = \frac{1}{N}\sum_{k=0}^{N-1} X(k) \sum_{n=0}^{N-1} \left(e^{j2\pi k/N}z^{-1}\right)^{n}$$

$$X(z) = \frac{1}{N}\sum_{k=0}^{N-1} X(k) \cdot \frac{1-z^{-N}}{1-e^{j2\pi k/N}z^{-1}} = \frac{1-z^{-N}}{N} \cdot \sum_{k=0}^{N-1} \frac{X(k)}{1-e^{j2\pi k/N} \cdot z^{-1}}$$

This equation is identical to that of frequency sampling form.
Now, when this is evaluated over a unit circle, then we write

$$X\left(e^{j\omega}\right) = \frac{1-e^{-j\omega N}}{N}\sum_{k=0}^{N-1} \frac{X(k)}{1-e^{j(\omega-2\pi k/N)}}$$

4.7 DISCUSS THE PROPERTY OF THE DFT:

1. Linearity

It states that the DFT of a combination of signals is equal to the sum of DFT of individual signals. Let us take two signals $x_1(n)$ and $x_2(n)$, whose DFT s are $X_1(\omega)$ and $X_2(\omega)$ respectively. So, if

$$x_1(n) \rightarrow X_1(\omega)$$

and

$$x_2(n) \rightarrow X2(\omega)$$

Then $ax_1(n) + bx_2(n) \rightarrow ax_1(\omega) + bx_2\ (\omega)ax_1(n) + bx_2(n) \rightarrow ax_1(\omega) + bx_2(\omega)$ where **a** and **b** are constants.

2.Symmetry

The symmetry properties of DFT can be derived in a similar way as we derived DTFT symmetry properties. We know that DFT of sequence xn is denoted by $X(K)$. Now, if x(n) and X(K) are complex valued sequence, then it can be represented as under

$$x(n) = x_R(n) + jx_1(n), 0 \leq n \leq N-1$$

And

$$X(K) = X_R(K) + jX_1(K), 0 \leq K \leq N-1$$

3. Duality Property

Let us consider a signal xn, whose DFT is given as XK. Let the finite duration sequence be $X(N)$. Then according to duality theorem,

If,

$$x(n) \leftrightarrow X(K)$$

Then,

$$X(N) \leftrightarrow Nx[((-k))_N]$$

So, by using this theorem if we know DFT, we can easily find the finite duration sequence.

4. Complex Conjugate Properties

Suppose, there is a signal xn, whose DFT is also known to us as XK. Now, if the complex conjugate of the signal is given as x^*n, then we can easily find the DFT without doing much calculation by using the theorem shown below.

If,

$$x(n) \leftrightarrow X(K)$$

Then,

$$x*(n) \leftrightarrow X*((K))_N = X*(N-K)$$

5.Circular Frequency Shift

The multiplication of the sequence xn with the complex exponential sequence $e^{\frac{j2\Pi kn}{N}}$ is equivalent to the circular shift of the DFT by L units in frequency. This is the dual to the circular time shifting property.

If,

$$x(n) \leftrightarrow X(K)$$

Then,

$$x(n)e^{\frac{j2\Pi Kn}{N}} \leftrightarrow X((K-L))_N$$

If there are two signals $x_1(n)$ and $x_2(n)$ and their respective DFTs are $X_1(k)$ and $X_2(K)$,then multiplication of signals in time sequence corresponds to circular convolution of their DFTs.

If,

$$x_1(n) \leftrightarrow X_1(K) \ \& \ x_2(n) \leftrightarrow X_2(K)$$

Then,

$$x_1(n) \times x_2(n) \leftrightarrow X_1(K) \odot X_2(K)$$

6.Parseval's Theorem

For complex valued sequences $x(n)$ and $y(n)$, in general

If,

$$x(n) \leftrightarrow X(K) \ \& \ y(n) \leftrightarrow Y(K)$$

Then

$$\sum_{n=0}^{N-1} x(n).y^*(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)y^*(k)$$

4.8 MULTIPLICATION OF TWO DFT & CIRCULAR CONVOLUTION:

This property states that the multiplication of two DFT is equivalent to the circular convolution of their sequences in time domain.

Mathematically, we have

If

$$x_1(n) \overset{DFT}{\longleftrightarrow} X_1(k)$$

and

$$x_2(n) \overset{DFT}{\longleftrightarrow} X_2(k) \text{ then,}$$

$$x_1(n) \odot x_2(n) \overset{DFT}{\longleftrightarrow} X_1(k) \cdot X_2(k)$$

Here, $\odot$ indicates circular convolution.

Let the result of circular convolution of $x_1(n)$ and $x_2(n)$ be $y(m)$ then the circular convolution can also be expressed as:

$$y(m) = \sum_{n=0}^{N-1} x_1(n)x_2((m-n))_N, \ m = 0,1,\dots N-1$$

Here, the term $x_2((m-n))_N$ indicates the circular convolution.

Proof:

Let us consider two discrete time sequences $x_1(n)$ and $x_2(n)$.
The DFT of $x_1(n)$ can be expressed as under:

$$X_1(k) = \sum_{n=0}^{N-1} x_1(n) e^{\frac{-j2\pi kn}{N}}, k = 0,1, \ldots N-1 \ldots \ldots (i)$$

To avoid the confusion, let us write the DFT of $x_2(n)$ using different index of summation i.e.

$$X_2(k) = \sum_{1=0}^{N-1} x_2(1) e^{\frac{-j2\pi kl}{N}}, k = 0,1, \ldots N-1 \ldots \ldots .. (ii)$$

It may be noted that in equation (ii), instead of n we have used 1 .

Let us denote the multiplication of two DFTs $X_1(k)$ and $X_2(k)$ by $Y(k)$. Therefore,

$$Y(k) = X_1(k) \cdot X_2(k) \ldots \ldots \ldots .. (iii)$$

Let IDFT of $Y(k)$ be $y(m)$. Then using definition of IDFT, we have

$$y(m) = \frac{1}{N} \sum_{k=0}^{N-I} Y(k) e^{\frac{j2\pi km}{N}} \ldots \ldots \ldots . (iv)$$

Substituting equation (iii) in equation (iv), we obtain

$$y(m) = \frac{1}{N} \sum_{k=0}^{N-1} X_1(k) \cdot X_2(k) e^{\frac{j2\pi km}{N}} \ldots \ldots .. (v)$$

Substituting the values of $X_1(k)$ and $X_2(k)$ from equations (i) and (ii) in equation (v), we obtain

$$y(m) = \frac{1}{N} \sum_{k=0}^{N-1} \left[\sum_{n=0}^{N-1} x_1(n) e^{-\frac{j2\pi kn}{N}} \right] \left[\sum_{1=0}^{N-1} x_2(1) e^{-\frac{j2\pi k1}{N}} \right] e^{\frac{i2\pi km}{N}} \ldots\ldots\ldots\ldots..(vi)$$

Rearranging the summations and terms in equation (vi), we get

$$y(m) = \frac{1}{N} \sum_{n=0}^{N-1} x_1(n) \sum_{1=0}^{N-1} x_2(1) \left[\sum_{k=0}^{N-1} e^{\frac{-j2\pi kl}{N}} \cdot e^{\frac{-j2nkn}{N}} . e^{\frac{12nkn}{N}} \right]$$

Therefore, we have

$$y(m) = \frac{1}{N} \sum_{n=0}^{N-1} x_1(n) \sum_{l=0}^{N-1} x_2(1) \left[\sum_{k=0}^{N-1} e^{+\frac{j2\pi k(m-n-1)}{N}} \right] \ldots \ldots . (vii)$$

Let us consider the last term of equation (vii). It may be written as under:

$$e^{j2\pi k(m-n-1)/N} = \left[e^{j2\pi(m-n-1)/N} \right]^k \ldots \ldots \ldots (viii)$$

Now, let us use the following standard summation expression:

$$\sum_{k=0}^{N-1} a^k = \begin{cases} N & \text{for } a = 1 \\ \dfrac{1-a^N}{1-a} & \text{for } a \neq 1 \end{cases} \ldots \ldots \ldots \ldots . (ix)$$

Let, here,

$$a = e^{+j2\pi\frac{(m-n-1)}{N}} \ldots\ldots\ldots\ldots\ldots(x)$$

Now, according to equation (ix), we shall consider two cases:

Case (i): When a = 1

If $(m-n-1)$ is multiple of N which means,
$(m-n-1) = N, 2N, 3N, \ldots$ then equation (x) becomes,
$a = e^{+j2\pi} = e^{+j2\pi(2)} = e^{+j2\pi(3)\ldots} = 1$
Thus, when $(m-n-1)$ is multiple of N (this means that $a = 1$), then according to equation(viii), the third summation in equation (vii) becomes equal to N.

Case (ii): When a ≠ 1

If $a \neq 1$, this means that if $m-n-1$ is not multiple of N then according to equation (ix). we have

$$\sum_{k=0}^{N-1} a^k = \frac{1-a^N}{1-a} \ldots\ldots\ldots\ldots(xi)$$

Substituting equation (x) in equation (xi), we obtain

$$\sum_{k=0}^{N-1} \left[e^{+j2\pi(m-n-1)}\right]^k = \frac{1-e^{+j2\pi(m-n-1)}}{1-e^{\frac{+j2\pi(m-n-1)}{N}}} \ldots\ldots\ldots\ldots\ldots(xii)$$

Here, m, n and l are integers.
Hence, $e^{+j2\pi(m-n-1)} = 1$ always. Therefore, R.H.S. of equation (xii) becomes zero when $a \neq 1$. Therefore to get the result of equation(ix), we have to consider the condition $a = 1$. This means that when $m-n-1$ is multiple of N. For this condition, we have the result of summation equals to N. Thus, equation (vii) becomes,

$$y(m) \quad = \frac{1}{N}\sum_{n=0}^{N-1} x_1(n) \sum_{1=0}^{N-1} x_2(1).N$$

$$\text{Therefore, we have, } y(m) \quad = \sum_{n=0}^{N-1} x_1(n) \sum_{1=0}^{N-1} x_2(1) \ldots\ldots\ldots(xiii)$$

We have obtained equation (xii) for the condition (m-n-1) is multiple of N. This condition can be expressed as:

$$m-n-1 = -pN \ldots\ldots\ldots(xiv)$$

Here, p is an integer and an integer can be positive or negatuve. For simplicity, we have considered negative integer. Now from equation (xiv), we obtain

$$1 = m-n+pN \ldots\ldots\ldots(xv)$$

Substituting this value in equation (xiii), we get,

$$y(m) = \sum_{n=0}^{N-1} x_1(n) \cdot x_2(\, m-n+pN) \ldots\ldots\ldots(xvi)$$

Here, we have not considered the second summation of equation (xiii). Because this summation is in terms of 1 and exponential term is absent in equation (xvi).

Now, the term $x_2(m - n + pN)$ indicates a periodic sequence with period N. This is because p is an integer. This term also indicates that the periodic sequence is delayed by n samples. Further, we know that if a sequence is periodic and delayed then it can be expressed as:

$$x_2(m - n + pN) = x_2((m - n))_N \dots\dots (xvii)$$

Here, the R.H. S term indicates circular shifting of $x_2(n)$.substituting this value in equation (xvii), we get

$$y(m) = \sum_{n=0}^{N-1} x_1(n) + x_2((m - n))_N m = 0.1. \dots N - 1 \dots\dots (xix)$$

Methods of Circular Convolution

Generally, there are two methods, which are adopted to perform circular convolution and they are −

- Concentric circle method,
- Matrix multiplication method.

Concentric Circle Method

Let $x_1(n)$ and $x_2(n)$be two given sequences. The steps followed for circular convolutionof $x_1(n)$and $x_2(n)$are

Take two concentric circles. Plot N samples of $x_1(n)$on the circumference of the outer circle maintaining equal distance successive points in anti-clockwise direction.

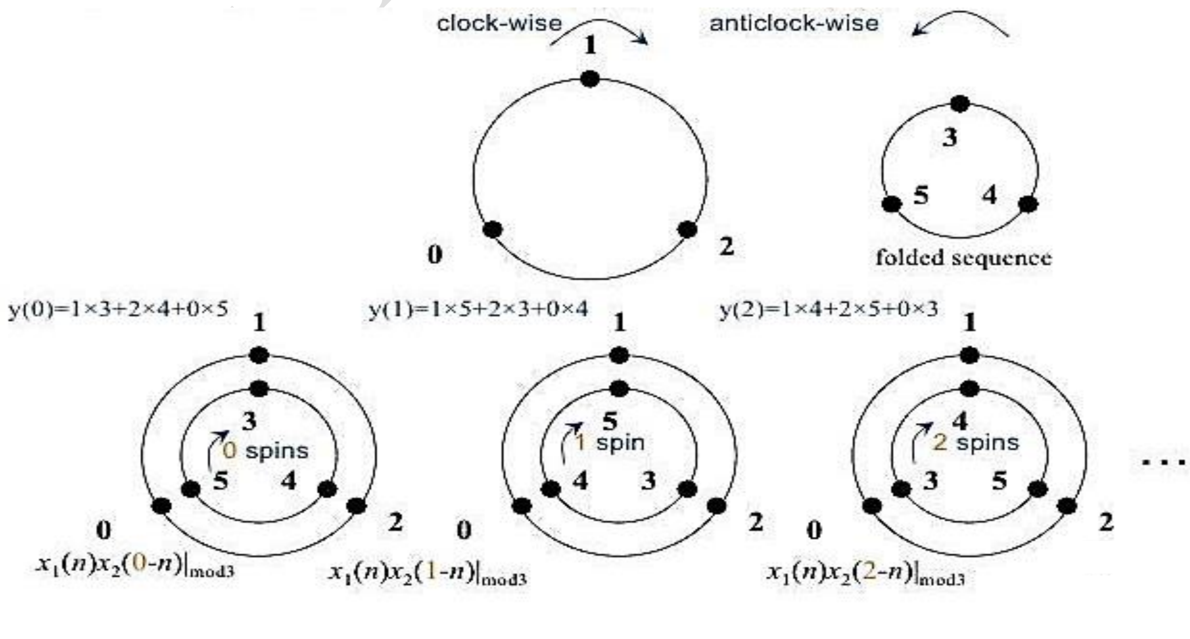
For plotting $x_2(n)$, plot N samples of $x_2(n)$ in clockwise direction on the inner circle, starting sample placed at the same point as 0^{th} sample of $x_1(n)$.

Multiply corresponding samples on the two circles and add them to get output.

Rotate the inner circle anti-clockwise with one sample at a time.

Example: Find circular convolution of given sequence

$$x_1 = \{1,2,0\} \qquad x_2 = \{3,5,4\}$$



y(n)={y(0),y(1),y(2)}
y(n)={11,11,14}

Matrix multiplication method

Matrix method represents the two-given sequence $x_1(n)$and $x_2(n)$in matrix form.

- One of the given sequences is repeated via circular shift of one sample at a time to form a N X N matrix.
- The other sequence is represented as column matrix.
- The multiplication of two matrices gives the result of circular convolution.
- In matrix method one sequence is repeated via circular shifting of samples.it represented as:

$$y(m) = x(n) \odot h(n)$$

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ \vdots \\ y(N-2) \\ y(N-1) \end{bmatrix} = \begin{bmatrix} h(0) & h(N-1) & h(N-2) & \cdots & h(2) & h(1) & x(0) \\ h(1) & h(0) & h(N-1) & \cdots & h(3) & h(2) & x(1) \\ h(2) & h(1) & h(0) & \cdots & h(4) & h(3) & x(2) \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ h(N-2) & h(N-3) & h(N-4) & \cdots & h(0) & h(N-1) & x(N-2) \\ h(N-1) & h(N-2) & h(N-3) & \cdots & h(1) & h(0) & x(N-1) \end{bmatrix}$$

Example: Find circular convolution of given sequence by matrix method

$$x_1 = \{1,2,0\} \qquad x_2 = \{3,5,4\}$$

Representing $x_2(n)$ as N×N matrix from and $x_1(n)$ as column matrix and multiplying we have

$$\begin{vmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{vmatrix} \begin{vmatrix} 3 \\ 5 \\ 4 \end{vmatrix} = \begin{vmatrix} 11 \\ 11 \\ 14 \end{vmatrix}$$

y(n)={11,11,14}

POSSIBLE SHORT TYPE QUESTIONS WITH ANSWER

1.**Write different properties of DFT?[2019(s-new)]**

ANS: Different properties of DFT are
Linearity
Symmetry
Duality
Complex conjugate
Circular frequency shift

2.Define Linearity

ANS.
It states that the DFT of a combination of signals is equal to the sum of DFT of individual signals.
Let us take two signals $x_1(n)$ and $x_2(n)$, whose DFT s are $X_1(\omega)$ and $X_2(\omega)$ respectively. So, if

$$x_1(n) \to X_1(\omega)$$
and
$$x_2(n) \to X2(\omega)$$

Then $ax_1(n) + bx_2(n) \to ax_1(\omega) + bx_2(\omega) ax_1(n) + bx_2(n) \to ax_1(\omega) + bx_2(\omega)$
where **a** and **b** are constants.

3.Define periodicity?

ANS: If $X(K)$ is an N-point DFT at $X(n)$, then $X(n+N) = X(n)$ for all value of n

$X(K+N) = X(K)$ For all value of K.

4.Define Parseval's Theorem?

ANS : For complex valued sequences xn and yn,

In general If, $x(n) \leftrightarrow X(K)$ & $y(n) \leftrightarrow Y(K)$

$$\sum_{n=0}^{N-1} X(n).y^{*}(n)$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} X(k).y^{*}(k)$$

5.**What is DFT?[2019(S-NEW)]**

ANS:

It is a finite duration discrete frequency sequence which is obtain by sampling one period of Fourier transform. Sampling is done at "N" equally spaced points over the period points over the period extending from $\boldsymbol{\omega = 0}$ **to** $\boldsymbol{\omega = 2\pi}$.

Mathematically:

The DFT of discrete sequence x(n) is denoted by X(k).it is given by

$$\mathbf{X(k) = \sum_{n=0}^{N-1} x(n).e^{-j2\pi kn/N}}$$

$$\mathbf{Here\ k = 0, 1, 2, \ldots N - 1}$$

6.**Name the number of complex multiplication and addition required to compute an N-point DFT?**

Basically, its number of multiplication and addition based on their proportion of N-point DFT. such as periodicity, linearity, circular convolution and multiplication of two sequence etc.

6.**What are the application of circular convolution?(S-24)**

1) Circular convolution can be used to implement digital filters, especially when the filter is periodic or when utilizing the FFT for fast computation.

2) In communication systems, circular convolution can be used to detect errors in data that is treated as periodic.

POSSIBLE LONG TYPE QUESTIONS

1. **Write down any 5 properties of DFT.(S-24)**
2. **Find the 4-point DFT of the sequence? [2019(S-NEW)]**

$$\mathbf{x(n) = \{2, 0, 1, 0\}}$$

$$\mathbf{x(n) = \{1, 0, 2, 1\}} \quad (\boldsymbol{S - 24})$$

3. **Difference between linear and circular convolution.(S-24)**
4. **Find the 4-point IDFT of the sequence? (S-24)**

$$\mathbf{x(k) = \{1, 0, 0, 1\}}$$

CHAPTER NO.-05
FAST FOURIER TRANSFORM ALGORITHM & DIGITAL FILTERS

LEARNING OBJECTIVES:

5.1 Compute DFT & FFT algorithm.
5.2 Direct computation of DFT.
5.3 Divide & Conquer approach to computation of DFT.
5.4 Radix-2 algorithm (Small Problems).
5.5 Application FFT algorithms.
5.6 Introduction to digital filters. (FIR filter) & General consideration.
5.7 Introduction to DSP architecture. familiarization of different Types of processors.

5.1 COMPUTE DFT & FFT ALGORITHM.

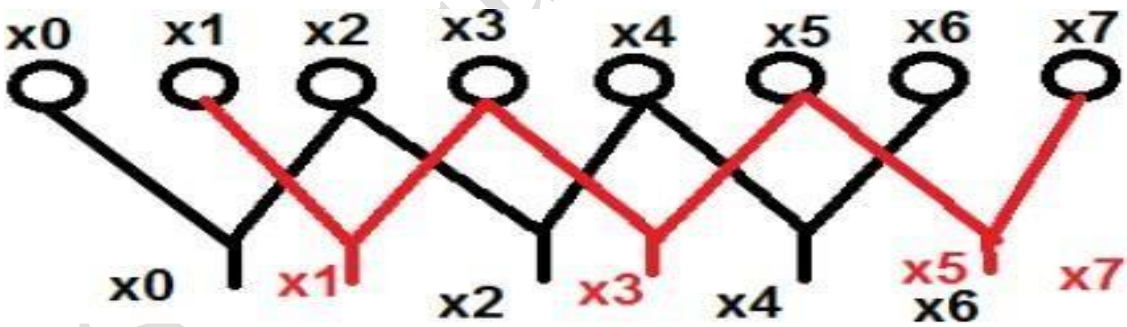
- In earlier DFT methods, we have seen that the computational part is too long.
- This can be reduced through FFT or fast Fourier transform.
- So, we can say FFT is nothing but computation of discrete Fourier transform in analgorithmic format, where the computational part will be reduced.

The main advantage of having FFT is that through it, we can design the FIR filters.
Mathematically, the FFT can be written as follows;

$$X(k) = \sum_{n=0}^{N-1} x(n).w_N^{kn} \qquad 0 \le k \le N-1$$

$$\text{Where } w_N = e^{-j2\pi/N}$$

Let us take an example to understand it better. We have considered eight points named from x_0 to x_7. We will choose the even terms in one group and the odd terms in the other.Diagrammatic view of the above said has been shown below –



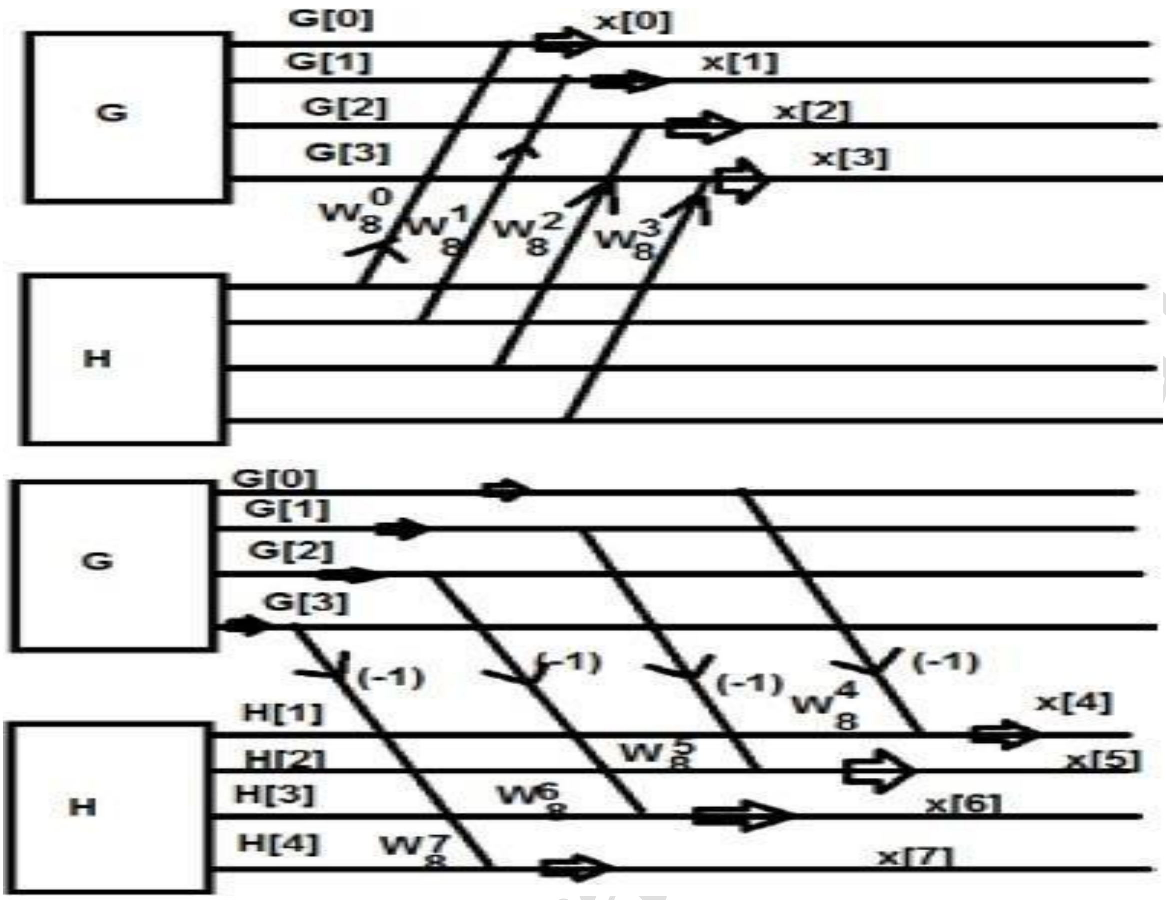
Here, points x_0, x_2, x_4 and x_6 have been grouped into one category and similarly, points x_1, x_3, $x_5$and x_7 has been put into another category. Now, we can further make them in a group of two and can proceed with the computation. Now, let us see how these breaking into further two are helping in computation.

$$x[k] = \sum_{r=0}^{\frac{N}{2}-1} x[2r]W_N^{(2r)k} + \sum_{r=0}^{\frac{N}{2}-1} x[2r+1]W_N^{(2r+1)k}$$

$$= \sum_{r=0}^{\frac{N}{2}-1} x[2r]W_{\frac{N}{2}}^{(2r)k} + \sum_{r=0}^{\frac{N}{2}-1} x[2r+1]W_{\frac{N}{2}}^{(2r+1)k} \times W_N^k$$

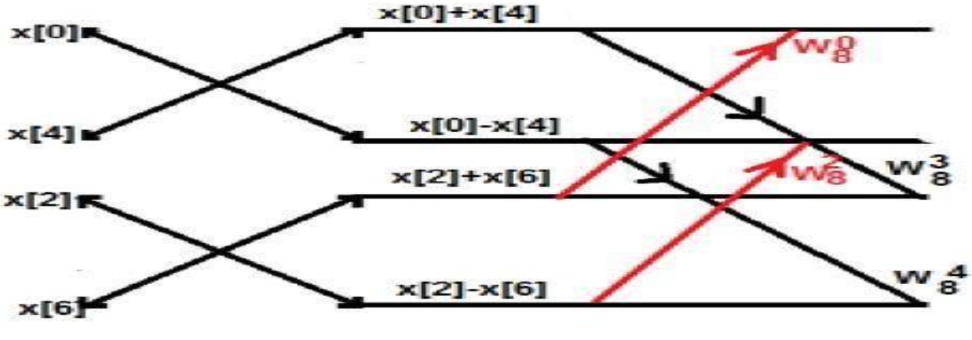
$$= G[k] + H[k] \times W_N^k$$

Initially, we took an eight-point sequence, but later we broke that one into two parts G[k] and H[k]. G[k] stands for the even part whereas H[k] stands for the odd part. If we want to realize it through a diagram, then it can be shown as below –

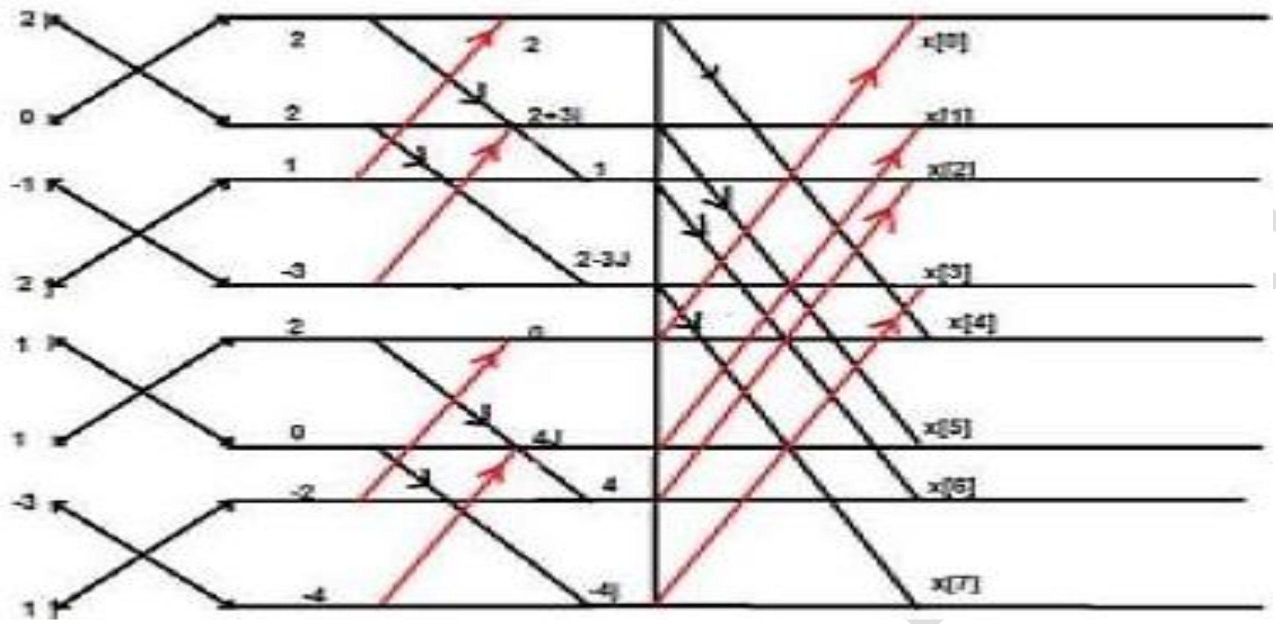


From the above figure, we can see that
W48=-W08
W58=-W18
W68=-W28
W78=-W38
Similarly, the final values can be written as follows −
$G[0] - W08H[1] = x[4]$
$G[1] - W18H[2] = x[5]$
$G[2] - W28H[3] = x[6]$
$G[1] - W38H[4] = x[7]$

The above one is a periodic series. The disadvantage of this system is that K cannot be broken beyond 4 point. Now Let us break down the above into further. We will get the structures something like this.



Example:
Consider the sequence x[n] = {2,1, −1, −3,0,1,2,1}. Calculate the FFT.
Solution - The given sequence is x[n] = {2,1, −1, −3,0,1,2,1}
Arrange the terms as shown below



FFT Algorithms can be of two types, such as:

1. Decimation in Time Sequence
2. Decimation in Frequency Sequence

1.Decimation in Time Sequence

This algorithm is also known as Radix-2 DIT FFT algorithm which means the number of output points N can be expressed as a power of 2 , that is, $N = 2^M$, where M is an integer.
Let $x(n)$ is an N-point sequence, where N is assumed to be a power of 2 . Decimate or break this sequence into two sequences of length $\frac{N}{2}$, where one sequence consisting of the even-indexed values of $x(n)$ and the other of odd-indexed values of $x(n)$.
That is

$$x_e(n) = x(2n) \quad n = 0,1, \dots, \frac{N}{2} - 1$$

$$\boldsymbol{x_o(n) = x(2n+1)\ n = 0,1, \dots, \frac{N}{2} - 1 \dots \dots (1)}$$

The N-point DFT of $x(n)$ can be written as

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{nk}\ k = 0,1, \dots, N-1 \dots \dots .(2)$$

Separating $x(n)$ into even and odd indexed values of $x(n)$, we obtain

$$X(k) \quad = \sum_{\substack{n=0 \\ (\text{ cven})}}^{N-1} x(n) W_N^{nk} + \sum_{n=0}^{N-1} x(n) W_N^{nk}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x(2n) W_N^{2nk} + \sum_{n=0}^{\frac{N}{2}-1} x(2n+1) W_N^{(2n+1)k} \dots \dots .(3)$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x(2n) W_N^{2nk} + W_N^k \sum_{n=0}^{\frac{N}{2}-1} x(2n+1) W_N^{2nk}$$

Substituting Eq.(1) in Eq. (3) we have

$$X(k) = \sum_{n=0}^{\frac{N}{2}-1} x_e(n) W_N^{2nk} + W_N^k \sum_{n=0}^{\frac{N}{2}-1} x_o(n) W_N^{2nk} \ldots\ldots (4)$$

We can write

$$W_N^2 = \left(e^{-j2\pi/N}\right)^2 = e^{-j2\pi/N/2} = W_{N/2} \ldots\ldots..(5)$$

i.e., $W_N^2 = W_{N/2} \ldots\ldots.(6)$

Substituting Eq. (5) in Eq. (4) we get

$$\mathrm{X_k} = \underbrace{\sum_{\mathrm{n}=0}^{\frac{\mathrm{N}}{2}-1} x_e(n) W_{N/2}^{nk}}_{\text{DFT of even-indexed part}} + W_N^k \underbrace{\sum_{\mathrm{n}=0}^{\frac{\mathrm{N}}{2}-1} x_0(n) W_{N/2}^{nk}}_{\text{DFT of odd-indexed part}} \ldots\ldots..(6)$$

$$= X_e(k) + W_N^k X_0(k) \ldots\ldots.(7)$$

Each of the sums in Eq. (6) is an $\frac{N}{2}$-point DFT, the first sum being the $\frac{N}{2}$-pcint DFT of the even-indexed sequence and the second being the $\frac{N}{2}$-point DFT of the odd-indexed sequence. Although the index k ranges from $k = 0,1 \ldots, \mathrm{N} - 1$, cach of the sums is computed only for $k = 0,1 \ldots, \frac{N}{2} - 1$, since $X_e(k)$ and $X_0(k)$ are periodic in k with period $\frac{N}{2}$. After the two DFTs are computed, they are combined according to Eq. (7) to get the N-point DFT of $X(k)$. So Eq. (7) holds for the values of $k = 0,1, \ldots, \frac{N}{2} - 1$.

For $k \geq N/2$

$$W_N^{k+N/2} = -W_N^k \ldots\ldots (8)$$

Now $X(k)$ for $k \geq N/2$ is given by

$$X(k) = X_c\left(k - \frac{N}{2}\right) - W_N^{k-\frac{N}{2}} X_0\left(k - \frac{N}{2}\right) \text{ for } k = \frac{N}{2}, \frac{N}{2} + 1, \ldots N - 1 \ldots.(9)$$

Let us find the number of complex multiplications and complex additions required to compute Eq.(4.12). For direct evaluation of DFT we know that the number of complex multiplications required is equal to N^2. In the same way to calculate $\frac{N}{2}$ - point DFT of $X_e(k)$ we need $\left(\frac{N}{2}\right)^2$ complex multiplications and to compute $X_0(k)$ we need another $\left(\frac{N}{2}\right)^2$ complex multiplications. That is we require $2\left(\frac{N}{2}\right)^2$ complex multiplications. Then the two $\frac{N}{2}$-point DFT's are combined to get $X(k)$. For this we need N complex multiplications. Thus the total number of complex multiplications required for computing $X(k)$ is

$$\left(\frac{N}{2}\right)^2 + \left(\frac{N}{2}\right)^2 + N = N + \frac{N^2}{2}$$

Similarly the total number of complex additions required is

$$\frac{N}{2}\left(\frac{N}{2} - 1\right) + \frac{N}{2}\left(\frac{N}{2} - 1\right) + N = \frac{N^2}{2}$$

The direct evaluation of X(k) requires N^2 complex multiplication and N(N-1) complex addition. When we decompose x(n) into two subsequences of length N/2 and compute X(k) using eq. (7) the number of computations are reduced by a factor of 2.

Now let us take $N = 8$. Then $X_e(k)$ and $X_o(k)$ are 4-point $(N/2)$ DFTs of even-indexed sequence $x_e(n)$ and odd-indexed sequence $x_o(n)$ respectively, where

$$x_e(0) = x(0); \quad x_o(0) = x(1)$$
$$x_e(1) = x(2); \quad x_o(1) = x(3)$$
$$x_e(2) = x(4); \quad x_o(2) = x(5)$$
$$x_e(3) = x(6); \quad x_o(3) = x(7)$$

From Eq. (7) and Eq. (9) we have

$$\begin{aligned} X(k) &= X_e(k) + W_8^k X_o(k) \text{ for } 0 \leq k \leq 3 \\ &= X_e(k-4) - W_8^{k-4} X_o(k-4) \text{ for } 4 \leq k \leq 7 \end{aligned} \ldots\ldots..(10)$$

By substituting different values of k we get

$$X(0) = X_e(0) + W_8^0 X_o(0); X(4) = X_e(0) - W_8^0 X_o(0)$$
$$X(1) = X_e(1) + W_8^1 X_o(1); X(5) = X_e(1) - W_8^1 X_o(1)$$

$$X(2) = X_e(2) + W_8^2 X_o(2); X(6) = X_e(2) - W_8^2 X_o(2)$$
$$X(3) = X_e(3) + W_8^3 X_o(3); X(7) = X_e(3) - W_8^3 X_o(3) \ldots\ldots (11)$$

From the above set of equations we can find that $X(0)$ and $X(4), X(1)$ and$X(5)$, $X(2)$ and $X(6), X(3)$ and $X(7)$ have same inputs. $X(0)$ is obtained by multiplying $X_0(0)$ with W_8^0 and adding the product to $X_e(0)$. Similarly $X(4)$ is obtained by multiplying $X_a(0)$ with W_8^0 and subtracting the product from$X_\varepsilon(0)$. This operation can be represented by a butterfly diagram as shown figure (1)

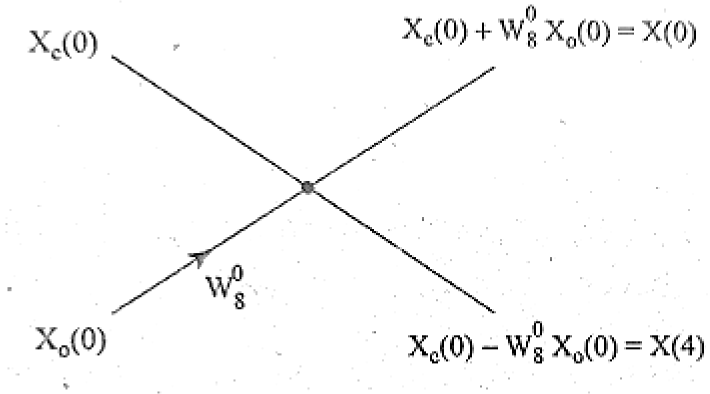
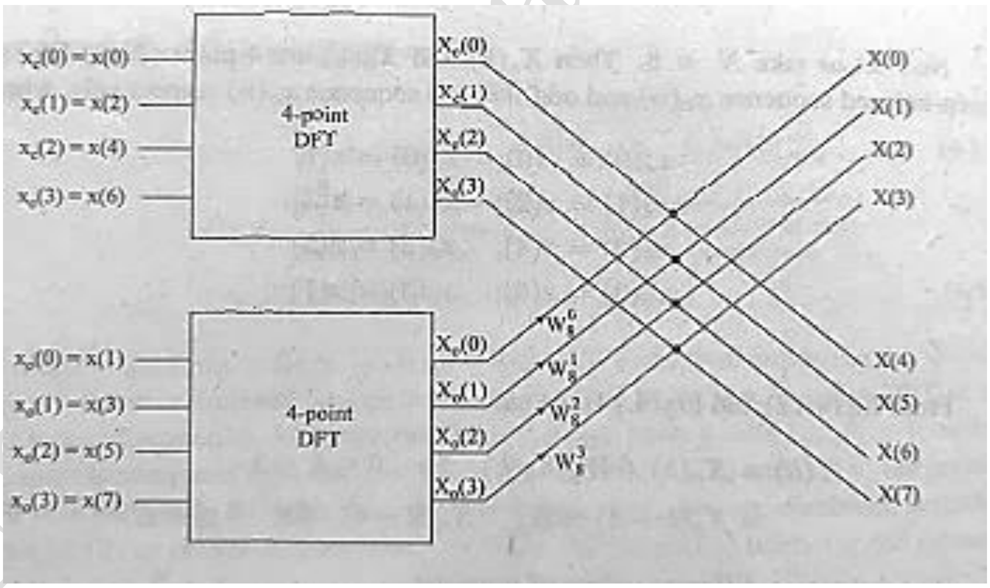


Fig.1 Flow graph of butterfly diagram of eq.(11)

Now the values $X(k)$ for $k = 1,2,3,4,5,6,7$ can be obtained and an 8-point DFT flow graph can be constructed from two 4-point DFTs as shown in Fig. 2.

From Fig.. 2 we can find that initially the sequence $x(n)$ is shuffled into evenindexed sequence $x_e(n)$ and odd-indexed sequence $x_o(n)$ and then transformed to give $X_e(k)$ and $X_o(k)$. For $k = 0,1,2,3$ the values $X_e(k)$ and $X_0(k)$ are combined according to Eqs. (11) and using butterfly structure shown in Fig. 1 the 8-point. DFT is obtained. The inputs to the butterfly are separated by $\frac{N}{2}$ samples i.e. 4 samples. And the powers of the twiddle factors associated in this set of butterflies are in natural order.



(Fig.2)Construction of an 8-point DFT from two 4-point DFTs

Now we apply the same approach to decompose each of $\frac{N}{2}$ sample DFT. This can be done by dividing the sequence $x_e(n)$ and $x_o(n)$ into two sequences consisting of even and odd members of the sequences. The $\frac{N}{2}$-point DFTs can be expressed as a combination of $\frac{N}{4}$ point DFTs. That is, $X_e(k)$ for $0 \le k \le \frac{N}{2} - 1$ can be written as:

$$X_e(k) \quad = X_{ee}(k) + W_N^{2k} X_{eo}(k) \text{ for } 0 \le k \le \frac{N}{2} - 1$$
$$= X_{ee}\left(k - \frac{N}{4}\right) - W_N^{2\left(k-\frac{N}{4}\right)} X_{eo}\left(k - \frac{N}{4}\right) \text{ for } \frac{N}{4} \le k \le \frac{N}{2} - 1 \ldots\ldots (12)$$

Where $X_{ee}(k)$ is $\frac{N}{4}$-point DFT of the even members of $x_e(n)$ and $X_{eo}(k)$ is $\frac{N}{4} -$ point of DFT of the odd

members of $x_e(n)$.
In the same way

$$X_o(k) = X_{oe}(k) + W_N^{2k} X_{oo}(k) \text{ for } 0 \le k \le \frac{N}{2} - 1$$

$$= X_{oe}\left(k - \frac{N}{4}\right) - W_N^{2\left(k-\frac{N}{4}\right)} X_{oo}\left(k - \frac{N}{4}\right) \text{ for } \frac{N}{4} \le k \le \frac{N}{2} - 1 \ldots \ldots (13)$$

Where $X_{oe}(k)$ is $\frac{N}{4}$-point DFT of the even members of $x_o(n)$ and $X_{oo}(k)$ is $\frac{N}{4} -$ point of DFT of the odd members of $x_o(n)$.

Bit-reversal of DIT algorithm

In this structure, we represent all the points in binary format i.e., in 0 and 1. Then, we reverse those structures. The sequence we get after that is known as bit reversal sequence. This is also known as decimation in time sequence. In-place computation of an eight-point DFT is shown ina tabular format as shown below –

POINTS	BINARY FORMAT	REVERSAL	EQUIVALENT POINTS
0	000	000	0
1	001	100	4
2	010	010	2
3	011	110	6
4	100	001	1
5	101	101	5
6	110	011	3
7	111	111	7

2.Decimation in Frequency Sequence

DIT algorithm is based on the decomposition of the DFT computation by forming smaller and smaller subsequences of the sequence$x(n)$. In DIF algorithm the output sequence $X(k)$ is divided into smaller and smaller subsequences. In this algorithm the input sequence $x(n)$ is partitioned into two sequences each of length $\frac{N}{2}$ samples. The first sequence $x_1(n)$ consists of first $\frac{N}{2}$ samples of $x(n)$ and the second sequence $x_2(n)$ consists of the last $\frac{N}{2}$ samples of $x(n)$ i.e.

$$x_1(n) = x(n),\ n = 0,1,2, \ldots \frac{N}{2} - 1 \ldots \ldots (1)$$

$$x_2(n) = x\left(n + \frac{N}{2}\right)\ n = 0,1,2, \ldots \frac{N}{2} - 1 \ldots \ldots (2)$$

If $N = 8$ the first sequence $x_1(n)$ has values for $0 \le n \le 3$ and $x_2(n)$ has values for$4 \le n \le 7$.
The N-point DFT of $x(n)$ can be written as

$$X(k) = \sum_{n=0}^{\frac{N}{2}-1} x(n) W_N^{nk} + \sum_{n=\frac{N}{2}}^{\mathrm{N}-1} x(n) W_N^{nk}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x_1(n) W_N^{nk} + \sum_{n=\frac{N}{2}}^{\mathrm{N}-1} x_2(n) W_N^{(n+N/2)k}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x_1(n) W_N^{nk} + W_N^{Nk/2} \sum_{n=0}^{\frac{N}{2}-1} x_2(n) W_N^{nk}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x_1(n) W_N^{nk} + e^{-j\pi k} \sum_{n=0}^{\frac{N}{2}-1} x_2(n) W_N^{nk}$$

When k is even $e^{-jxk} = 1$

$$X(2k) = \sum_{n=0}^{\frac{N}{2}-1} [x_1(n) + x_2(n)] W_N^{2nk}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} [x_1(n) + x_2(n)] W_{N/2}^{nk}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} f(n) W_{\frac{N}{2}}^{nk} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. (3)$$

Where

$$f(n) = x_1(n) + x_2(n) \ldots\ldots\ldots\ldots\ldots (4)$$

Eq. (3) is the $\frac{N}{2}$-point DFT of the $\frac{N}{2}$-point sequence $f(n)$ obtained by adding the first-half and the last-half of the input sequence. When k is odd

$$e^{-j\pi k} = -1$$

$$X(2k+1) = \sum_{n=0}^{\frac{N}{2}-1} [x_1(n) - x_2(n)] W_N^{(2k+1)n}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} [x_1(n) - x_2(n)] W_N^{n} W_{N/2}^{nk}$$

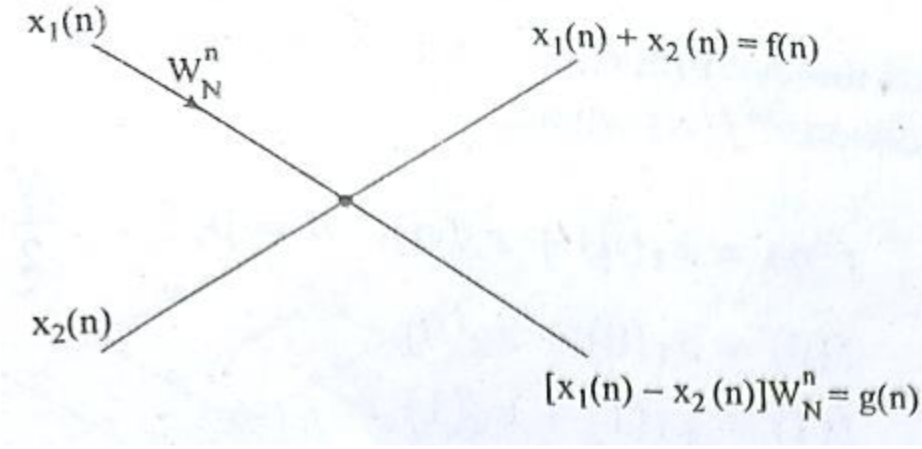
$$= \sum_{n=0}^{\frac{N}{2}-1} g(n) W_{\frac{N}{2}}^{nk} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Where

$$g(n) = [x_1(n) - x_2(n)] W_N^n \ldots\ldots\ldots.. (6)$$

Eq. (5) is the $\frac{N}{2}$-point DFT of the sequence $g(n)$ obtained by subtracting the seeond half of the input sequence from the first half and then multiplying the resulting sequence withW_N.
From Eq.(3) and Eq. (5) we find that the even and odd samples of the DFT can be obtained from the $\frac{N}{2}$-point DFTs of $f(n)$ and $g(n)$ respectively.
The Eq.(4) and Eq. (6) can be represented by a butterfly as shown in Fig, 4.10. This is the basic operation of DIF algorithm.
From Eq. (4), for $N = 8$, we have

$$X(0) = \sum_{n=0}^{3} [x_1(n) + x_2(n)] = \sum_{n=0}^{3} f(n) = f(0) + f(1) + f(2) + f(3) \ldots\ldots. (7)$$



(Fig.3) Flow graph of basic butterfly diagram for DIF algorithm

$$X(2) = \sum_{n=0}^{3}[x_1(n) + x_2(n)]W_8^{2n} = \sum_{n=0}^{3} f(n)W_8^{2n}$$
$$= f(0) + f(1)W_8^2 - f(2) - f(3)W_8^2 \ \dots\dots\dots\dots (8)$$
$$X(4) = \sum_{n=0}^{3}[x_1(n) + x_2(n)]W_8^{4n} = \sum_{n=0}^{3} f(n)W_8^{4n} = \sum_{n=0}^{3} f(n)-1^n$$
$$= f(0) - f(1) + f(2) - f(3) \dots\dots\dots\dots.(9)$$
$$X(6) = \sum_{n=0}^{3}[x_1(n) + x_2(n)]W_8^{6n} = \sum_{n=0}^{3} f(n)(-W_8^{2n})$$
$$= f(0) - f(1)W_8^2 - f(2) + f(3)W_8^2 \dots\dots\dots\dots (10)$$
$$\therefore \ W_8^4 = \left(e^{\frac{j2\pi}{8}}\right)^4 = e^{j\pi} = -1;$$
$$\therefore W_8^8 = \left(e^{j2\pi/8}\right)^8 = e^{j2\pi} = 1$$

From Eq. (5) we have

$$X(1) = \sum_{n=0}^{3} \ [x_1(n) - x_2(n)]W_8^n = \sum_{n=0}^{3} \ g(n) = g(0) + g(1) + g(2) + g(3) \dots\dots\dots.(11)$$
$$X(3) = \sum_{n=0}^{3} \ [x_1(n) - x_2(n)]W_8^{3n} = \sum_{n=0}^{3} g(n)W_8^{2n}$$
$$= g(0) + g(1)W_8^2 - g(2) - g(3)W_8^2 \dots\dots..(12)$$
$$X(5) = \sum_{n=0}^{3} \ [x_1(n) - x_2(n)]W_8^{5n} = \sum_{n=0}^{3} g(n)W_8^{4n} = \sum_{n=0}^{3} \ g(n)(-1)^n$$
$$= g(0) - g(1) + g(2) - g(3) \dots\dots..(13)$$
$$X(7) = \sum_{n=0}^{3} \ [x_1(n) - x_2(n)]W_8^{7n} = \sum_{n=0}^{3} \ g(n)(-W_8^2)^n$$
$$= g(0) - g(1)W_8^2 - g(2) + g(3)W_8^2 \dots\dots\dots (14)$$

We have seen that the even-indexed samples of $X(k)$ can be obtained from the 4-point DFT of the sequence $f(n)$ where

$$f(n) \quad = x_1(n) + x_2(n) \ n = 0,1, \dots \frac{N}{2} - 1$$

i.e., $f(0) = x_1(0) + x_2(0)$

$f(1) = x_1(1) + x_2(1)$ $\dots\dots\dots\dots\dots (15)$

$f(2) = x_1(2) + x_2(2)$

$f(3) = x_1(3) + x_2(3)$

The odd-indexed samples of $X(k)$ can be obtained from the 4-point DFT of the sequence $g(n)$
Where

$$g(n) = \ [x_1(n) - x_2(n)]W_8^n$$

i.e.,

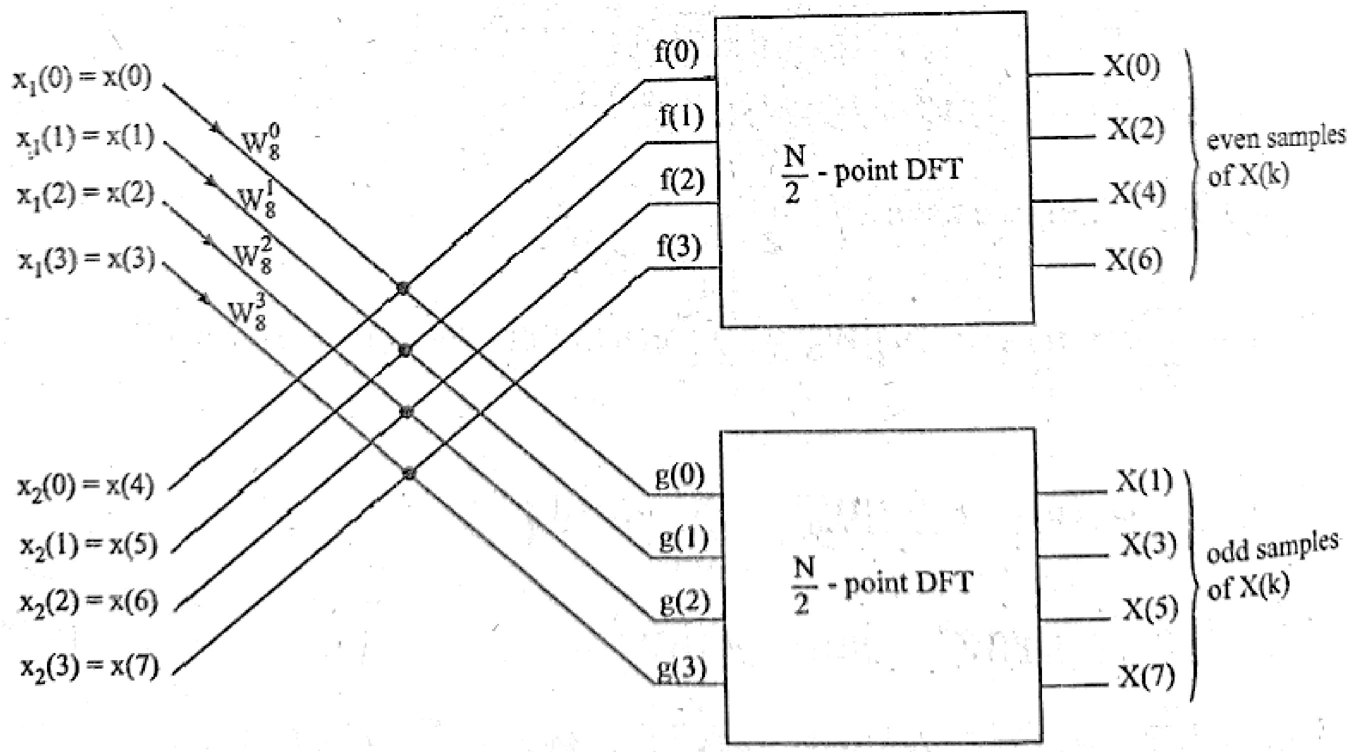
$g(0) = [x_1(0) - x_2(0)]W_8^0$

$g(1) = [x_1(1) - x_2(1)]W_8^1 \dots\dots\dots\dots\dots.(16)$

$g(2) = [x_1(2) - x_2(2)]W_8^2$

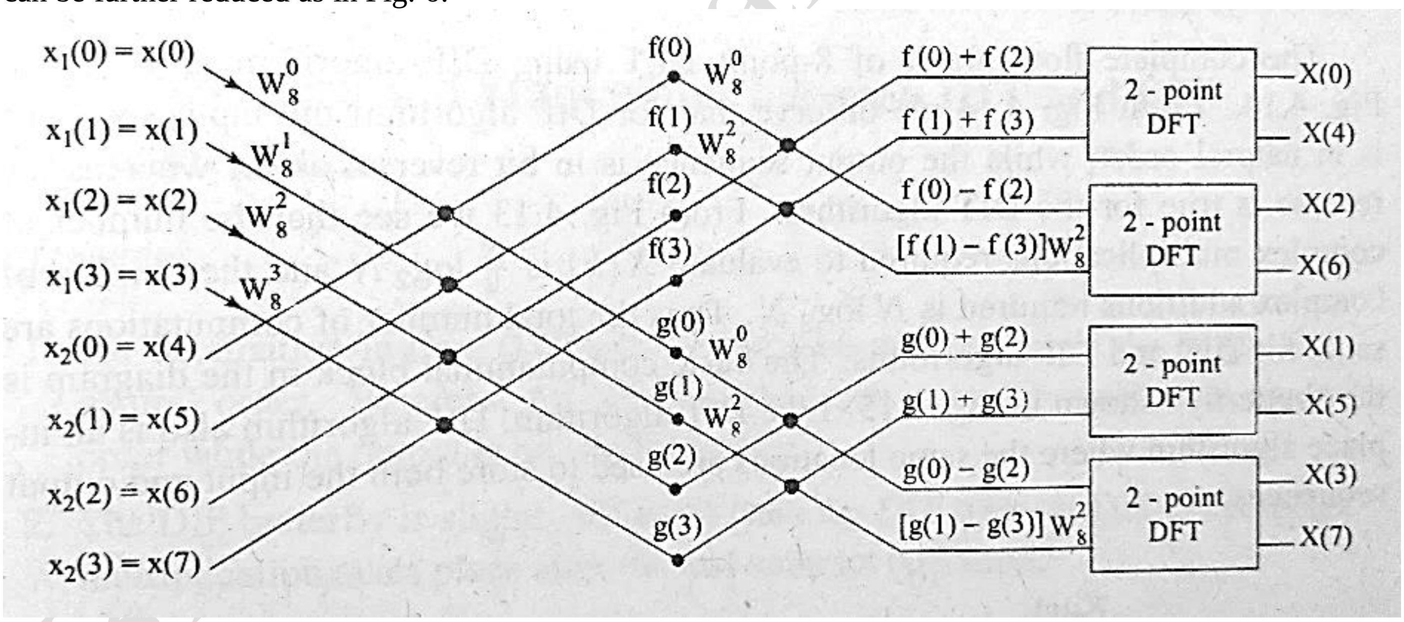
g(3)= $[x_2(3) - x_2(3)]W_8^3$

Using the above information and the butterfly structure shown in Fig. 3 we can draw the flow graph of 8-point DFT shown figure no.4.

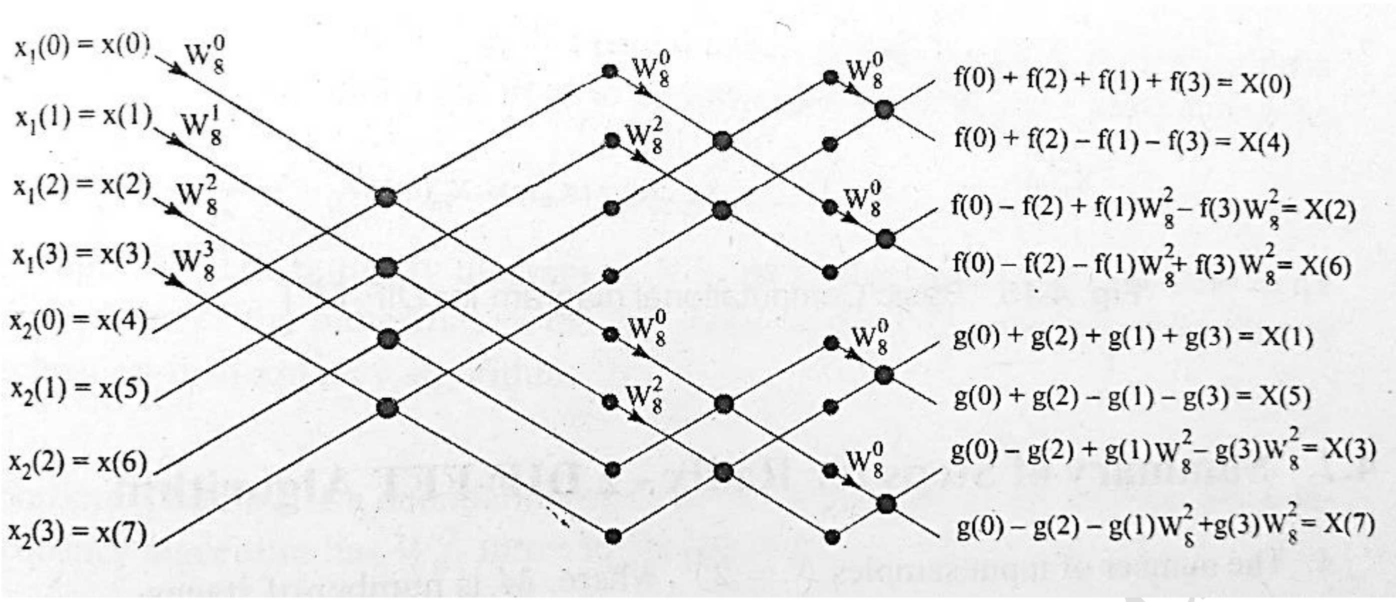


(Fig.4) Reduction of an 8-point DFT to two 4-point DFT by decimation in frequency

Now each $\frac{N}{2}$-point DFT can be computed by combining the first half and the last haif of the input points for each of the $\frac{N}{2}$-point DFTs and then computing $\frac{N}{4}$-point DFTs. For the 8-point DFT example the resultant flow graph is shown in Fig. 5. The 2-point DFT can be found by adding and subtracting the input points. The Fig. 5 can be further reduced as in Fig. 6.



(Fig.5) Flow graph of decimation in frequency decomposition of an 8-point DFT into four 2-point DFT computations.



(Fig.6) Flow graph of 8-point DIF-FFT algorithm

5.2. DIRECT COMPUTATION OF DFT

The DFT of a sequence can be evaluated using the formula

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-\frac{j2\pi nk}{N}} \quad 0 \le k \le N-1 \ldots\ldots (1)$$

Substituting $W_N = e^{-j2\pi/N}$, we have

$$X(k) = \sum_{n=0}^{N-1} x(n)W_N^{nk} \quad 0 \le k \le N-1 \ldots\ldots\ldots (2)$$

$$= \sum_{n=0}^{N-1} \{\mathrm{Re}[x(n)] + j\,\mathrm{Im}[x(n)]\}\{\mathrm{Re}[W_N^{nk}] + \mathrm{jIm}[W_N^{nk}]\} \ldots\ldots (3)$$

$$= \sum_{n=0}^{N-1} Re[x(n)]Re[W_N^{nk}] - \sum_{n=0}^{N-1} Im[x(n)]Im[W_N^{nk}] + j\{\sum_{n=0}^{N-1} Im[x(n)]Re[W_N^{nk}] + \sum_{n=0}^{N-1} Re[x(n)]Im[W_N^{nk}] \ldots (4)$$

From Eq.(3) we can see that to evaluate one value of $X(k)$, the number of complex multiplications required is N. Therefore to evaluate all N value of $X(k)$, the number of complex multiplications required is N^2. In the same way, to evaluate one value of $X(k)$ the number of complex additions required is $(N-1)$. To evaluate all N values of $X(k)$, the total number of complex additions required is $N(N-1)$. From Eq. (4), we can observe that the computation of $X(k)$ for each k requires $4N$ real multiplications. Therefore, to evaluate $X(k)$ for all k from 0 to $N-1$ requires $4N^2$ real multiplications.

Each of the four sums of N terms requires $N-1$ real two-input additions, and to combine the sum to get the real part and imaginary part requires two more. Therefore, to evaluate $X(k)$ for each k requires $4(N-1)+2$ real additions. For all values of k a total number of real additions $N(4N-2)$ required for direct evaluation of the DFT. The above results are obtained by assuming the value of W_N^{kn} as always complex, even though for some values of kn, it equals to $1, -1, j$ or $-j$. The direct evaluation of the DFT is basically inefficient because it does not use the symmetry and periodicity properties of the twiddle factor W_N. These two properties are
Symmetry property: $W_N^{k+N/2} = -W_N^k$

Periodicity property: $W_N^{k+N/2} = W_N^k$

Example:

Determine the 4-point DFT of following sequence x(n)={0,1,2,3}

Using DIT ,DIF algorithm

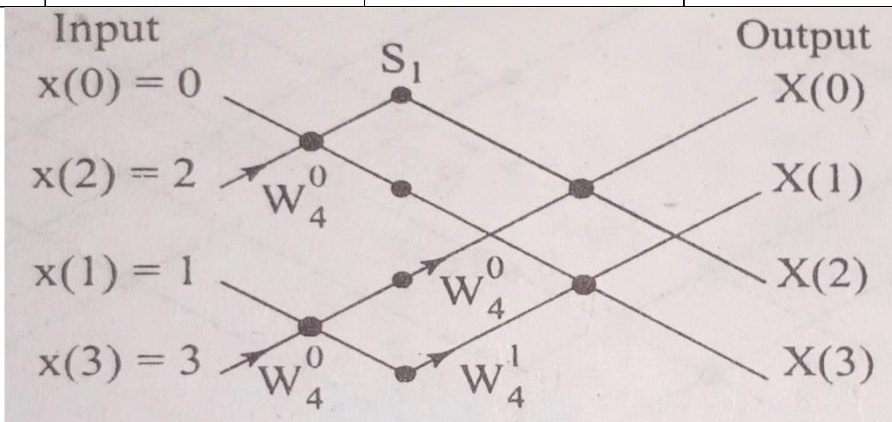
1)DIT algorithm

Twiddle factors associated with butterflies are

$$\omega_4^0 = 1; \omega_4^1 = e^{-j2\pi/4} = -j$$

Bit reversal of input given by

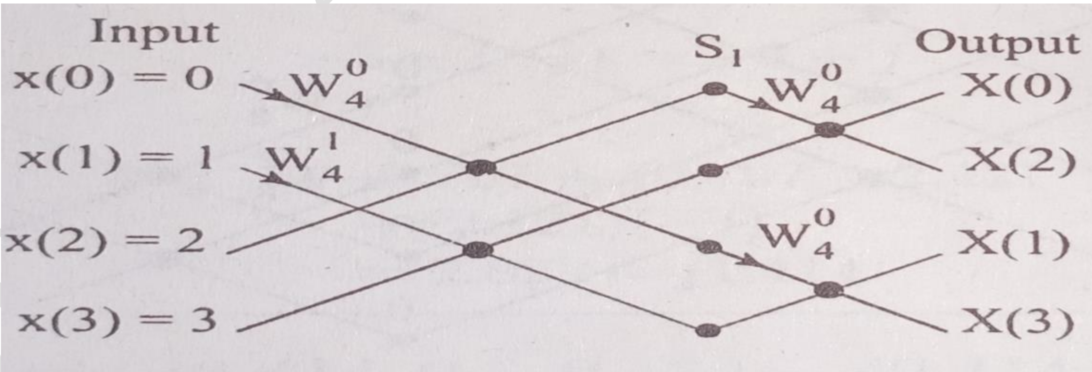
Input index	Binary index	Bit reversal	Bit reversal index
0	00	00	0
1	01	10	2
2	10	01	1
3	11	11	3



input	S_1	output
0	0+2= 2	2+4=6
2	0-2= -2	-2+(-j)(-2)= -2+2j
1	1+3= 4	2-4=-2
3	1-3= -2	-2-(-j)(-2)= -2-2j

$$X(k) = \{6, -2 + 2j\ , -2, -2 - 2j\ \}$$

2) DIF algorithm



input	S_1	output
0	0+2=2	2+4= 6
1	1+3=4	2-4= -2
2	0-2=-2	-2+2j
3	(1-3)(-j)=2j	-2-2j

$$X(k) = \{\ 6, -2, -2 + 2j, -2 - 2j\}$$

5.3 DIVIDE AND CONQUER APPROACH TO COMPUTATION OF DFT

We will define the DFT as,

$$X_N[k] = \sum_{n=0}^{N-1} x[n] e^{-\frac{j2\pi kn}{N}} \quad \ldots\ldots\ldots (1)$$

It is fairly easy to visualize this 1 point DFT, but how does it look when x[n] has 8 points, 256 points, 1024 points, etc. That's where matrices come in. For an N point DFT, we will define our input as x[n] where $n = 0,1,2, \ldots N-1$. Similarly, the output will be defined as X[k] where $k = 0,1,2, \ldots N-1$. Referring to our definition of the Discrete Fourier Transform above, to compute an N point DFT, all we need to do is simply repeat Eq. 1, N times. For every value of x[n] in the discrete time domain, there is a corresponding value, X[k], in the frequency domain.

Input x[n] =

$$[x[0] \quad x[1] \quad \cdots \quad x[N-1]]$$

Output X[k] =

$$\begin{bmatrix} X[0] \\ X[1] \\ \vdots \\ X[N-1] \end{bmatrix}$$

To solve for X[K], means simply repeating Eq. (1), N times, where x[n] is a real scalar value for each entry. We represent this in the matrices below.

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ \vdots \\ X[N-1] \end{bmatrix} = x[0] \begin{bmatrix} e^{-\frac{j2\pi(0)}{N}} \\ e^{-\frac{j2\pi(0)}{N}} \\ e^{-\frac{j2\pi(0)}{N}} \\ \vdots \\ e^{-\frac{j2\pi(0)}{N}} \end{bmatrix} + x[1] \begin{bmatrix} e^{-\frac{j2\pi(0)}{N}} \\ e^{-\frac{j2\pi(1)}{N}} \\ e^{-\frac{j2\pi(2)}{N}} \\ \vdots \\ e^{-\frac{j2\pi(N-1)}{N}} \end{bmatrix} + x[2] \begin{bmatrix} e^{-\frac{j2\pi(0)}{N}} \\ e^{-\frac{j2\pi(2)}{N}} \\ e^{-\frac{j2\pi(4)}{N}} \\ \vdots \\ e^{-\frac{j2\pi 2(N-1)}{N}} \end{bmatrix} + \cdots + x[N-1] \begin{bmatrix} e^{-\frac{j2\pi(0)}{N}} \\ e^{-\frac{j2\pi(N-1)}{N}} \\ e^{-\frac{j2\pi 2(N-1)}{N}} \\ \vdots \\ e^{-j2\pi\frac{(N-1)^2}{N}} \end{bmatrix} \ldots\ldots\ldots (2)$$

From this matrix representation of the DFT, you can see that N^2 complex multiplications and N^2 − N complex additions are necessary to fully compute the discrete fourier transform. There are a few simplifications that can be made right away. The first column vector of complex exponentials can be reduced to a vector of 1's. This is possible because, e^(0* anything) will always equal 1 . This also applies for the first entry in each vector of complex exponentials because n always begins at 0 .

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ \vdots \\ X[N-1] \end{bmatrix} = x[0] \begin{bmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} + x[1] \begin{bmatrix} 1 \\ e^{-\frac{j2\pi(1)}{N}} \\ e^{-\frac{j2\pi(2)}{N}} \\ \vdots \\ e^{-\frac{j2\pi(N-1)}{N}} \end{bmatrix} + x[2] \begin{bmatrix} 1 \\ e^{-\frac{j2\pi(2)}{N}} \\ e^{-\frac{j2\pi(4)}{N}} \\ \vdots \\ e^{-\frac{j2\pi 2(N-1)}{N}} \end{bmatrix} + \cdots + x[N-1] \begin{bmatrix} 1 \\ e^{-\frac{j2\pi(N-1)}{N}} \\ e^{-\frac{j2\pi 2(N-1)}{N}} \\ \vdots \\ e^{-j2\pi\frac{(N-1)^2}{N}} \end{bmatrix} \ldots\ldots\ldots\ldots (3)$$

One of the key building blocks of the Fast Fourier Transform, is the Divide and Conquer DFT. As the name implies, we will divide Eq. 1 into two separate summations. The first summation processes the even components of x[n] while the second summation processes the odd components of x[n]. This produces,

$$X_N[k] = \sum_{m=0}^{\frac{N}{2}-1} x[2m]e^{-\frac{j2\pi k(m)}{\frac{N}{2}}} + e^{-\frac{j2\pi k}{N}} \sum_{m=0}^{\frac{N}{2}-1} x[2m+1]e^{-\frac{j2\pi k(m)}{\frac{N}{2}}} \quad \ldots\ldots..(4)$$

Our DFT has now been successfully split into two $N/2ptDFT's$. For simplification purposes, the first summation will be defined as $X0[k]$ and the second summation as $X1[k]$. We can now simplify Eq. 4 to the following form.

$$X[k] = X_0[k] + e^{-\frac{j2\pi k}{n}} X_1[k] \quad \ldots\ldots\ldots(5)$$

where

$$X_0[k] = \sum_{m=0}^{\frac{N}{2}-1} x[2m]e^{-\frac{j2\pi k(m)}{\frac{N}{2}}} \quad \ldots\ldots..(6)$$

and

$$X_1[k] = \sum_{m=0}^{\frac{N}{2}-1} x[2m+1]e^{-\frac{j2\pi k(m)}{\frac{N}{2}}} \quad \ldots\ldots..(7)$$

The complex exponential preceding $X1[k]$ in Eq. 4 is generally called the "twiddle factor" and represented by

$$W_N^k = e^{-j2\pi k/N}$$

By definition of the discrete fourier transform, $X0[k]$ and $X1[k]$ are periodic with period $N/2$. Therefore, we can split Eq. 4 into two separate equations.

$$X[k] = X_0[k] + W_N^k X_1[k]$$
$$X[k + (N/2)] = X_0[k] - W_N^k X_1[k]$$

Once again, we are left with a number of $\mathbf{1pt}$. DFT's. First, note that we have two separate equations and therefore need two separate equations of matrices. Similar to Eq. 2, we will repeat the DFT for the entire length of the input signal. However, since we split $\mathbf{x[n]}$ into even and odd components, we will only repeat the DFT ($\mathbf{N/2}$) times for $\mathbf{X0}$ and $\mathbf{X1}$. The first equation solves for the first half of $\mathbf{X[k]}$.

From Eq. 6 and 7,

The second equation solves for the second half of X[k].

$$\begin{bmatrix} X_0[0] \\ X_0[1] \\ X_0[2] \\ \vdots \\ X_0[N/2-1] \end{bmatrix} - \begin{bmatrix} W_N^0 & 0 & \cdots & 0 \\ 0 & W_N^1 & & \vdots \\ \vdots & & W_N^2 & \\ & & \ddots & 0 \\ 0 & \cdots & & W_N(N/2)-1 \end{bmatrix} \begin{bmatrix} X_1[0] \\ X_1[1] \\ X_1[2] \\ \vdots \\ X_1[N/2-1] \end{bmatrix} = \begin{bmatrix} X[N/2] \\ X[(N/2)+1] \\ X[(N/2)+2] \\ \vdots \\ X[N-1] \end{bmatrix}$$

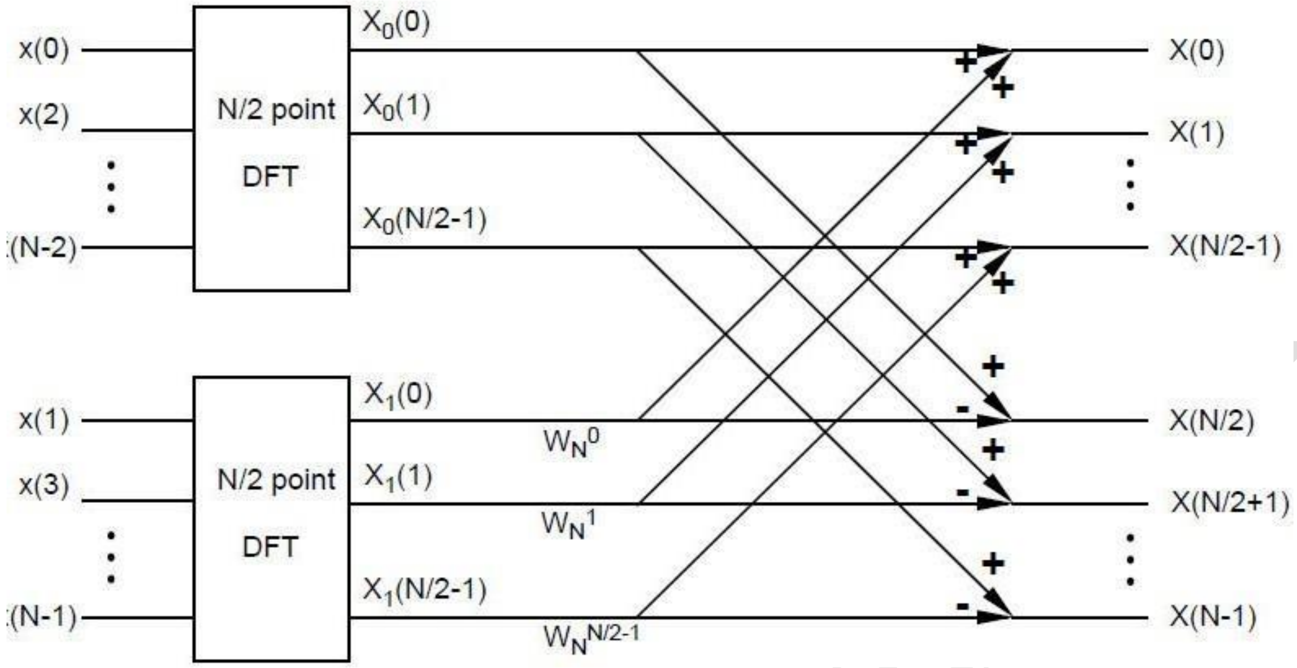
where

$$k = 0,1,2, \ldots (N/2) - 1$$

After analyzing the two matrices above, there are a few concepts you should understand. The matrix X0 in each equation is just the condensed form of Eq. 2 and then cut in half. It is simply the DFT repeated (N/2) times where the input is the even indices of x[n].

The matrix X1 in each equation, is also the condensed form of Eq. 2 cut in half. However, the input is now the odd indices of x[n].

Now compare these two equations to the DFT signal path below, as shown in



5.4 RADIX-2 ALGORITHM

Radix-2 algorithm is a member of the family of so-called Fast Fourier transform (FFT) algorithms. It computes separately the DFTs of the even-indexed inputs ($\mathbf{x_0}, \mathbf{x_1}, \ldots, \mathbf{x_{N-2}}$) and of the odd-indexed inputs ($\mathbf{x_1}, \mathbf{x_3}, \ldots, \mathbf{x_{N-1}}$), and then combines those two results to produce the DFT of the whole sequence. This idea can then be performed recursively to reduce the overall runtime from O ($\mathbf{N_2}$) to O (N log N). Radix-2 algorithm requires that N is a power of two; since the number of sample points N can usually be chosen freely by the application, this is often not an important restriction. To derive the algorithm, lets rearrange the DFT of x, into two parts: a sum over the even-numbered indices and a sum over the odd-numbered indices:

$$X_k = \sum_{m=0}^{N/2-1} x_{2m} e^{-\frac{2\pi i}{N}(2m)k} + \sum_{m=0}^{N/2-1} x_{2m+1} e^{-\frac{2\pi i}{N}(2m+1)k}$$

One can factor a common multiplier $e^{-\frac{2\pi i}{N}k}$ out of the second sum.

$$X_k = \sum_{m=0}^{N/2-1} x_{2m} e^{-\frac{2\pi i}{N}(2m)k} + e^{-\frac{2\pi i}{N}k} \sum_{m=0}^{N/2-1} x_{2m+1} e^{-\frac{2\pi i}{N}(2m)k} \ldots\ldots(1)$$

The two sums in Eq. (1) are the DFT of the even-indexed part and the DFT of odd-indexed part of x_n. Denote the DFT of the even-indexed inputs by E_k and the DFT of the oddindexed inputs by O_k and we obtain:

$$X_k = \underbrace{\sum_{m=0}^{N/2-1} x_{2m} e^{-\frac{2\pi i}{(N/2)}mk}}_{\text{DFT of even-indexed part}} + e^{-\frac{2\pi i}{N}k} \underbrace{\sum_{m=0}^{N/2-1} x_{2m+1} e^{-\frac{2\pi i}{(N/2)}mk}}_{\text{DFT of odd-indexed part}} = E_k + e^{-\frac{2\pi i}{N}k} O_k \ldots\ldots(2)$$

As the functions of kE_k and O_k are periodic with the period $N/2$:

$$E_{k+\frac{N}{2}} = E_k$$

and

$$O_{k+\frac{N}{2}} = O_k$$

Therefore, we can rewrite Eq. (2) as

$$X_k = \begin{cases} E_k + e^{-\frac{2\pi i}{N}k} O_k & \text{for } 0 \le k < N/2 \\ E_{k-N/2} + e^{-\frac{2\pi i}{N}k} O_{k-N/2} & \text{for } N/2 \le k < N \end{cases}$$

where we used the periodicity of O_k and E_k to translate the index k.

Using the following property of the twiddle factor $e^{-2\pi ik/N}$,

$$e^{\frac{-2\pi i}{N}(k+N/2)} = e^{\frac{-2\pi ik}{N} - \pi i} = e^{-\pi i} e^{\frac{-2\pi ik}{N}} = -e^{\frac{-2\pi ik}{N}}$$

we can rewrite X_k as:

$$\begin{aligned} X_k &= E_k + e^{-\frac{2\pi i}{N}k} O_k, \\ X_{k+\frac{N}{2}} &= E_k - e^{-\frac{2\pi i}{N}k} O_k. \end{aligned}$$

This result, expressing the DFT of length N recursively in terms of two DFTs of size N/2, is the core of the radix-2 fast Fourier transform.

Decimation in Time (DIT) Radix 2 FFT algorithm

Decimation in Time (DIT) Radix 2 FFT algorithm converts the time domain N point sequence x(n) to a frequency domain N-point sequence X(k). In Decimation in Time algorithm the time domain sequence x(n) is decimated and smaller point DFT are performed. The results of smaller point DFTs are combined to get the result of N-point DFT.

In DIT radix -2 FFT the time domain sequence is decimated into 2-point sequences. For each 2-point sequence,2 -point DFT can be computed. From the result of 2-point DFT the 4-point DFT can be calculated. From the result of 4-point DFT the 8- point DFT can be calculated. This process is continued until we get N point DFT. This FFT algorithm is called radix-2 FFT.

In decimation in time algorithm the N point DFT can be realized from two numbers of N/2-point DFTs, The N/2-point DFT can be calculated from two numbers of N/4-point DFTs and so on.

Let x (n) be N sample sequence, we can decimate x (n) into two sequences of N/2 samples.

Let the two sequences be f1 (n) and f2 (n).

Let f1 (n) consists of even numbered samples of x (n) and f2(n) consists of odd numbered samples of x(n).

$$f\,1(n) = x(2n) \text{ for } n = 0,1,2,3 \ldots\ldots\ldots\ldots N/2 - 1$$

$$f\,2(n) = x(2n+1) \text{ for } n = 0,1,2,3 \ldots\ldots\ldots\ldots N/2 - 1$$

$$\text{Let } X\,(k) = N - \text{point DFT of } x\,(n)$$

$$F1\,(k) = N/2 \text{ point DFT of } f\,1(n)$$

$$F2\,(k) = N/2 \text{ point DFT of } f\,2(n)$$

$$\text{By definition of DFT the } N/2 \text{ point DFT of } f1(n) \text{ and } f2(n) \text{ are given by}$$

$$F1\,(k) = \sum f1\; N\; 2\; -\; 1\; n = 0\; (n)WN/2\; kn\;;$$

$$F2\,(k) = \sum f2\; N\; 2\; -\; 1\; n = 0\; (n)WN/2\; kn$$

$$\text{Now} - \text{point DFT } X\,(k), \text{in terms of } N/2 \text{ point DFTs } F1\,(k) \text{ and } F2\,(k) \text{ is given by}$$

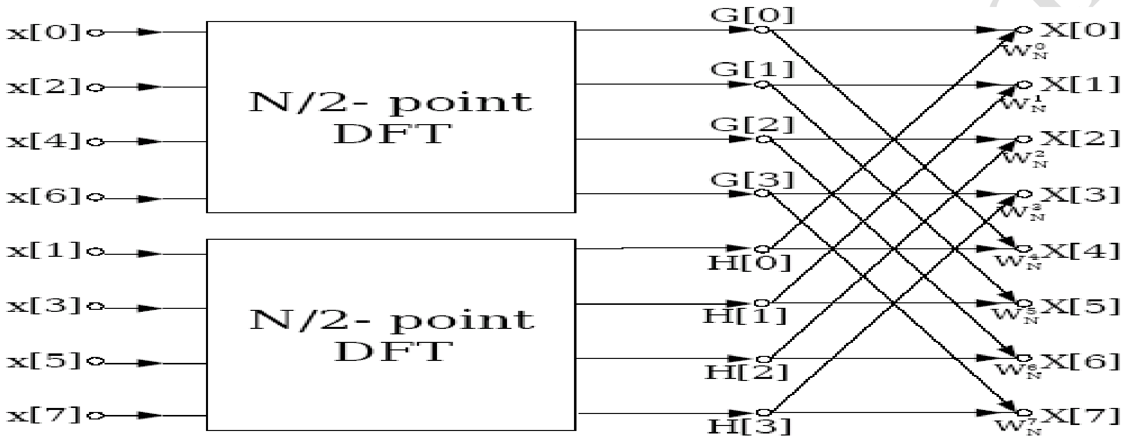
$$X\,(k) = F1\,(k) + Wk\,N\,F2(k), \text{where}, k = 0,1,2, \ldots\ldots\ldots\ldots\ldots.. (N-1)$$

Having performed the decimation in time once, we can repeat the process for each of the sequences f1 (n) and f2 (n). Thus f1(n) would result in the two N/4-point sequences and f2(n) would result in another two N/4-point sequences.

Let the decimated N/4-point sequences of f1 (n) be V11 (n) and V12 (n).

V11 (n) = f1 (2n); for n = 0, 1, 2, ... N/ 4 − 1
V12 (n) = f1 (2n + 1) ; for n = 0,1,2, ... N/ 4 – 1
Let the decimated N/4 point sequences of f2(n) be V21(n) and V22(n).
V 21(n) = f1 (2n) ; for n = 0,1,2, ... N/ 4 – 1
V 22(n) = f1 (2n + 1) ; for n = 0,1,2, ... N /4 – 1
Let V11 (k) = N/4 point DFT of V11 (n);
V12 (k) = N/4 point DFT of V12 (n)
V 21(k) = N/4 point DFT of V 21(n)
V 22(k) = N/4 point DFT of V 22(n)
Then like earlier analysis we can show that,
F1 (k) = V11 (k) + Wk N/2 V12 (k) ; for k = 0,1,2,3, ... N/ 2 − 1
F2 (k) = V21 (k) + Wk N/2 V 22(k) ; for k = 0,1,2,3, ... N /2 − 1

Hence the N/2-point DFTs are obtained from the results of N/4-point DFTs. The decimation of the data sequence can be repeated again and again until the resulting sequences are reduced to 2-point sequences



Steps of radix-2 DIT-FFT algorithm;

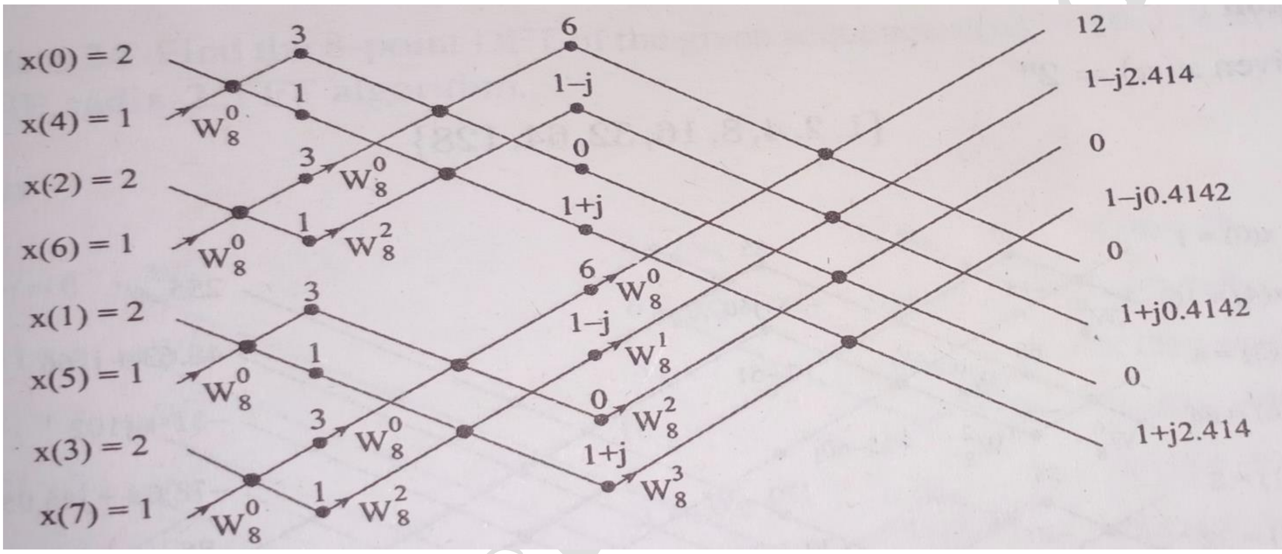
1. The number of input samples N=2^M,where ,M is an integer
2. The input sequence is shuffled through bit reversal.
3. The number of stages in the flow graph is given by $M = \log_2 N$.
4. Each stage consists of $\frac{N}{2}$ butterflies.
5. Inputs/outputs for each butterfly are separated by 2^{m-1} samples, where m represents the stage index, i.e, for first stage $m = 1$ and for second stage $m = 2$ so on.
6. The number of complex multiplications is given by $\frac{N}{2}\log_2 N$.
7. The number of complex additions is given by $N\log_2 N$.
8. The twiddle factor exponents are a function of the stage index m and is given by
9. $k = \frac{Nt}{2^m}\ t = 0,1,2, \dots 2^{m-1} - 1$
10. The number of sets or sections of butterflies in each stage is given by the formula2^{M-m}.
11. The exponent repeat factor (ERF), which is the number of times the exponent sequence associated with m is repeated is given by 2^{M-m}.

Steps of radix-2 DIF FFT algorithm;

1. The number of input samples $N = 2^M$, where, M is number of stages.
2. The input sequence is in natural order.
3. The number of stages in the flow graph is given by $M = \log_2 N$.

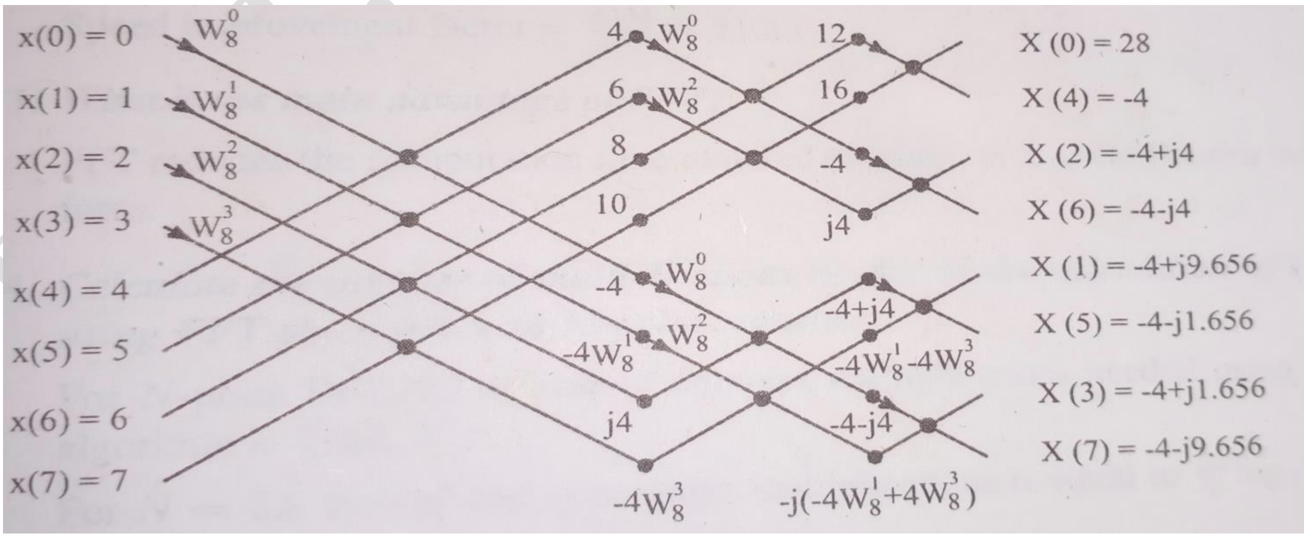
4. Each stage consists of $\frac{N}{2}$ butterflies.
5. Inputs/outputs for each butterfly are separated by 2^{M-m} samples, where m represents the stage index i.e., for first stage $m = 1$ and for second stage $m = 2$ so on.
6. The number of complex multiplications is given by $\frac{N}{2}\log_2 N$.
7. The number of complex additions is given by $N\log_2 N$.
8. The twiddle factor exponents are a function of the stage index m and is given by
9. $k = \frac{Nt}{2^{M-m+1}}, \ t = 0,1,2, \dots, 2^{M-m} - 1$
10. The number of sets or sections of butterflies in each stage is given by the formula 2^{m-1}.
11. The exponent repeat factor (ERF), which is the number of times the exponent sequence associated with m repeated, is given by 2^{m-1}.

Q. An 8-point sequence is given by x(n)={2,2,2,2,1,1,1,1}compute 8-point DFT of x(n) by radix-2 DIT –FFT.



$$X(K) = \{12, 1 - j2.414, 0, 1 - j0.4142, 0, 1 + j0.4142, 0, 1 + j2.414\}$$

Q. An 8-point sequence is given by x(n)={0,1,2,3,4,5,6,7}.compute 8-point DFT of x(n) by radix-2 DIF –FFT.



$$X(K) = \{28, -4 + j9.656, -4 + j4, -4 + j1.656, -4, -4 - j1.656, -4 - j4, -4 - j9.656\}$$

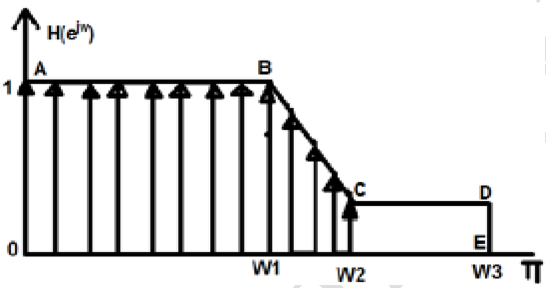
5.5. APPLICATIONS OF FFT ALGORITHMS

The fast Fourier transform (FFT) is a computationally efficient method of generating a Fourier transform. The main advantage of an FFT is speed, which it gets by decreasing thenumber of calculations needed to analyze a waveform.

- It covers FFTs, frequency domain filtering, and applications to video and audio signalprocessing.
- As fields like communications, speech and image processing, and related areas are rapidly developing, the FFT as one of the essential parts in digital signal processing has been widelyused.
- In the fields like cross-correlation, matched filtering, system identification, power spectrumestimation, and coherence function measurement.
- Used to create additional measurement functions such as frequency response, impulseresponse, coherence, amplitude spectrum, and phase spectrum.

5.6 INTRODUCTION TO DIGITAL FILTERS(FIR FILTERS) & GENERAL CONSIDERATIONS

FIR filters can be useful in making computer-aided design of the filters. Let us take an example andsee how it works. Given below is a figure of desired filter.



While doing computer designing, we break the whole continuous graph figures into discretevalues. Within certain limits, we break it into 64, 256 or 512 and so on.
we have taken limits between $-\pi$ to $+\pi$. We have divided it into 256 parts. The points can berepresented as $H_0, H_1 \ldots$ up to H256
Here, we apply IDFT algorithm and this will give us linear phase characteristics. Sometimes, we may be interested in some particular order of filter. Let us say we want to realize the above given design through 9th order filter. So, we take filter values as h_0, h_1, $h_2 \ldots h_9$. Mathematically, it can be shown as below

$$H(ej\omega) = h + h1_e - j\omega + h2_e - 2j\omega + \cdots + h9_e - 9j\omega$$

Where there is large number of dislocations, we take maximum points.

For example, in the above figure, there is a sudden drop of slopping between the points B and C. So, we try to take more discrete values at this point, but there is a constant slope betweenpoint C and D. There we take less number of discrete values.
For designing the above filter, we go through minimization process as follows;

$$H(ej\omega 1) = h + h1_e - j\omega + h2_e - 2j\omega + \cdots + h9_e - 9j\omega$$
$$H(ej\omega 2) = h + h1_e - j\omega + h2_e - 2j\omega + \cdots + h9_e - 9j\omega$$

Similarly,

$$H(ej\omega 1000) = h0 + h1_e - j\omega 1000 + h2_e - 2j\omega 1000 + \cdots + h9_e - 9j\omega 1000$$

Representing the above equation in matrix form, we have –

$$\begin{bmatrix} H(e^{jw_1}) \\ \vdots \\ H(e^{jw_{1000}}) \end{bmatrix} = \begin{bmatrix} e^{-jw_1} & \ldots & e^{-j9w_1} \\ \vdots & \ddots & \vdots \\ e^{jw_{1000}} & \ldots & e^{j9w_{1000}} \end{bmatrix} \begin{bmatrix} h_0 \\ \vdots \\ h_9 \end{bmatrix}$$

Let us take the 1000×1 matrix as B, 1000×9 matrix as A and 9×1 matrix as *h*^

So, for solving the above matrix, we will write

$$\hat{h} = [ATA] - 1ATB$$

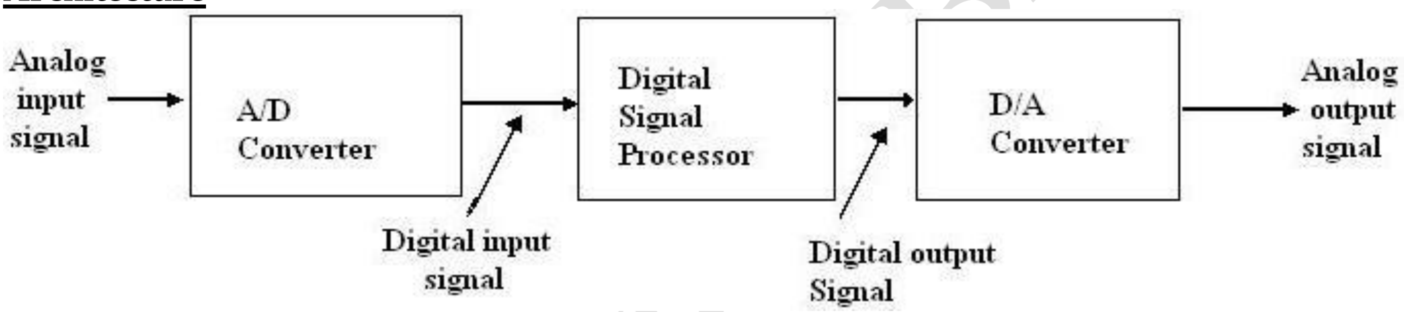
$$= [A^{*}\mathrm{TA}] - 1A^{*}\mathrm{TB}$$

where A^* represents the complex conjugate of the matrix A.

5.6. INTRODUCTION TO DSP ARCHITECTURE, FAMILIARIZATION OF DIFFERENT TYPES OF PROCESSORS

- Digital signal processing algorithms typically require a large number of mathematicaloperations to be performed quickly and repeatedly on a series of data samples.
- Signals (perhaps from audio or video sensors) are constantly converted from analogto digital, manipulated digitally, and then converted back to analog form.
- Many DSP applications have constraints on latency; that is, for the system to work, the DSP operation must be completed within some fixed time, and deferred (or batch)processing is not viable.
- Most general-purpose microprocessors and operating systems can execute DSP algorithms successfully, but are not suitable for use in portable devices such as mobilephones and PDAs because of power efficiency constraints.
- A specialized DSP, however, will tend to provide a lower-cost solution, with betterperformance, lower latency, and no requirements for specialized cooling or large batteries.

Architecture



Software architecture

- By the standards of general-purpose processors, DSP instruction sets are often highlyirregular; while traditional instruction sets are made up of more general instructions that allow them to perform a wider variety of operations, instruction sets optimized for digital signal processing contain instructions for common mathematical operationsthat occur frequently in DSP calculations.
- Both traditional and DSP-optimized instruction sets are able to compute any arbitraryoperation but an operation that might require multiple ARM or x86 instructions to compute might require only one instruction in a DSP optimized instruction set.

Instruction sets

- Multiply–accumulates (MACs, including fused multiply–add, FMA) operations used extensively in all kinds of matrix operations convolution for filtering dot product polynomial evaluation
- Fundamental DSP algorithms depend heavily on multiply–accumulate performance FIR filters Fast Fourier transform (FFT)
- Other related instructions:

1)SIMD
2)VLIW

Data instructions

- Saturation arithmetic, in which operations that produce overflows will accumulate at themaximum (or minimum) values that the register can hold rather than wrapping around (maximum+1 doesn't overflow to minimum as in many general-purpose CPUs, instead itstays at maximum). Sometimes various sticky bits operation modes are available.
- Fixed-point arithmetic is often used to speed up arithmetic processing.
- Single-cycle operations to increase the benefits of pipelining.

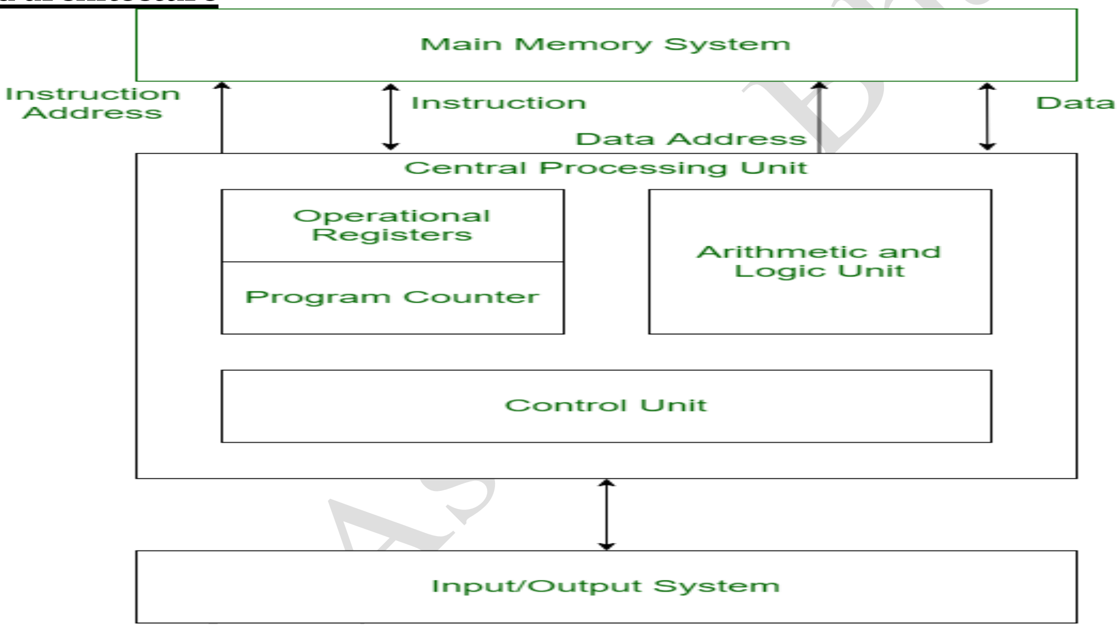
Program flow

- Floating-point unit integrated directly into the data path
- Pipelined architecture
- Highly parallel multiplier–accumulators (MAC units)
- Hardware-controlled looping, to reduce or eliminate the overhead required for looping operations

Hardware architecture

- In engineering, hardware architecture refers to the identification of a system's physical components and their interrelationships.
- This description, often called a hardware design model, allows hardware designers to understand how their components fit into a system architecture and provides to software component designers important information needed for software development and integration.
- Hardware architecture allows the various traditional engineering disciplines (e.g., electrical and mechanical engineering) to work more effectively together to develop and manufacture new machines, devices and components.
- DSPs are usually optimized for streaming data and use special memory architectures that are able to fetch multiple data or instructions at the same time, such as the Harvard architecture or modified von Neumann architecture, which use separate program and data memories.

Harvard architecture



Harvard Architecture is the computer architecture that contains separate storage and separate buses (signal path) for instruction and data. It was basically developed to overcome the bottleneck of Von Neumann Architecture. The main advantage of having separate buses for instruction and data is that the CPU can access instructions and read/write data at the same time.

Buses

Buses are used as signal pathways. In Harvard architecture, there are separate buses for both instruction and data. Types of Buses:

- Data Bus**:** It carries data among the main memory system, processor, and I/O devices.
- Data Address Bus**:** It carries the address of data from the processor to the main memory system.
- Instruction Bus**:** It carries instructions among the main memory system, processor, and I/O devices.
- Instruction Address Bus**:** It carries the address of instructions from the processor to the main memory system.

Operational Registers

There are different types of registers involved in it which are used for storing addresses of different types of instructions.

For example, the Memory Address Register and Memory Data Register are operational registers.

Program Counter
It has the location of the next instruction to be executed. The program counter then passes this next address to the memory address register.

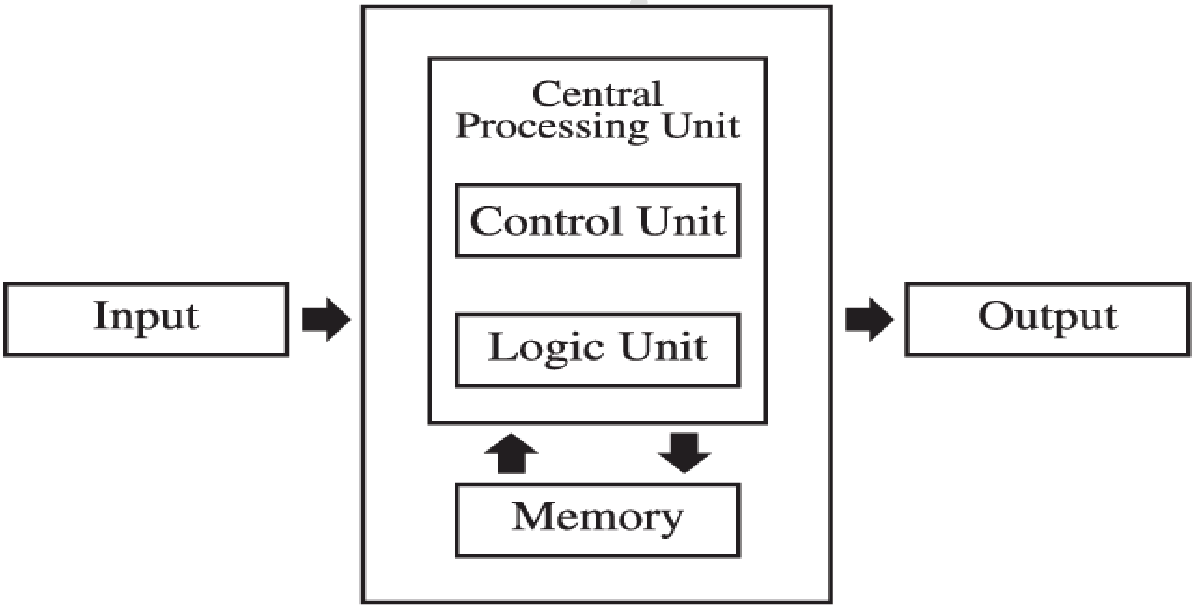
Arithmetic and Logic Unit
The arithmetic logic unit is that part of the CPU that operates all the calculations needed. It performs addition, subtraction, comparison, logical Operations, bit Shifting Operations, and various arithmetic operations.

Control Unit
The Control Unit is the part of the CPU that operates all processor control signals. It controls the input and output devices and also controls the movement of instructions and data within the system.

Input/Output System
Input devices are used to read data into main memory with the help of CPU input instruction. The information from a computer as output is given through Output devices. The computer gives the results of computation with the help of output devices.

Advantage of Harvard Architecture:
Harvard architecture has two separate buses for instruction and data. Hence, the CPU can access instructions and read/write data at the same time. This is the major advantage of Harvard architecture.
In practice, Modified Harvard Architecture is used where we have two separate caches (data and instruction). This is common and used in X86 and ARM processors.

Von Neumann architecture



Control Unit –
A control unit (CU) handles all processor control signals. It directs all input and output flow, fetches code for instructions, and controls how data moves around the system.
Arithmetic and Logic Unit (ALU) –
The arithmetic logic unit is that part of the CPU that handles all the calculations the CPU may need, e.g. Addition, Subtraction, Comparisons. It performs Logical Operations, Bit Shifting Operations, and Arithmetic operations.
Main Memory Unit (Registers) –

- **Accumulator:** Stores the results of calculations made by ALU.
- **Program Counter (PC):** Keeps track of the memory location of the next instructions to be dealt with. The PC then passes this next address to Memory Address Register (MAR).
- **Memory Address Register (MAR):** It stores the memory locations of instructions that need to be fetched from memory or stored into memory.

- **Memory Data Register (MDR):** It stores instructions fetched from memory or any data that is to be transferred to, and stored in, memory.
- **Current Instruction Register (CIR):** It stores the most recently fetched instructions while it is waiting to be coded and executed.
- **Instruction Buffer Register (IBR):** The instruction that is not to be executed immediately is placed in the instruction buffer register IBR.

Input/Output Devices – Program or data is read into main memory from the input device or secondary storage under the control of CPU input instruction. Output devices are used to output the information from a computer. If some results are evaluated by computer and it is stored in the computer, then with the help of output devices, we can present them to the user.

POSSIBLE SHORT TYPE QUESTIONS WITH ANSWERS

1.Define FFT.Is it a z transform?(S-24)

Ans- A fast Fourier transform (FFT) is an algorithm that computes the discrete Fouriertransform (DFT) of a sequence, or its inverse (IDFT).it is a z-transform

2.What are the Applications of FFT?

Ans:

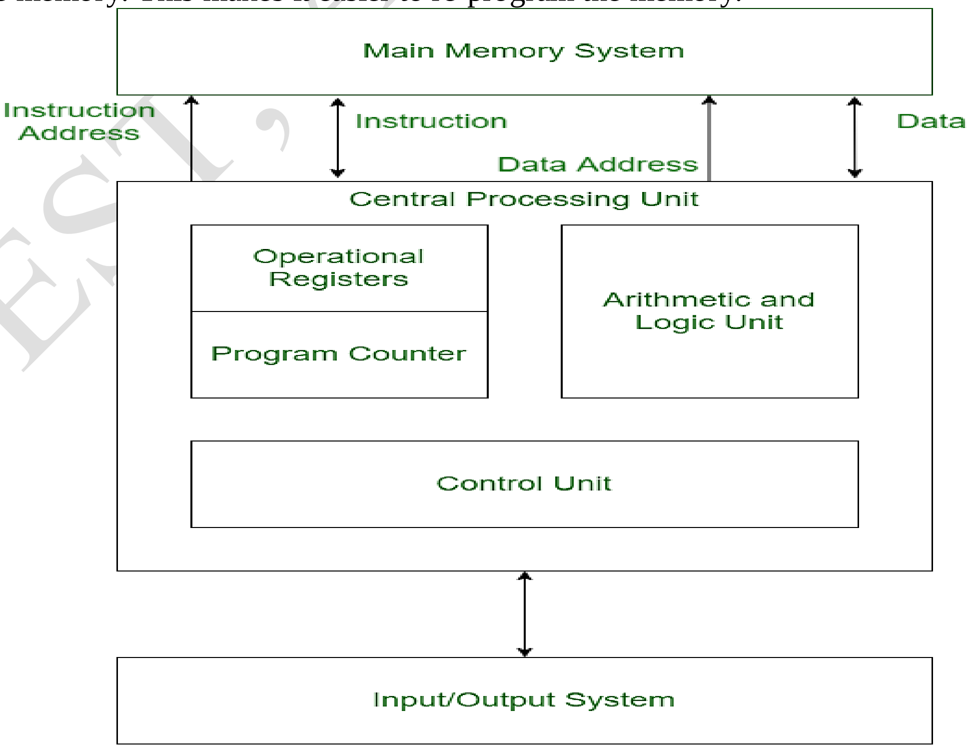
- Digital signal processing algorithms typically require a large number of mathematicaloperations to be performed quickly and repeatedly on a series of data samples.
- Signals (perhaps from audio or video sensors) are constantly converted from analog to digital, manipulated digitally, and then converted back to analog form.

3.Define FIR.

Ans- The Frequency-Domain FIR Filter block implements frequency-domain, fast Fourier transform(FFT)-based filtering to filter a streaming input signal. In the time domain, the filtering operation involves a convolution between the input and the impulse response of the finite impulse response (FIR) filter.

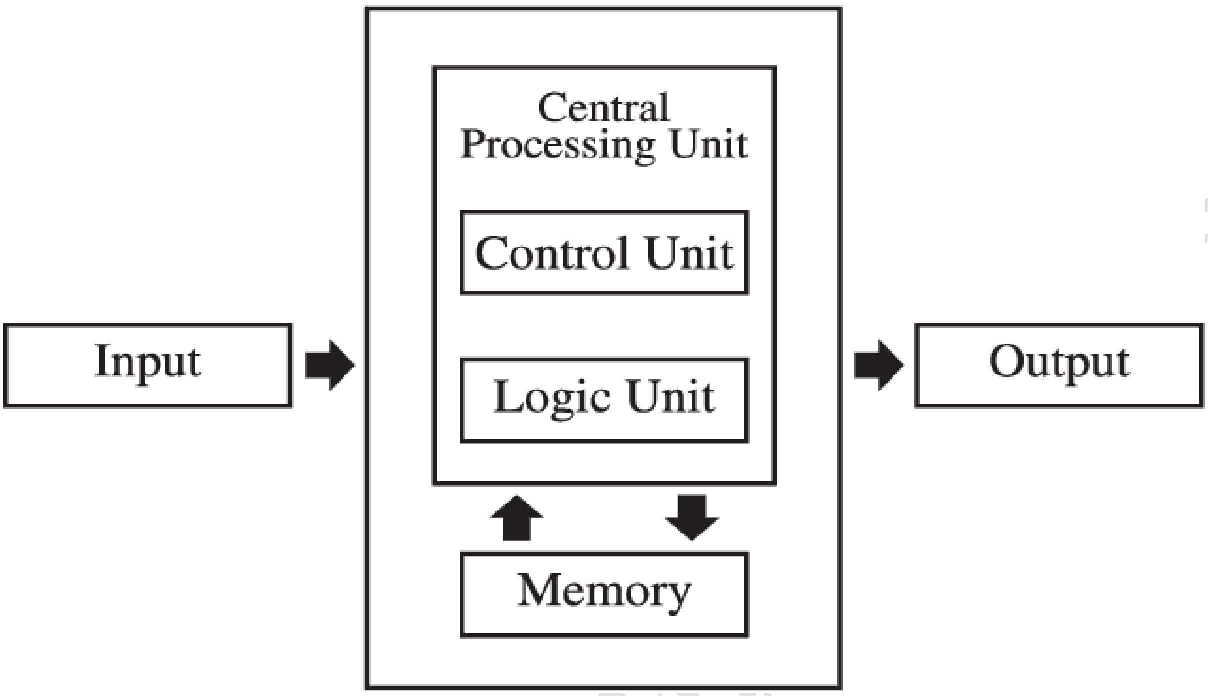
4.What is Harvard Architecture ?

Ans- Harvard architecture is used primarily for small embedded computers and signal processing. Commonly used within CPUs to handle the cache. Not only data but also instructions of programs are stored within the same memory. This makes it easier to re-program the memory.



5.What is Von Neumann Architecture ?

Ans- The von Neumann architecture—also known as the von Neumann model or Princeton architecture—is a computer architecture based on a 1945 description by John von Neumann andothers in the First Draft of a Report on the EDVAC.



POSSIBLE LONG TYPE QUESTIONS

1.Find the DFT of the sequence using DIF-FFT algorithm.

$$\boldsymbol{x(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}}$$

2.Explain the Radix-2 Algorithm. [2019(S-NEW)]

3.Write short note on DSP Architecture.

4.What are the different applications of FFT algorithm?

5.Write short note on FIR Filter.

6/What are the different FFT Algorithms? Explain each in detail? [2019(S-NEW)]