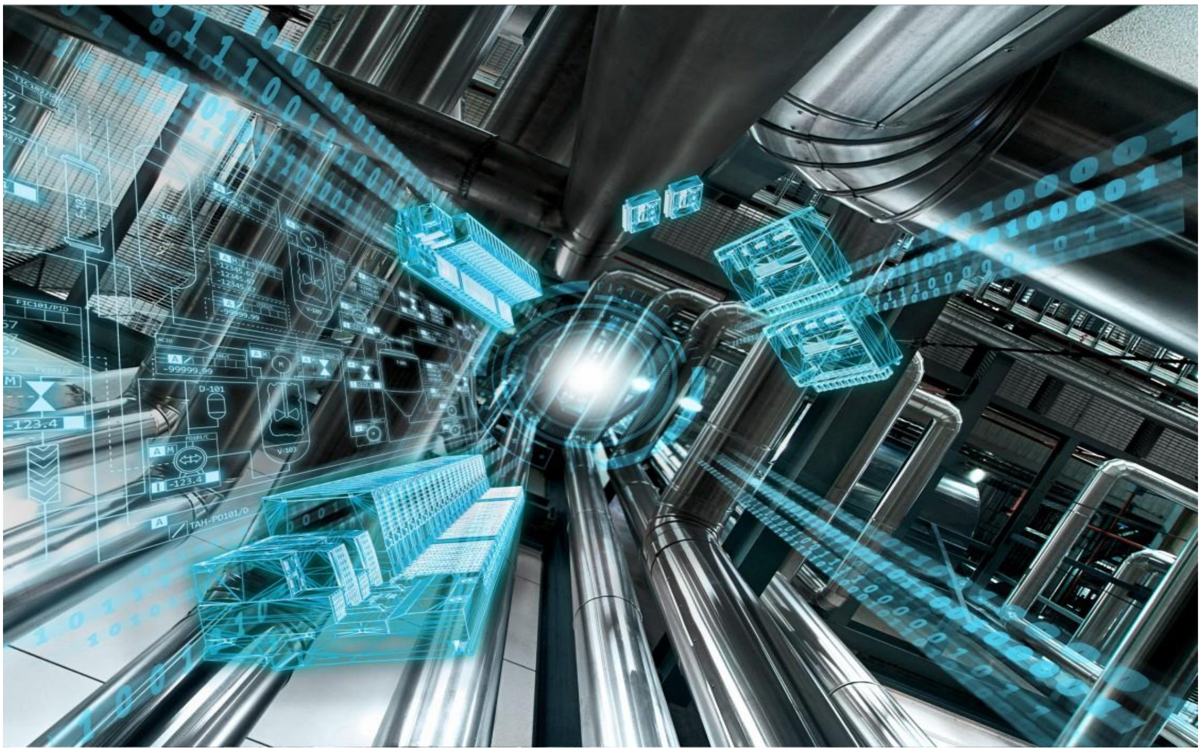
BHADRAK ENGINEERING SCHOOL & TECHNOLOGY (BEST), ASURALI, BHADRAK

Control Systems & Components (Th- 02)

(As per the 2020-21 syllabus of the SCTE&VT, Bhubaneswar, Odisha)



Sixth Semester

E&TC Engg.

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CONTROL SYSTEM ENGINEERING

CHAPTER-WISE DISTRIBUTION OF PERIODS & MARKS

Sl. No.	Chapter/ Unit No.	Topics	Periods as per Syllabus	Expected Marks
01	01	Fundamental of control system	05	08
02	02	TRANSFER FUNCTIONS	08	14
03	03	CONTROL SYSTEM COMPONENTS & MATHEMATICAL MODELLING OF PHYSICAL SYSTEM	05	08
04	04	BLOCK DIAGRAM & SIGNAL FLOW GRAPHS(SFG)	08	14
05	05	TIME DOMAIN ANALYSIS OF CONTROL SYSTEMS	08	13
06	06	FEEDBACKCHARACTERISTICS OF CONTROL SYSTEMS	06	10
07	07	STABILITY CONCEPT, & ROOT LOCUS METHOD	08	13
08	08	FREQUENCY-RESPONSE ANALYSIS&BODE PLOT	07	12
09	09	STATE VARIABLE ANALYSIS	05	08
TOTAL			60	100

CHAPTER NO.-01
FUNDAMENTAL CONTROL SYSTEM

Learning Objectives:

1.1 Classification of Control system
1.2 Open loop system & Closed loop system and its comparison
1.3 Effects of Feed back
1.4 Standard test Signals (Step, Ramp, Parabolic, Impulse Functions)
1.5 Servomechanism
1.6 Regulators (Regulating systems)

System:

- A system is a combination of physical components arranged in such a way that it gives a proper output to a given reference input or simple input.
- Proper output may or may not be desired.
- For Example: Fan with Blade

Control System:

- A control system is a combination of physical components arranged in such a way that it gives a
- desired output to given reference input. For Example: Fan without blade is not a system
- Fan with blade without regulator is a system
- Fan with blade with regulator is a control system

1.1 Classification of Control Systems:

Control System are two types

- Open loop control system
- Closed loop control system
  - Open loop control system in which the control action is independent of the output. Closed loop control system in which the control action is somehow dependent upon the output and are generally called as feedback Control Systems.

1.2 Open loop system and Closed Loop system and its comparison:

Open-loop control system:

- It is a control system where its control action only depends on input signal and does not depend on its output response as shown in Fig.1

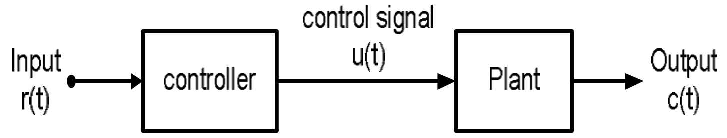


Fig.1 An open-loop system

Examples: traffic signal, washing machine, bread toaster, etc.

Advantages:

- Simple design and easy to construct
- Economical

- Easy for maintenance
- Highly stable operation

Dis-advantages:

- Not accurate and reliable when input or system parameters are variable in nature
- Recalibration of the parameters are required time to time

Closed-loop control system:

- It is a control system where its control action depends on both of its input signal and output response as shown in Fig. 2

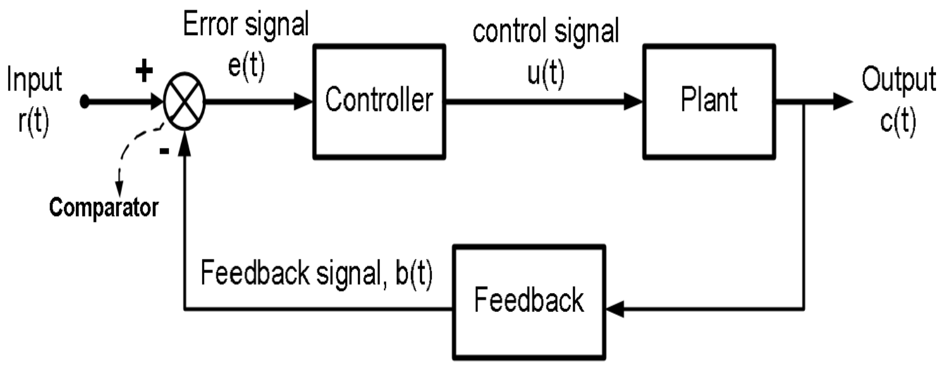


Fig-2

Examples: automatic electric iron, missile launcher, speed control of DC motor, etc.

Advantages:

- More accurate operation than that of open-loop control system.
- Can operate efficiently when input or system parameters are variable in nature.
- Less nonlinearity effect of these systems on output response.
- High bandwidth of operation
- There is facility of automation
- Time to time recalibration of the parameters is not required

Disadvantages:

- Complex design and difficult to construct
- Expensive than that of open-loop control system
- Complicate for maintenance
- Less stable operation than that of open-loop control system

Comparison between Open-loop and Closed-loop control systems:

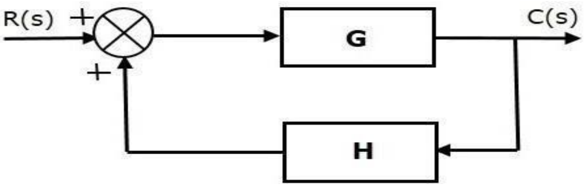
- It is a control system where its control action depends on both of its input signal and output response.

Sl. No.	Open-loop control systems	Closed-loop control systems
1	No feedback is given to the control system	Feedback is given to the control system
2	Cannot be intelligent	Intelligent controlling action

3	There is no possibility of undesirable system oscillation(hunting)	Closed loop control introduces the possibility of undesirable system oscillation(hunting)
4	The output will not very for a constant input, provided the system parameters remain unaltered	In the system the output may vary for a constant input, depending upon the feedback
5	System output variation due to variation in parameters of the system is greater and the output very in an uncontrolled way	System output variation due to variation in parameters of the system is less.
6	Error detection is not present	Error detection is present
7	Small bandwidth	Large bandwidth
8	More stable	Less stable or prone to instability
9	Affected by non-linearities	Not affected by non-linearities
10	Very sensitive in nature	Less sensitive to disturbances
11	Simple design	Complex design
12	Cheap	Costly

1.3 Effects of Feedback:

- If either the output or some part of the output is returned to the input side and utilized as partof the system input, then it is known as feedback. Feedback plays an important role in order to improve the performance of the control systems



- The effects of feedback are as follows:
- Gain is reduced by a factor $\frac{1}{1+G(S)H(S)}$.
- There is improvement in sensitivity.
- There is reduction of parameter variation by a factor $1 + G(s)H(s)$
- There may be reduction of stability.

1.4 Standard test Signals (Step, Ramp, Parabolic, Impulse Functions):

- The standard test signals are impulse, step, ramp and parabolic. These signals are used to know theperformance of the control systems using time response of the output.

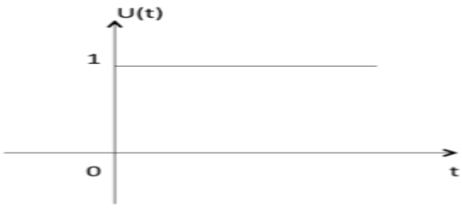
Step function:

- A unit step function the value of the function is zero for t< 0 and its value is A for ≥ 0.
- If A=1, the function r(t) = u(t) =1, and it known as unit step function.

Mathematically, it is given as

$$r(t) = 0 \; for \; t < 0$$
$$r(t) = A \; for \; t \geq 0$$

Following figure shows unit step signal

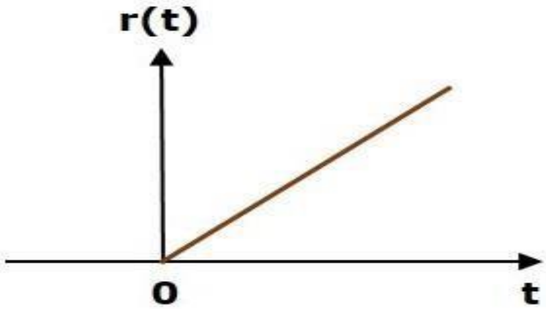


- A unit ramp signal, r(t) is defined as the value of ramp function is zero for t < 0 and after t ≥ 0,it nearly increases with time.

  Mathematically, it is given as

$$r(t) = 0\ for\ t < 0$$
$$r(t) = At\ for\ t \geq 0$$

Following figure shows unit ramp signal



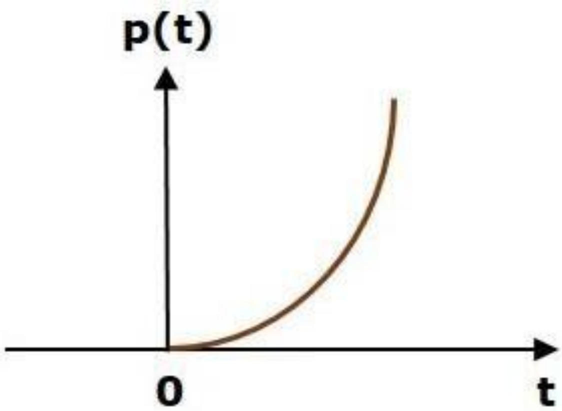
Parabolic function

- A unit parabolic signal, p(t) is defined as, the value of a parabolic function is zero for t < 0, and it is equal to $\frac{At^2}{2}\ for\ t \geq 0$.

  Mathematically, it is given as

$$p(t) = 0\ for\ t < 0$$
$$p(t) = \frac{At}{2}\ for\ t \geq 0$$

The following figure shows the unit parabolic signal.

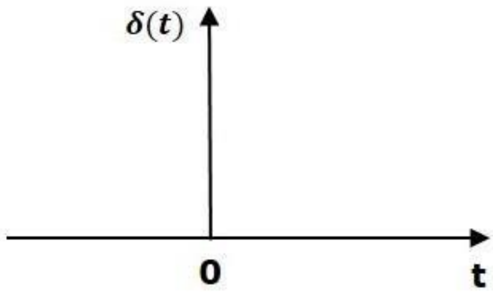


Unit Impulse Signal

- A unit impulse signal, δ(t) is defined as the width of the function is A and its height is $\frac{1}{A}$ will increase. In the limit t→0, A→0, $\frac{1}{A} \rightarrow \infty$. The pulse will be of infinite magnitude and it is termed as an impulse of magnitude unity. It is denoted by δ(t) and is shown in the figure below.

δ(t)=0 for t≠0

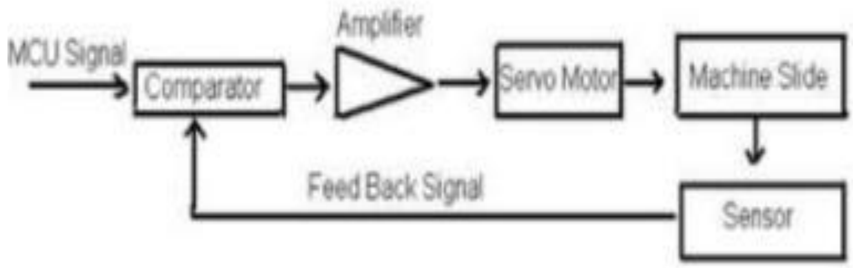
$$\delta(t) = 1, t = 0$$



The following figure shows unit impulse signal.

1.5 Servomechanism:

- A servo system primarily consists of three basic components – a-controlled device, an output sensor, a feedback system.
- This is an automatic closed loop control system. Here instead of controlling a device by applying the variable input signal, the device is controlled by a feedback signal generated by comparing output signal and reference input signal.
- When reference input signal or command signal is applied to the system, it is compared with output reference signal of the system produced by output sensor, and a third signal produced by a feedback system. This third signal acts as an input signal of controlled device.
- This input signal to the device presents as long as there is a logical difference between reference input signal and the output signal of the system.
- After the device achieves its desired output, there will be no longer the logical difference between reference input signal and reference output signal of the system. Then, the third signal produced by comparing theses above said signals will not remain enough to operate the device further and to produce a further output of the system until the next reference input signal or command signal is applied to the system.



POSSIBLE SHORT TYPE QUESTIONS WITH ANSWERS

1. **Name three applications of control systems**.

Ans- Guided missiles, Fighter plane stability, Satellitetracking antenna.

2. **What is meant by System?**

Ans-When the number of elements connected performs a specific function then the group of elements is said to constitute a system or interconnection of various components for a specific task is called system. Example: Automobile.

3. **What is meant by Control System?**

Ans-Any set of mechanical or electronic devices that manages, regulates or commands the behavior of the system using control loop is called the Control System. It can range from a small controlling device to alarge industrial controlling device which is used for controlling processes or machines.

4. **What is open loop and control loop systems?**

Ans-Open loop control System: An open-loop control system is a system in which the control action is independent of the desired output signal. Examples: Automatic washing machine, Immersion rod. Closed loop control System: A closed-loop control system is a system in which control action is dependent on the desired output. Examples: Automatic electric iron, Servo voltage stabilizer, an air conditioner.

5. **What are the necessary components of the feedback control system?**

Ans-The processing system (open loop system), feedback path element, an error detector, and controller arethe necessary components of the feedback control system.

6. **What is the feedback in the control system?**

Ans-When the input is fed to the system and the output received is sampled, and the proportional signal is then fed back to the input for automatic correction of the error for further processing to get the desired output is called as feedback in control system.

7. **Write the effect of negative feedback in control system. [S-23]**

   Ans: gain is reduced by a factor

$$\frac{1}{1+G(s)H(s)}$$

   Negative feedback in a control system reduces the overall gain and increases stability. It reduces the sensitivity of output to input variation, distortion, and noise reduction.

8. **Give two disadvantages of closed loop control over open loop control system. [S-24]**

   Ans: Higher cost: Due to the additional components and complexity, closed loop systems are generally more expensive than open loop systems.

   Response time: Closed loop systems may have a slight delay in response due to the feedback loop, which can be a critical factor in time-sensitive applications.

POSSIBLE LONG TYPE QUESTIONS

1. What are the advantages and disadvantages of open loop control system?
2. What are the advantages and disadvantages of closed-loop control System?
3. Name the three major design criteria for control systems.
4. Explain the effect of feedback in a closed loop control system. [S-22]
5. State difference between open loop and closed loop control system. [S-23,24]

CHAPTER NO.-02
TRANSFER FUNCTIONS

Learning Objectives:

2.1 Transfer Function of a system & Impulse response,
2.2 Properties, Advantages & Disadvantages of Transfer Function
2.3 Poles & Zeroes of transfer Function
2.4 Representation of poles & Zero on the s-plane
2.5 Simple problems of transfer function of network

2.1 Transfer Function & Impulse response:(TF)

The transfer function of a control system is defined as the ratio of the Laplace transform of the output to Laplace transform of the input assumingall initial conditions to be zero

Or

Transfer function is a mathematical model of a system

Or

It is defined as Laplace transform of impulse response of the system by taking zero initial condition. Thus, the cause-and-effect relationship between the outputand input is related to each other through a transfer function.

In a Laplace Transform, if the input is represented by R(s) and the output isrepresented by C(s), then the transfer function will be:

$$G(s) = \frac{C(s)}{R(S)} => R(s) \cdot G(s) = C(s)$$

That is, the transfer function of the system multiplied by the input function gives theoutput function of the system.

$$G(s) = \frac{C(s)}{R(s)}$$

2.2 Properties, Advantages & Disadvantages of Transfer Function

Properties of transfer function (T.F)

The properties of transfer function are given below:

1. The ratio of Laplace transforms of output to Laplace transform of input assuming allinitial conditions to be zero.
2. The transfer function of a system is the Laplace transform of its impulse responseunder assumption

of zero initial conditions.

3. Replacing 's' variable with linear operation $D = \frac{d}{dt}$ in transfer function of a system,the differential equation of the system can be obtained.
4. The transfer function of a system does not depend on the inputs to the system.
5. The system poles and zeros can be determined from its transfer function.
6. Stability can be found from ...characteristics equations.

Advantages of Transfer function:

1. If transfer function of a system is known, the response of the system to any input can be determined very easily.
2. A transfer function is a mathematical model and it gives the gain of the system.
3. Since it involves the Laplace transform, the terms are simple algebraic expressions and no differential terms are present.
4. Poles and zeroes of a system can be determined from the knowledge of the transfer function of the system.

Disadvantages of Transfer function

1. Transfer function does not take into account the initial conditions.
2. The transfer function can be defined for linear systems only.
3. No inferences can be drawn about the physical structure of the system.

2.3 Poles & Zeroes of transfer Function

Definition of zeros of the system: Zeros of the system are calculated by putting magnitude of transfer function equal to zeros.

Definition of poles of the system: Poles of the system are calculated by putting magnitude of transfer function equal to infinite

Transfer function of a control system can also be represented as-

$$G(s) = \frac{C(s)}{R(s)}$$

$$= \frac{C_0S^n + C_1S^{n-1} + C_2S^{n-2} + \cdots \cdot\cdot + C_{n-1}S + C_n}{R_0S^m + R_1S^{m-1} + R_2S^{m-2} + \cdots\cdots + R_{m-1} + R_m}$$

$$= \frac{K(s-z_1)(s-z_2)(s-z_3)\cdots\cdots(s-z_n)}{(s-p_1)(s-P_2)(s-p_3)\cdots\cdots(S-p_m)}$$

Where K is known as the gain factor of the transfer function.

- Now in the above function if s = z_1, or s = z_2, or s = $z_3 \ldots$. s = z_n, the value of transfer function becomes zero. These $z_{1,}z_2, z_3 \ldots z_n$, are roots of the numerator polynomial. As for these roots the numerator polynomial, the transfer function becomes zero, these roots are called zeros of the transfer function.
- Now, if s = p_1 or s = p_2 or s = $p_3 \ldots$. s = p_m, the value of transfer function becomes infinite. Thus,

the roots of denominator are called the poles of the function.

2.4.Simple problems of transfer function of network:

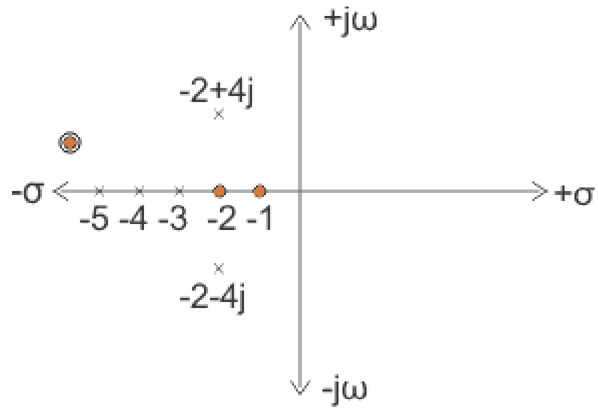
Example-1

Find out and sketch pole and zero of the given transfer function

$$G(s) = \frac{(s+1)(s+2)}{(s+3)(+4)(s+5)(s+2-4j)(s+2+4j)}$$

Solution

The zeros of the function are, -1, -2 and the poles of the functions are -3, -4, -5, -2 + 4j, -2 – 4j.Here n = 2 and m = 5, as n < m and m – n = 3, the function will have 3 zeros at s→ ∞, The poles and zeros are plotted in the figure below



Example-2

Find out and sketch pole and zero of the given transfer function

$$G(s) = \frac{(s-2)(s+5)(s+8)}{S(s+1)(s+6)(S+9)(s+1-j3)(s+1+j3)}$$

Solution

In the above transfer function, if the value of numerator is zero, then

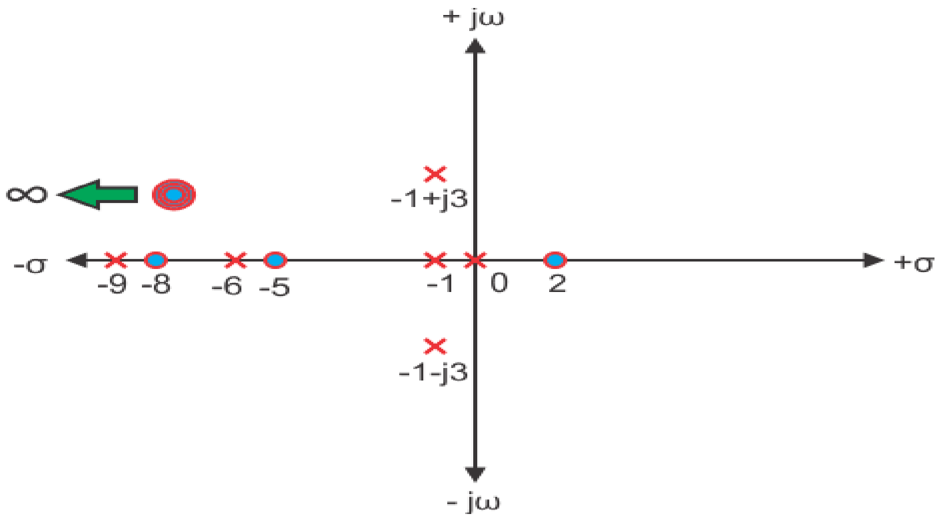
$$(s-2)(s+5)(s+8) = 0$$
$$s = 2, -5, -8$$

These are the location of zeros of the function.

Similarly, in the above transfer function, if the value of denominator is zero,then

$$s(s+1)(s+6)(s+9)(s+1-j3)(s+1+j3) = 0$$
$$s = 0, -1, -6, -1+j3, -1-j3$$

These are the location of poles of the function.



As the number of zeros should be equal to number of poles, the remaining three zeros are located at s →∞.

2.5Mathematical Modeling of Electrical System (R,L,C analogous system)

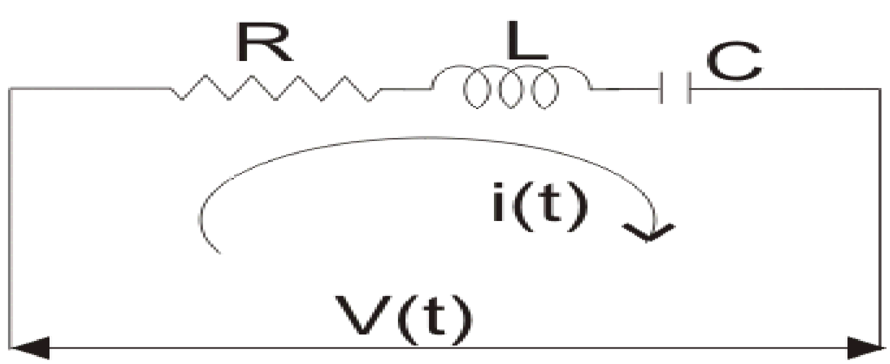
In an electrical type of system, we have three variables –

1. Voltage which is represented by ‘V’.
2. Current which is represented by ‘I’.
3. Charge which is represented by ‘Q’.

And also we have three parameters which are active and passive components:

1. Resistance which is represented by ‘R’.
2. Capacitance which is represented by ‘C’.
3. Inductance which is represented by ‘L’.

- Now we are in condition to derive analogy between electrical and mechanical typesof systems. There are two types of analogies and they are written below:
- Force Voltage Analogy: In order to understand this type of analogy, let us consider acircuit which consists of series combination of resistor, inductor and capacitor.



A voltage V is connected in series with these elements as shown in the circuit diagram. Now from the circuit diagram and with the help of KVL equation we writethe expression for voltage in terms of charge, resistance, capacitor and inductor as,

$$V = L\frac{dt^2}{d^2q} + R\frac{dt}{dq} + \frac{q}{c}$$

POSSIBLE SHORT TYPE QUESTIONS WITH ANSWERS

1. **What mathematical model permits easy interconnection of physical systems?**

Ans: Transfer Function

2. **To what classification of systems can the transfer function be best applied?**

Ans: Linear time invariant

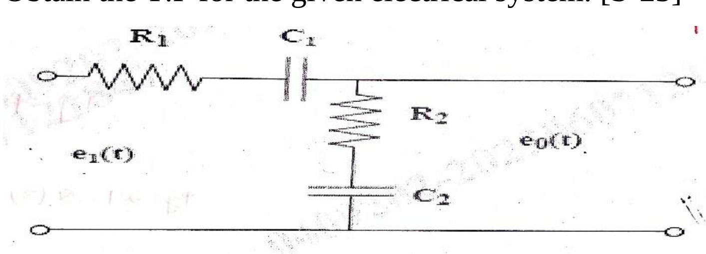
3. **What transformation turns the solution of differential equations into algebraic manipulations?**

Ans: Laplace

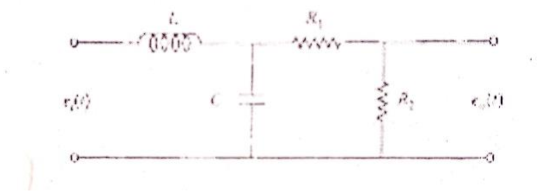
4. **What is transfer function? [S-19,23,24]**
5. **Ans:** A transfer function (also known as system function or network function) of a system, sub-system, or component is a mathematical function that models the system's output for each possible input. It is widely used in electronic engineering tools like circuit simulators and control systems.
6. **Define signal flow graph (SFG) & write two properties of SFG. [S-24]**
   Ans: A signal-flow graph or signal-flowgraph (SFG), is a specialized flow graph, a directed graph in which nodes represent system variables, and branches (edges, arcs, or arrows) represent functional connections.
   Properties: The important properties of the SFG are as follows: SFG's are applicable only to linear systems. The equations for which an SFG drawn must be algebraic equations in the form of cause-and-effect relationship. Nodes are used to represent variables.
7. **How do you mean by order and type of a system? [S-23,24]**
   Ans: The total number of poles of the closed-loop system gives the order of the system. The type of the system can be defined as the number of poles located exactly at s=0 of the open-loop transfer function.

POSSIBLE LONG TYPE QUESTIONS

1. What assumption is made concerning initial conditions when dealing with transferfunctions?
2. What do we call the mechanical equations written in order to evaluate the transferfunction?
3. Why do transfer functions for mechanical networks look identical to transferfunctions for electrical networks?
4. Obtain the T.F for the given electrical system. [S-23]



5. Obtain the T.F for the given electrical system. [S-24]



CHAPTER NO-03
CONTROL SYSTEM COMPONENTS & MATHEMATICAL MODELING OF PHYSICAL SYSTEM

Learning Objectives:

3.1 Components of Control System
3.2 Potentiometer, Synchros, Diode modulator & demodulator ,
3.3 DC motors, AC Servomotors
3.4 Modelling of Electrical Systems (R, L, C, Analogous systems)

3.1 Components of control system

- Transducer which is the first major component in a control system is a device that senses the output in one form and convert it into another form, the sensing may be temperature, pressure, position, and conversion is generally into electrical

3.2 Gyroscope, Synchro's, Tachometer, DC servomotors, Ac servomotors

Gyroscope:

- It is an instrument used in space ships and aircrafts. The input is the angular velocityand the output is the angular displacement. The action of gyroscope is based on following principles.
- If no external torque acts on it the spinning wheel maintains the direction of its spin axis in space and this type of spinning is known asfree gyro type.
- If torque is applied to an axis inclined to the spin axis of a wheel the wheel rotate about an axis at an angle 90^0 to both the spin axis as well as the input torque axis. this type of rotation is known as precision type.

Synchro's:

Definition:

- The Synchro's is a type of transducer which transforms the angular position of the shaft intoan electric signal. It is used as an error detector and as a rotary position sensor. The error occurs in the system because of the misalignment of the shaft. The transmitter and the control transformer are the two main parts of the synchro's

Synchro's System Types

The synchro system is of two types. They are

- Control Type Synchro.
- Torque Transmission Type Synchro.

Torque Transmission Type Synchro's

- This type of synchro's has small output torque, and hence they are used for running the very light loadlike a pointer. The control type Synchro is used for driving the large loads.

Control Type Synchro's System

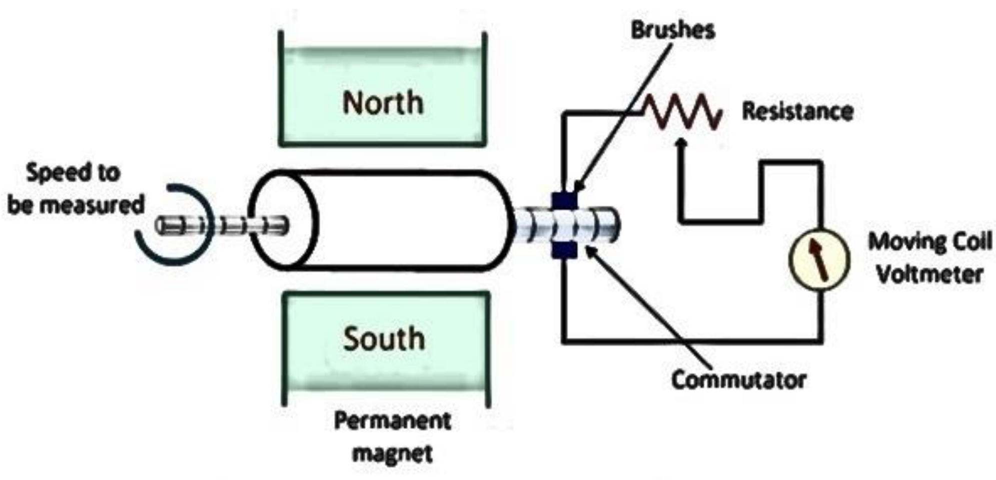
- The controls synchro's is used for error detection in positional control systems. Their systems consist twounits. They are
    - Synchro Transmitter
    - Synchro Receiver

Tachometer

- The tachometer uses for measuring the rotational speed or angular velocity of the machine which is coupled to it. It works on the principle of relative motion between the magnetic field and shaft of the coupled device. The relative motion induces the EMF in the coil which is placed between the constant magnetic field of the permanent magnet. The develops EMF is directly proportional to the speed of the shaft.
- Mechanical and electrical are the two types of the tachometer. The mechanical tachometer measures the speed of shaft regarding revolution per minutes.
- The electrical tachometer converts the angular velocity into an electrical voltage. The electrical tachometer has more advantages over the mechanical tachometer. Thus, it is mostly used for measuringthe rotational speed of the shaft. Depends on the natures of the induced voltage the electrical tachometer is categorized into two types.
    - AC Tachometer Generator
    - DC Tachometer Generator

DC Tachometer Generator

- Permanent magnet, armature, commutator, brushes, variable resistor, and the moving coil voltmeter are the main parts of the DC tachometer generator. The machine whose speed is to be measured is coupled with the shaft of the DC tachometer generator.



- The DC tachometer works on the principle that when the closed conductor moves in the magnetic field,EMF induces in the conductor. The magnitude of the induces emf depends on the flux link with the conductor and the speed of the shaft

.

- Armature of the DC generator revolves between the constant field of the permanent magnet. The rotation induces the emf in the coil. The magnitude of the induced emf is proportional to the shaftspeed.
- The commutator converts the alternating current of the armature coil to the direct current with the helpof the brushes. The moving coil voltmeter measures the induced emf. The polarity of the induces voltagedetermines the direction of motion of the shaft. The resistance is connected in series with the voltmeter for controlling the heavy current of the armature.
- The emf induces in the dc tachometer generator is given as,

$$E = \frac{\varphi PN}{60} \times \frac{Z}{a}, V$$

Where,
E = generated voltage in volt
Φ = flux per poles in, WeberP= number of poles
N = speed in revolution per minutes
Z = the number of the conductor in armature windings.
a = number of the parallel path in the armature windings.

$$E \propto N$$
$$E = KN$$
K=Constant,$\frac{\varphi P}{60} \times \frac{Z}{a}$

Advantages of the DC Tachometer Generator:

The following are the advantages of the DC Tachometer.

- The polarity of the induces voltages indicates the direction of rotation of the shaft.
- The conventional DC type voltmeter is used for measuring the induces voltage.

Disadvantages of DC Tachometer Generator:

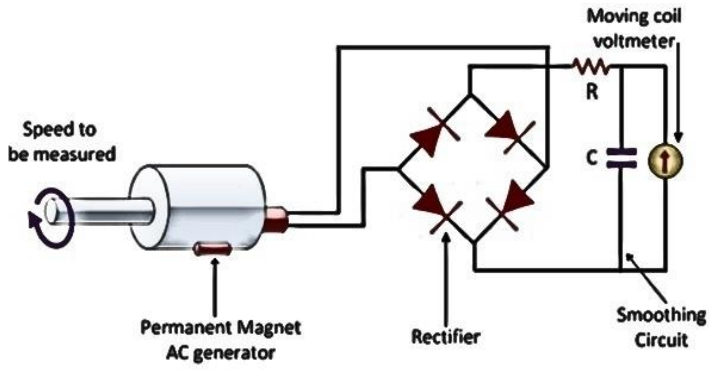
- The commutator and brushes require the periodic maintenance.
- The output resistance of the DC tachometer is kept high as compared to the input resistance. If the large current is induced in the armature conductor, the constant field ofthe permanent magnet will be distorted.

AC Tachometer Generator

- The DC tachometer generator uses the commutator and brushes which have many disadvantages. TheAC tachometer generator designs for reducing the problems. The AC tachometer has stationary armature and rotating magnetic field. Thus, the commutator and brushes are absent in AC tachometergenerator

- The rotating magnetic field induces the EMF in the stationary coil of the stator. The amplitude andfrequency of the induced emf are equivalent to the speed of the shaft. Thus, either amplitude or frequency is used for measuring the angular velocity.

- The below mention circuit is used for measuring the speed of the rotor by considering the amplitude ofthe induced voltage. The induces voltages are rectified and then passes to the capacitor filter for smoothening the ripples of rectified voltages.



A.C Tachometer Generator

Advantages

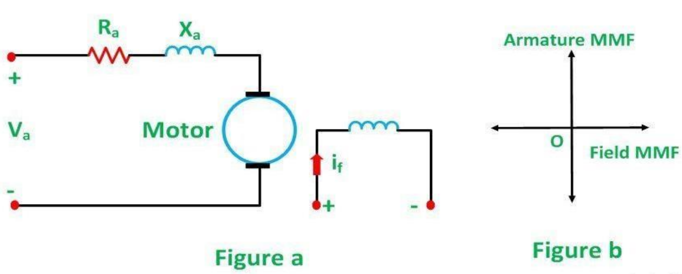
- The drag cup Tacho-generator generates the ripple free output voltage.
- The cost of the generator is also very less.

Disadvantages:

- The nonlinear relationship obtains between the output voltage and input speed when the rotor rotates at high speed.

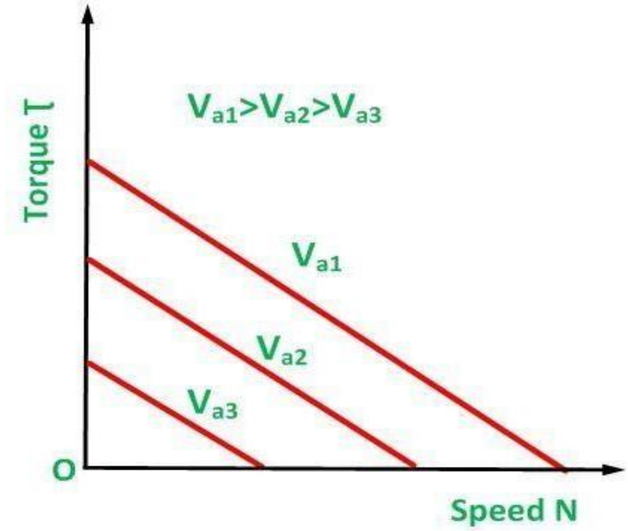
DC servomotor

- DC Servo Motors are separately excited DC motor or permanent magnet DC motors. The figure (a) shows the connection of Separately Excited DC Servo motor and the figure (b) shows the armature MMFand the excitation field MMF in quadrature in a DC machine.



- This provides a fast torque response because torque and flux are decoupled. Therefore, a small change in the armature voltage or current brings a significant shift in the position or speed of the rotor. Most ofthe high-power servo motors are mainly DC.

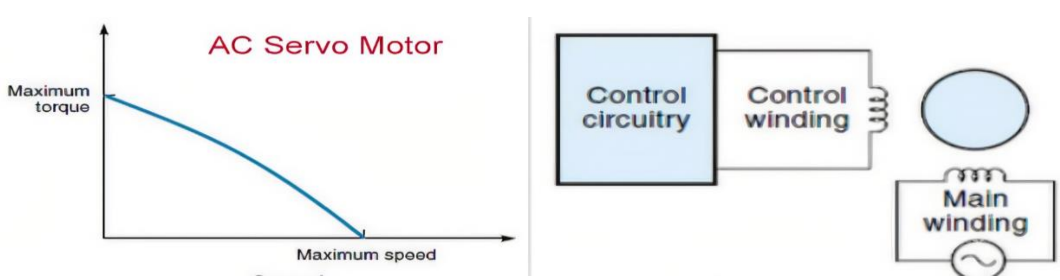
- The Torque-Speed Characteristics of the Motor is shown below



- As from the above characteristics, it is seen that the slope is negative. Thus, a negative slope provides viscous damping for the servo drive system.

AC Servo Motor:

- AC servo motors are basically two-phase squirrel cage induction motors and are used for low power applications. Nowadays, three phase squirrel cage induction motors have been modified such that they can be used in high power servo systems.
- The main difference between a standard split-phase induction motor and AC motor is that the squirrel cage rotor of a servo motor has made with thinner conducting bars, so that the motor resistance is higher.
- Based on the construction there are two distinct types of AC servo motors, they are synchronous type AC servo motor and induction type AC servo motor.



POSSIBLE SHORT TYPE QUESTIONS WITH ANSWERS

1. **Define Gyroscope**

Ans-It is an instrument used in spaceship and aircrafts. the input is the angular velocity and the output is the angular displacement

2. **Define tachometer?**

Ans-It is a miniature low voltage generator where the output voltage of generator is given by $Eg=kf\omega$

3. **Define synchro?**

Ans-It is an electromechanical device that produce an output voltage depending on the angular position of rotor and not on rotor speed

POSSIBLE LONG TYPE QUESTIONS

1.Explain synchro transmitter.

2.Explain synchro receiver.

3.Explain with diagram dc and ac servo motor.

4. Describe the construction & working principle of synchro's also explain how it is used in servo application. [S-23,24]

5. describe with block diagram the working of armature-controlled DC motor as a control system. [S-23,24]

CHAPTER NO.-04
BLOCK DIAGRAM AND SIGNAL FLOW GRAPHS(SFG)

Learning Objectives:

4.1 Definition of Basic Elements of a Block Diagram
4.2 Canonical Form of Closed loop Systems
4.3 Rules for Block diagram reduction
4.4 Procedure for of Reduction of Block Diagram
4.5 Simple Problem for equivalent transfer function
4.6 Basic Definition in Signal Flow Graph & properties
4.7 Mason 's Gain formula
4.8 Steps for solving Signal Flow graph
4.9 simple problems in signal flow graph for network

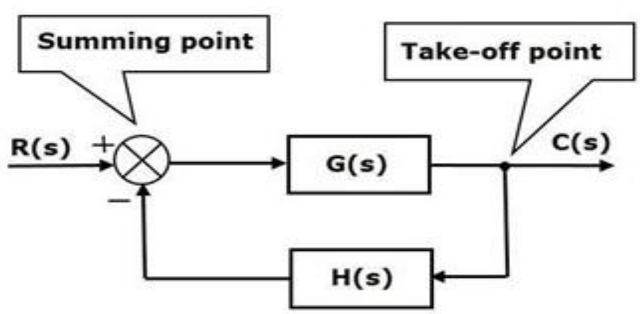
4.1 Definition: Basic Elements of Block Diagram:

Definition Block Diagram:

- Block diagram is a method to represent a system with the help of blocks.
- Some elements are used in order to represent the block diagram of a system mainly the elements are error detector and rectangular blocks.
- An error detector simply adds or subtract to signals and gives an error signal
- The rectangular block multiplies the input given to it with the gain of the rectangular blocks and gives output.

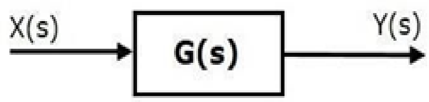
Basic elements of block diagram:

- The basic elements of a block diagram are a block, the summing point and the take-off point. Let us consider the block diagram of a closed loop control system as shown in the following figure to identify these elements.



- The above block diagram consists of two blocks having transfer functions G(s) and H(s). It is alsohaving one summing point and one take-off point. Arrows indicate the direction of the flow of signals. Let us now discuss these elements one by one.

Block:



- The transfer function of a component is represented by a block. Block has single input and single output. The following figure shows a block having input X(s), output Y(s) and the transfer function G(s).
- Transfer Function, $G(s) = Y(s)/X(s)$

$$\Rightarrow Y(s) = G(s)X(s)$$

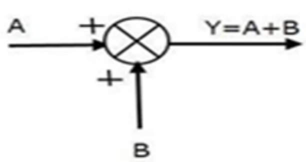
Output of the block is obtained by multiplying transfer function of the block with input.

Summing Point:

- The summing point is represented with a circle having cross (X) inside it. It has two or more inputs and single output. It produces the algebraic sum of the inputs. It also performs the summation or subtraction or combination of summation and subtraction of the inputs based on the polarity of the inputs. Let us see these three operations one by one.

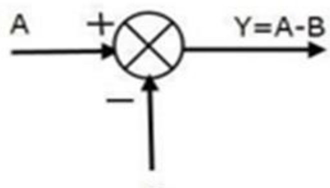
- The following figure shows the summing point with two inputs (A, B) and one output (Y). Here, the inputs A and B have a positive sign. So, the summing point produces the output, Y sum of A and B.

i.e., Y = A + B.



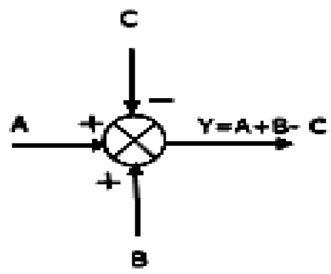
- The following figure shows the summing point with two inputs (A, B) and one output (Y). Here, the inputs A and B are having opposite signs, i.e., A is having positive sign and B is having negativesign. So, the summing point produces the output Y as the difference of A and B.

i.e., $Y = A + (-B) = A - B$



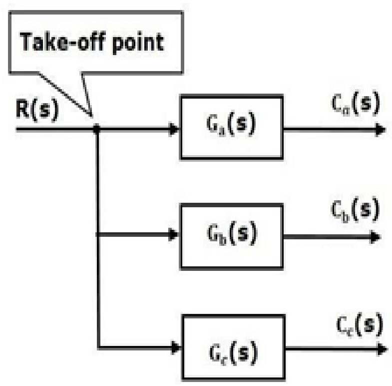
The following figure shows the summing point with three inputs (A, B, C) and one output (Y). Here, the inputs A and B are having positive signs and C is having a negative sign. So, the summing point produces the output Y

i.e., $Y = A + B + (-C) = A + B - C$

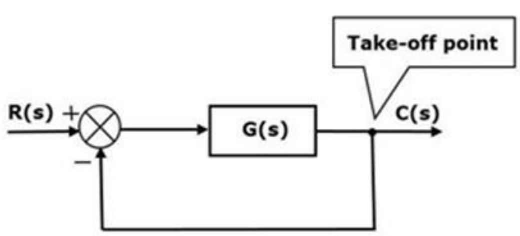


Take off point:

- The take-off point is a point from which the same input signal can be passed through more than one branch. That means with the help of take-off point, we can apply the same input to one or more blocks, summing points.
- In the following figure, the take-off point is used to connect the same input, R(s) to two more blocks.

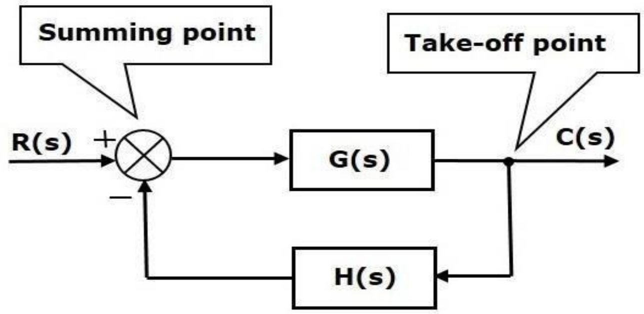


- In the following figure, the take-off point is used to connect the output C(s), as one of theinputs to the summing point.



- Block diagram algebra is nothing but the algebra involved with the basic elements of the block diagram. This algebra deals with the pictorial representation of algebraic equations.

4.2 Canonical form of closed loop system:



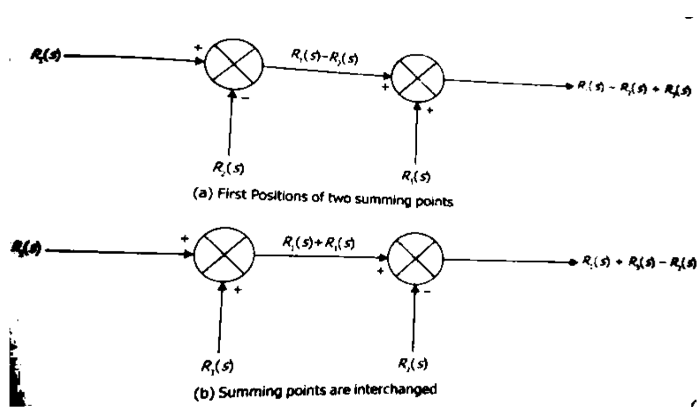
- This fig shows a block diagram which consist of forward path having one block, feedback path having one block, take off point and summing point.it represents a canonical form of close loop system(s) Laplace transform of reference input(s) is the Laplace transform of controlled output c(t), E(s) is the Laplace transform of error signal e(t),B(s) is the Laplace transform of feedback signal b(t).c(s) is the equivalent forward path transfer function(s) equivalent feedbackpath transfer function.

4.3: Rules Block Diagram Reduction Rules

Any complicated system can be brought into simple form by reduction of block diagram. The following rules are used in block diagram reduction.

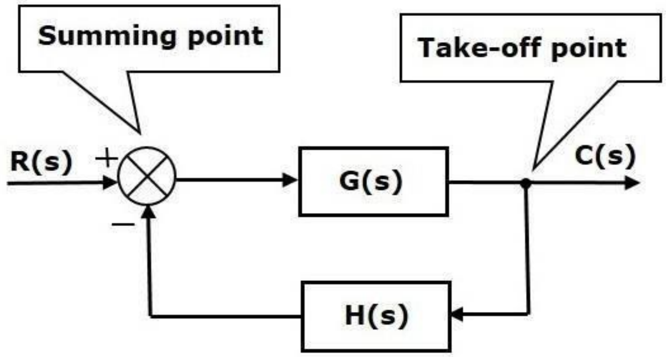
Rule 1: Associative law

- In fig.(a) two summing points have been taken into account in the 1st case the output is $R_1(s) - R_2(s) + R_3(S)$.in fig b the position of summing point is interchanged. the output is $R_1(s) + R_3(s) - R_2(s)$. From figs.(a) & (b) we have$R_1(s) - R_2(s) + R_3(s) = R_1(s) + R_3(s) - R_2(s)$
- If any block is present in between the summing Point ,by interchanging the summing points, it can be Shown that output will not.



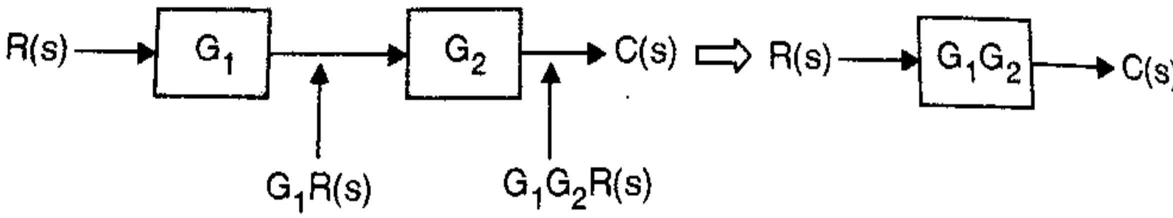
(a) First Positions of two summing points

(b) Summing points are interchanged



Rule 2: Blocks in Series/cascade

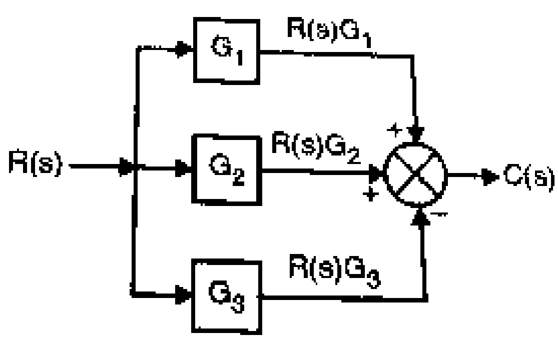
- Any finite specific number of blocks arranged in series can be combined together by multiplication asshown below:

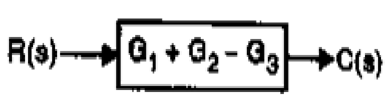


- The above blocks shown can be combined together and replaced with single block as Output $C(s) = G_1 \times G_2 \times R(s)$
- If there is a take-off point or summing point between the blocks, the blocks cannot be said to be in cascade/series. (The take-off/summing point has to be shifted before or after the block using anotherrule)

Rule 3:Blocks In Parallel

- When the blocks are connected in parallel combination, they get added algebraically (considering thesign of the signal) this can be combined as (refer both the diagrams)



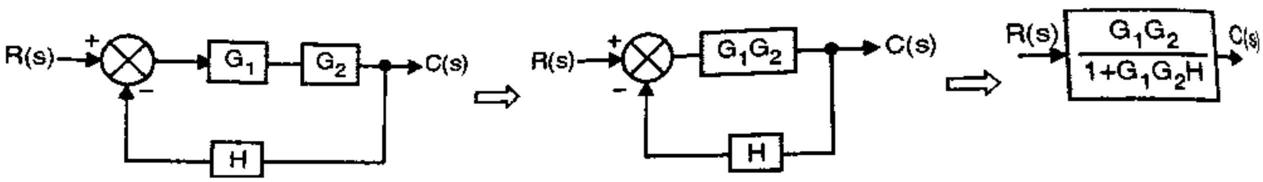


- The above blocks can be replaced with a single block as

$$C(s) = R(s)G_1 + R(s)G_2 - R(s)G_3$$
$$C(s) = R(s)(G_1 + G_2 - G_3)$$

- If any summing point/take-off point is present in between the blocks, then that has to be shifted first. (In aparallel arrangement, the direction of signal flow must be in the same direction through all the blocks).
- It is always better to avoid shifting the take-off point after the summing point.

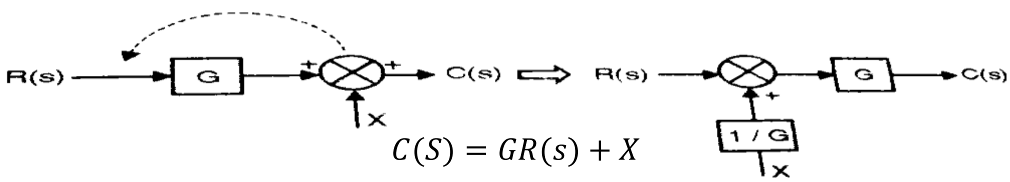
Rule 4: Elimination of feedback Loop



- We can use Closed loop transfer function to eliminate the feedback loop present.(Always remember for applying this method the direction of flow of signals should be in opposite direction, otherwise, if they are in the same direction, then we need to apply parallel reduction technique discussed above) Now consider the application of the above three rules together and refer to the block diagram above.

Rule 5: Shifting of a Summing Point before a block

- When we shift the summing point before a block, we need to do the transformation in order to achievethe same result. Please refer to the diagram below:



$$C(S) = GR(s) + X$$

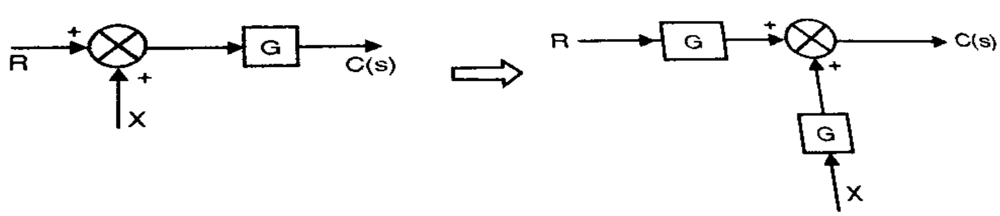
After shifting the summing point, we will get

$C(s) = \left[R + \left(\frac{X}{G}\right)\right] G = GR + X$ which is same as output in the first case.

- Hence to shift a summing point before a block, we need o to add another block of transfer function '$\frac{1}{G}$'. Before the summing point as shown in figure.

Rule 6: Shifting of the Summing Point after a block

When we generally shift the summing point after any block, we required to do the transformation to attain the same (required) result. Please refer the below diagram.



$$C(s) = (R + X)G$$

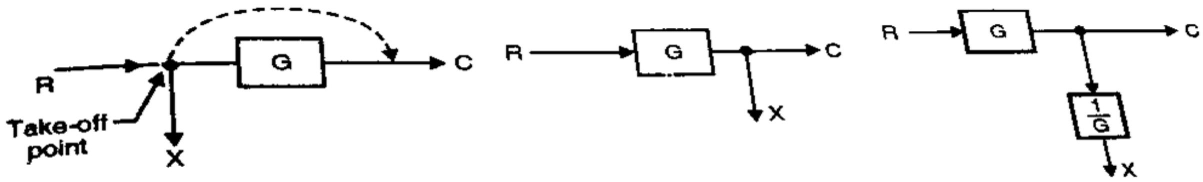
After shifting the summing point, we will get

$$C(s) = (R + X)\, G = GR + XG \text{ which is same as output in the first case.}$$

- Hence to shift a summing point before a block, we need to add another block having the same transferfunction at the summing point as shown in fig.

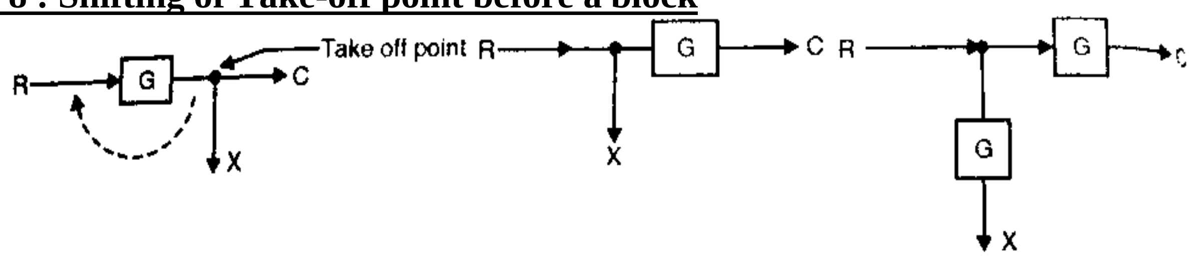
Rule 7:Shifting of Take-off point after a block

- Here we want to shift the take – off point after a block, as shown in the diagram



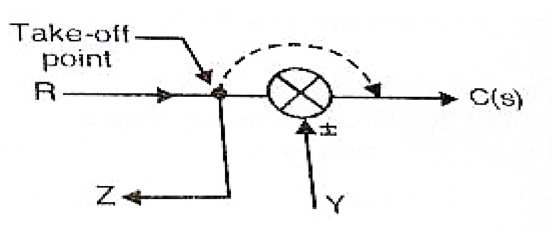
- Here we have X = R and C = RG (initially).In order to achieve this, we need to add a block of transfer function '$\frac{1}{G}$' in series with signal taking offfrom that point

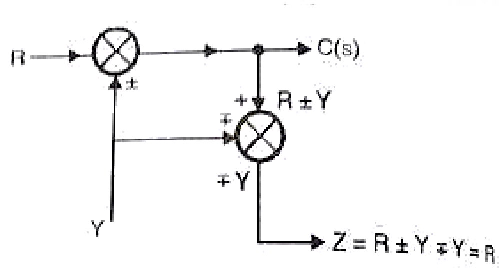
Rule 8 : Shifting of Take-off point before a block



- Here we want to shift the take – off point before a block, as shown in the diagram
- Here we have X = R and C = RG (initially)
- In order to achieve this, we need to add a block of transfer function 'G' in series with X signal taking offfrom that point.

Rule 9: Shifting a Take-off point after a Summing Point





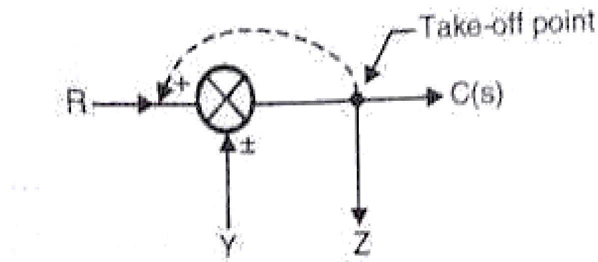
- It can be transformed to (refer both the diagrams). Before shifting take-off point, initially, we have:

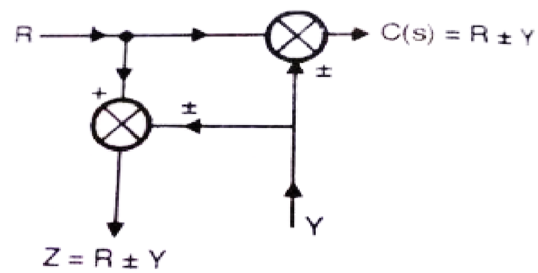
$$C(s)=R \pm Y$$
$$\text{and } Z = R \pm Y \text{ (initially)}$$

- Hence if we want to shift a take-off point after a summing point, one more summing point needs to beadded in series with take-off point

Rule 10: Shifting a take-off point before a summing point

- Suppose if we want to shift take-off point before a summing point, then initially we have $C(s)=R \pm Y$.and $Z = R \pm Y$ (initially). this can be transformed to (refer both the diagrams). In order to satisfy this condition, we need to add a summing point in series with the take-off point





4.4: Procedure for reduction of block diagram

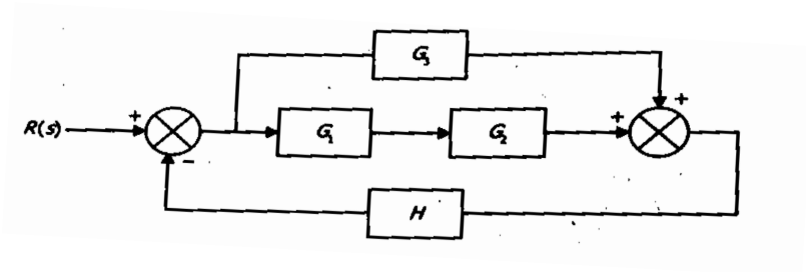
- Step-1-reduce the cascade blocks
- Step-2-reduce the parallel blocks
- Step-3-Reduce the internal feedback loops
- Step-4-shift take off point towards right and summing point towards left
- Step-5-repeat step 1 and step 4 until the simple form is obtained
- Step-6-find transfer function of the overall system using the formula C(s)/R(s)

Procedure for multiple input

- Step-1-Here reduce all but one input is zero. Find resultant output
- Step-2- Reduce step 1 until all input is covered.
- Step-3-Find the resultant output by superposition

4.5 Simple Problem for equivalent transfer function:

Example: Determine the ratio C(s)/R(s)of the block diagram shown in fig.



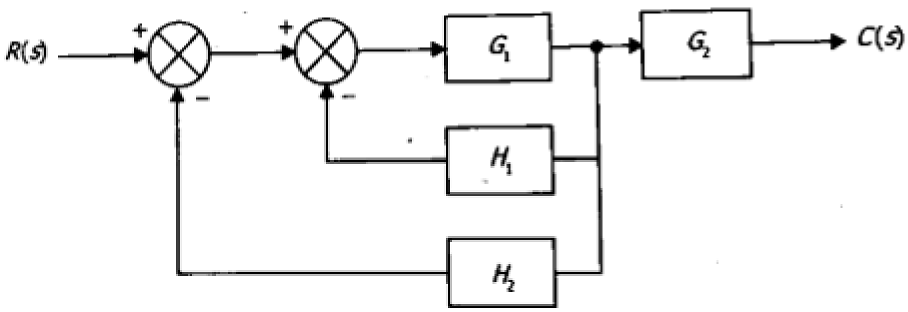
Solution

- G_1 and G_2 are connected in cascade and their equivalent is connected in parallel with G_3

$G = G_1G_2 + G_3$

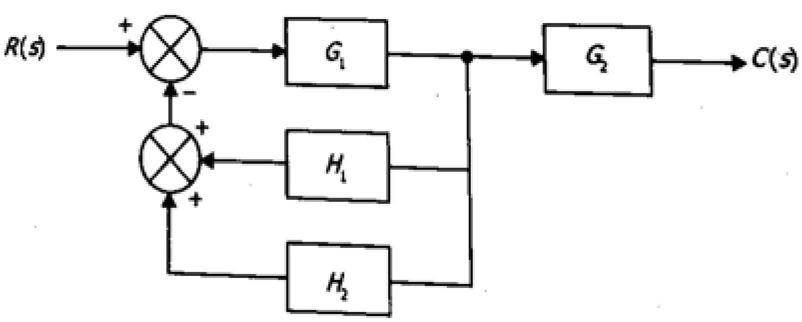
$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 - G(s)H(s)} = \frac{G_1G_2 + G_3}{1 - (G_1G_2 + G_3)H}$$

Example 2: Find C(s)R(s)of the block diagram shows in fig. 1
.



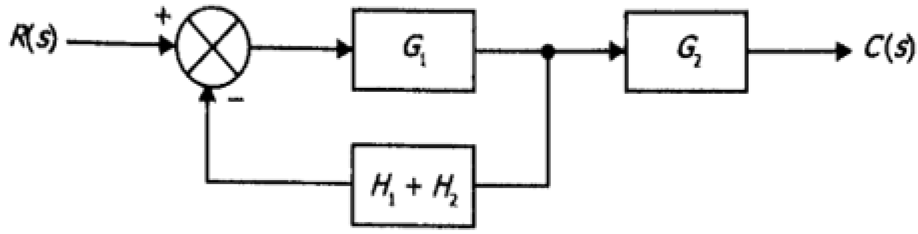
(Fig-1)

Solution:
Figure -1 is redrawn and shown in figure-2.



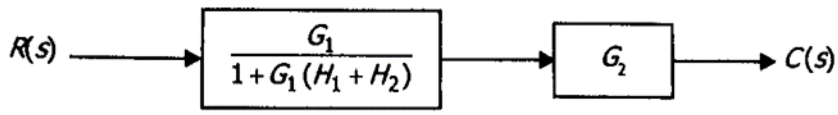
(Fig-2)

Since the two feedback loops are parallel, this fig.2 becomes.



(Fig-3)

Replacing the feedback loop of fig.3 by its equivalent block, fig.3 becomes.



(Fig-4)

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2}{1 + G_1(H_1 + H_2)}$$

Example 3: Find the single block equivalent of figure. (a)

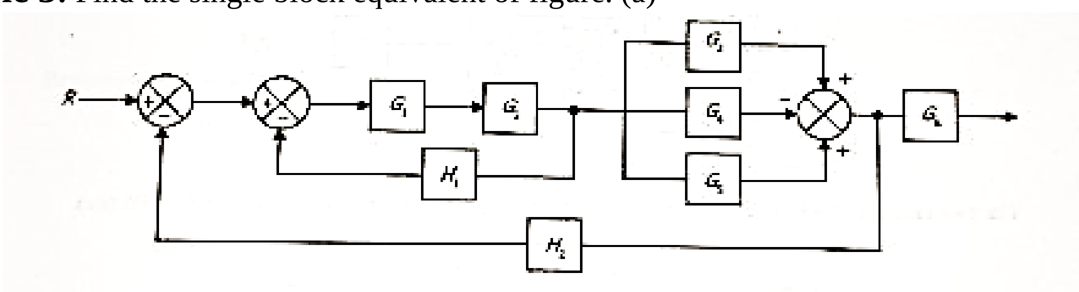


Fig-(a)

Solution: At first the cascade and parallel of blocks of fig(a) are reduces as shown in figure (b).

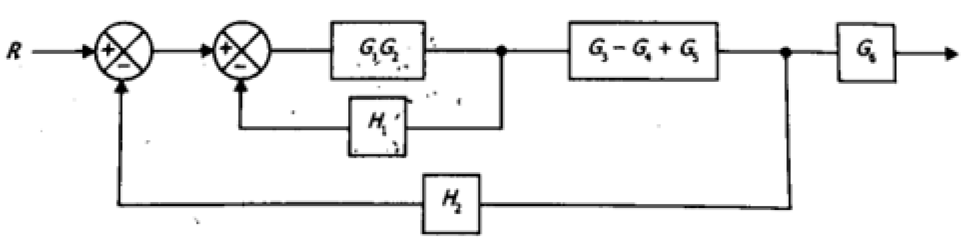


Fig-(b)

The first internal feedback loop of fig.(b) is reduced by its equivalent block shown in fig.(c).

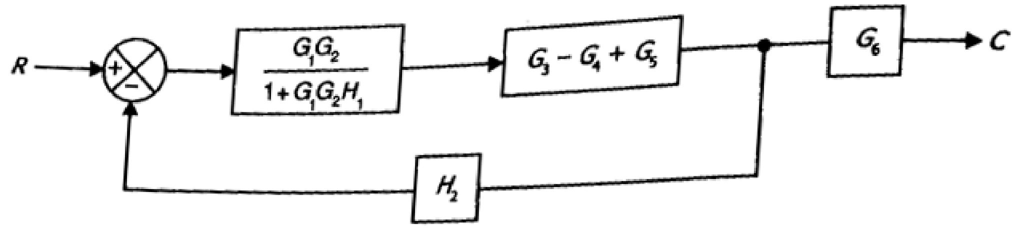


Fig-(c)

The cascade block of fig (c) is replaced by its equivalent block as shown in fig.(d)

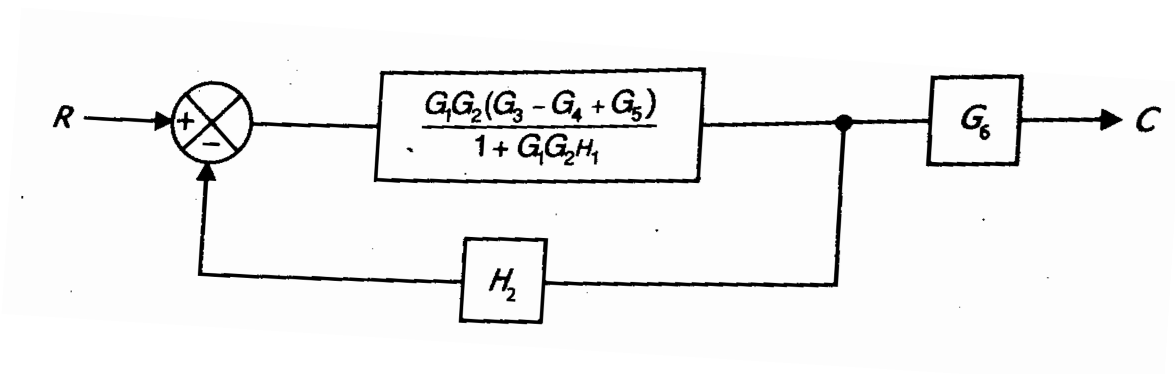


Fig-(d)

The feedback loop of fig.(d) is replaced by its equivalent block as shown in fig.(e).

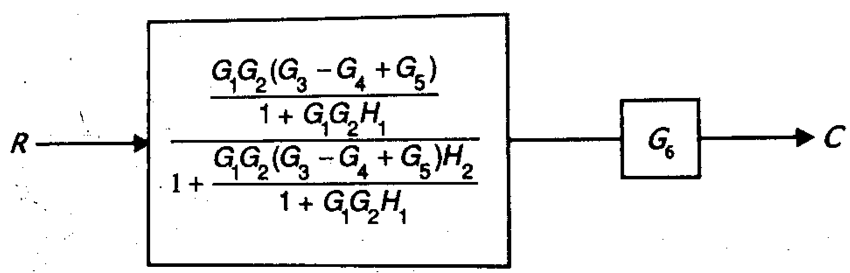


Fig-(e)

The two cascade blocks in fig((e) are replaced by equivalent block Shown in fig (f).

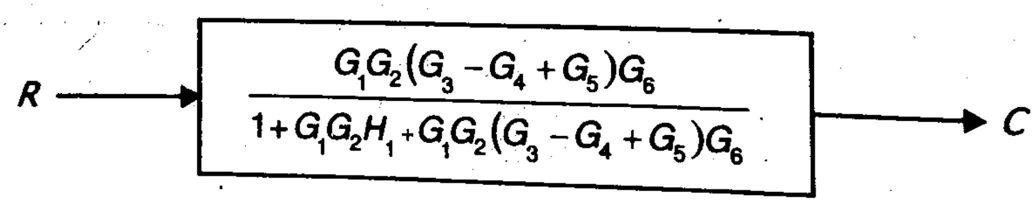


Fig-(f)

4.6 Basic definition in signal flow graph and properties:

Signal flow graph (SFG):

- Block diagram reduction technique is a time-consuming process to remove this difficulty to solve problem by using signal flow graph.
- In signal flow graph, instead of using blocks we use nodes and branches.

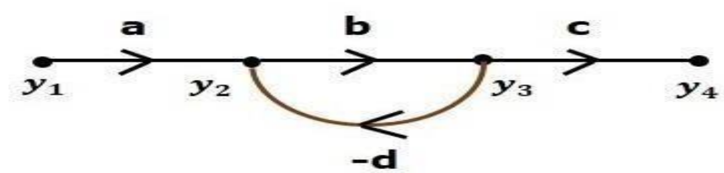
Some common term using in signal flow graph.

1. **Node:**

System variables are represented by nodes in SFG. There are three types of nodes -input node, output node and mixed node.

- **Input Node** − It is a node, which has only outgoing branches.
- **Output Node** − It is a node, which has only incoming branches.
- **Mixed Node** − It is a node, which has both incoming and outgoing branches
.

Example Let us consider the following signal flow graph to identify these nodes.



- The nodes present in this signal flow graph are Y_1, Y_2, Y_3 & Y_4
- Y_1 & Y_4 are the input node and output node respectively.
- Y_2 & Y_3 are mixed nodes.

2.Branch

- Signal flows from one node to another node in the indicated direction through branches. It has both gain and direction. For example, there are four branches in the above signal flow graph. These branches have gains of a, b, c and d.

3.path:

- Path represents the traversal of different branches in the indicated arrow marks or direction without repeating any node.

4.Forward path:

- It represents the traversal are from input node to output node without repeating any node. (Input nodes means the node which possess only out going branches and output node means the node which possess only incoming branches.)

5.Forward path gain:

- It is the multiplication of path gain during the traversal.

6.Loop:

- Loop represents the path whose starting point and terminating point is a single loop.

7.Loop Gain:

- It is the multiplication of path gain present in the loop.

8.Non touching loops:

- In different loops if there is no common node then the loops are considered as non-touching loops.

Note:

1. Nodes are represented by **'.'** Marks.
2. A branch is a simple line which connects two nodes.
3. The arrow mark present in the branch shows the direction of signal flow.
4. The branch gain is written above the arrow mark.

Properties of SFG:

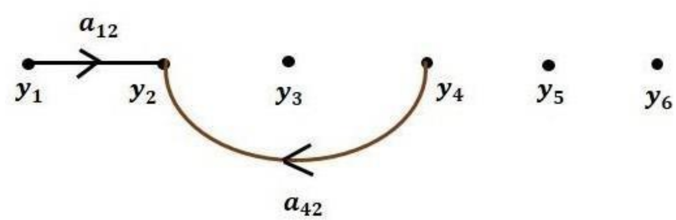
- It is applicable to linear system.
- Arrow indicate signal flow.
- There is transmission of the value of a variable on each node which leaves it.
- The algebraic sum of all signals entering a node gives the value of a variable at that node.
- Since different system equations can be written, therefore the signal flow graph of a system is not unique.
- Using mason's gain formula, the overall gain of a signal-flow graph can be determined.
- A signal -flow graph can represent a block diagram, but the reverse is true.

Construction of Signal Flow Graph

Let us construct a signal flow graph by considering the following algebraic equations –

$$Y_2 = a_{12}Y_1 + a_{42}Y_4$$
$$Y_3 = a_{23}Y_2 + a_{53}Y_5$$
$$Y_4 = a_{34}Y_3$$
$$Y_5 = a_{45}Y_4 + a_{35}Y_3$$
$$Y_6 = a_{56}Y_5$$

- There will be six nodes (Y_1, Y_2 , Y_3, Y_4, Y_5 & Y_6) and eight branches in this signal flow graph.The gains of the branches are ($a_{12}, a_{23}, a_{34}, a_{45,} a_{56}, a_{42}$, a_{53} & a_{35})
- To get the overall signal flow graph, draw the signal flow graph for each equation, thencombine all these signal flow graphs and then follow the steps given below –
- **Step 1** − Signal flow graph for$Y_2 = a_{12}Y_1 + a_{42}Y_4$ is shown in the following figure



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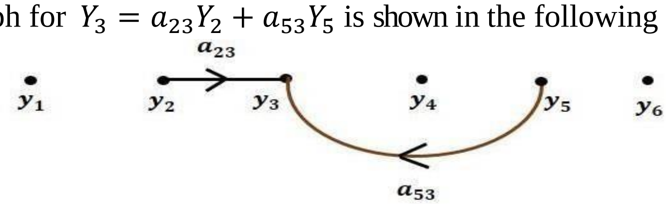
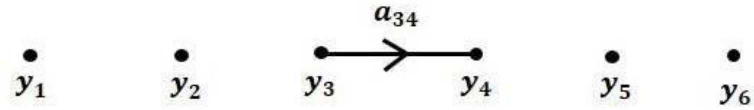
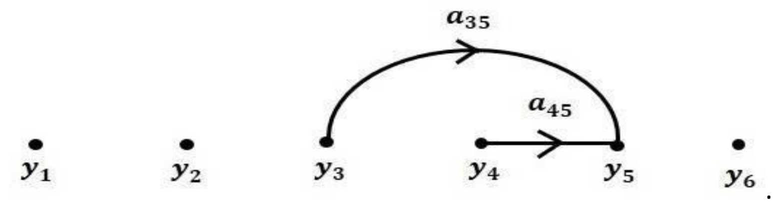
Step 2 − Signal flow graph for $Y_3 = a_{23}Y_2 + a_{53}Y_5$ is shown in the following figure.



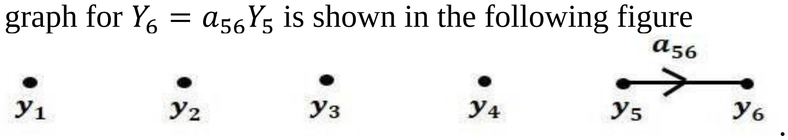
Step 3 − Signal flow graph for $Y_4 = a_{34}Y_3$ is shown in the following figure.



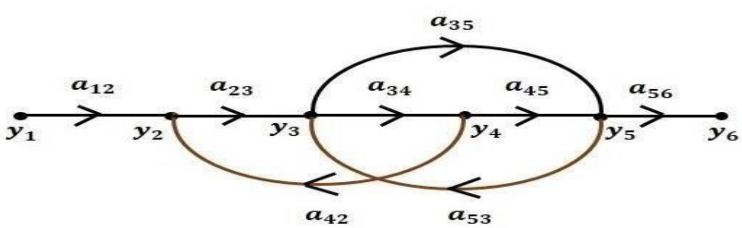
Step 4 − Signal flow graph for $Y_5 = a_{45}Y_4 + a_{35}Y_3$ is shown in the following figure



Step 5 − Signal flow graph for $Y_6 = a_{56}Y_5$ is shown in the following figure



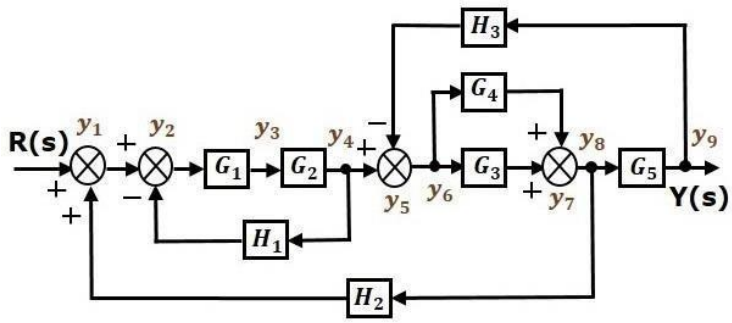
Step 6 − Signal flow graph of overall system is shown in the following figure.



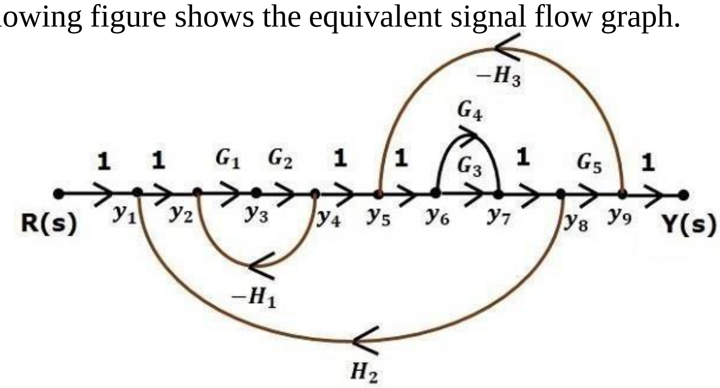
4.7: Construction of Signal flow graph from Block Diagram

1. Follow these steps for converting a block diagram into its equivalent signal flow graph.
2. Represent all the signals, variables, summing points and take-off points of block diagram as nodes in signal flow graph.
3. Represent the blocks of block diagram as branches in signal flow graph.
4. Represent the transfer functions inside the blocks of block diagram as gains of the branches in signal flow graph.
5. nodes as per the block diagram. If there is connection between two nodes (but there is no block in between), then represent the gain of the branch as one.
6. For example, between summing points, between summing point and takeoff point, between input and summing point, between take-off point and output.

Example: Let us convert the following block diagram into its equivalent signal flow graph.



- Represent the input signal $R(s)$ and output signal $C(s)$ of block diagram as input node $R(s)$ and output node $C(s)$ of signal flow graph.
- Just for reference, the remaining nodes (Y_1 to Y_9) are labelled in the block diagram. There are nine nodes other than input and output nodes. That is four nodes for four summing points, fournodes for four take-off points and one node for the variable between blocks G_1 and G_2 .
- The following figure shows the equivalent signal flow graph.



- With the help of Mason's gain formula (discussed in the next chapter), you can calculate the transfer function of this signal flow graph. This is the advantage of signal flow graphs. Here, weno need to simplify (reduce) the signal flow graphs for calculating the transfer function

4.8 Mason's Gain Formula:

- To find transfer function of a complicated system of a block diagram, reduction technique is a cumbersome process because step-by-step reduction of block diagram is required. On the other hand, it is possible to obtain the transfer function very easily by using mason's gain formula given below:

$$Transfer\ function = \frac{C(s)}{R(s)} = \frac{\sum_{i=1}^{N} P_i \Delta_i}{\Delta}$$

Where,

'N'=total number of forward paths.

P_i= Gain of the i^{th} forward path.

Δ=1− (sum of all individual loop gains)

+ (sum of gain of two non-touching loops)

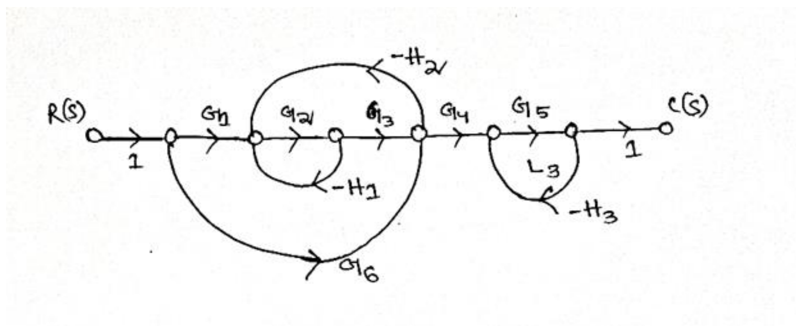
− (sum of gain of three non-touching loops) +…….

Δ_i =Is the similar of Δ by it is calculated by removing i^{th}forward path. By it is calculated by removing

4.9 Simple problems in signal flow graph for network:

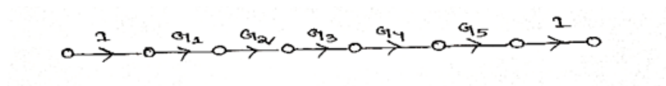
Problem.1

Using mason's gain formula determine the transfer function of the following SFG.



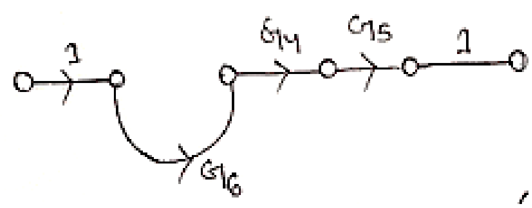
Solution:-

Step-1:first forward path.



$$P_i = P_1 = G_1G_2G_3G_4G_5$$

second forward path**.**

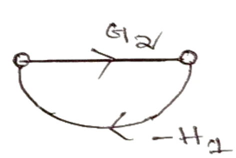


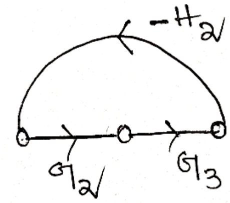
$$P_2 = G_4G_5G_6$$

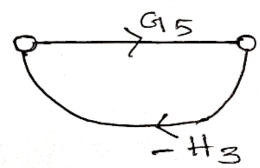
There are two forward -paths.

$$\frac{C(s)}{R(s)} = \frac{\sum_{i=1}^{2} P_i\Delta_i}{\Delta} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

Step-2: Determination of loops and loop gains.







$$L_1 = -G_2H_1$$
$$L_2 = -G_2G_3H_2$$
$$L_3 = -G_5H_3$$
$$L_1L_2L_3$$
$$L_1L_2 \rightarrow Touching$$
$$L_1L_3 \rightarrow Non-Touching$$

$L_2L_3 \rightarrow Non-touching$ L_1L_3 are non-touching loops similarly L_2L_3 are non-touching loops here there is no three non-touching loops.

So $$\Delta = 1-(L_1+L_2+L_3)+(L_1L_2+L_2L_3)$$
$$\Delta = 1-[(-G_2H_1)+(-G_2G_3H_2)+(-G_5H_3)]$$

$$+[(G_2H_1)(-G_5H_3)+(-G_2G_3H_3)(-G_5H_3)]$$
$$\Delta = 1-(-G_2H_1-G_2G_3H_2-G_5H_3)$$
$$+(G_2G_5H_1H_3+G_2G_3G_5H_2H_3)$$
$$= 1+(G_2H_1+G_2G_3H_2+G_5H_3)$$
$$+(G_2G_5H_1H_3+G_2G_3G_5H_2H_3)$$

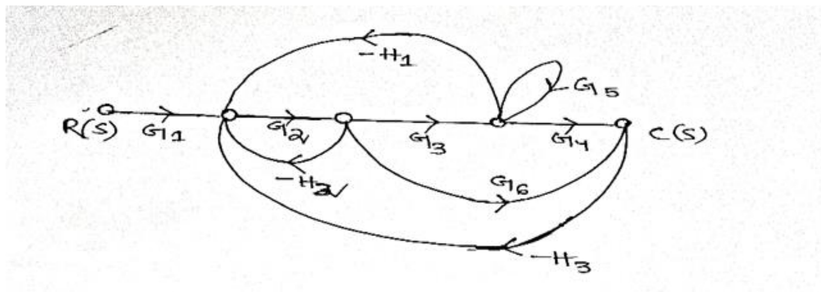
Determination of Δ_1,
$$\Delta_1 = 1-(0)$$
$$= 1$$

Determination of Δ_2,
$$\Delta_2 = 1-(-G_2H_1)$$
$$= 1+G_2H_1$$

$$T.F = \frac{C(s)}{R(S)} = \frac{(G_1G_2G_3G_4G_5)\cdot 1+(G_4G_5G_6)(1+G_2H_1)}{1+G_2H_1+G_2G_3H_2+G_5H_3+G_2G_5H_1H_3+G_2G_3G_5H_2H_3}$$

Problem.2

Find the overall transfer function for the following signal flow graph.

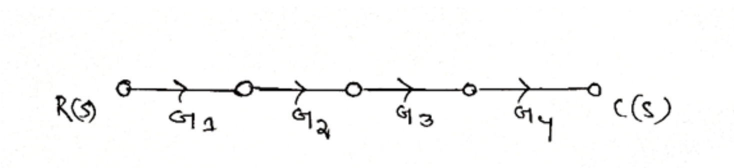


Solution.

Two forward path.

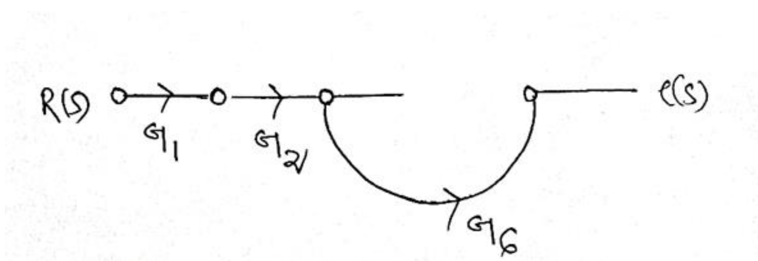
Step-1,

First forward path.



$$P_i = P_1 = G_1 G_2 G_3 G_4$$

Second forward path.

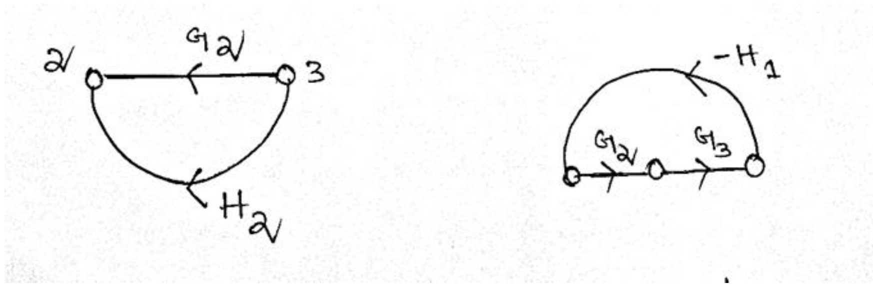


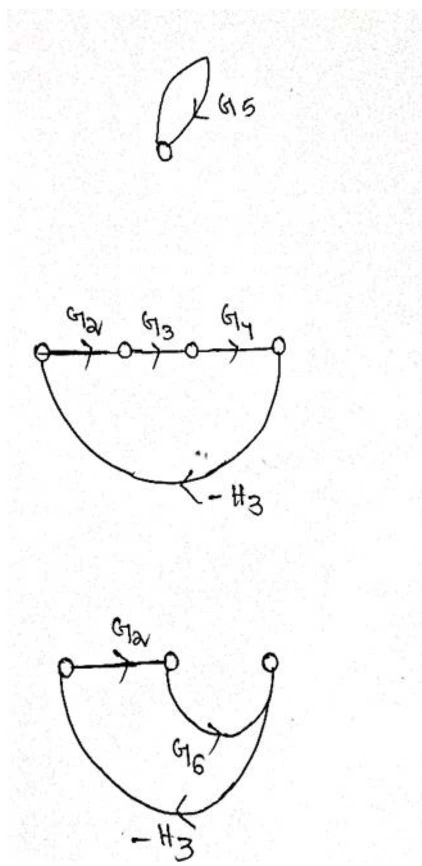
$$P_2 = G_1 G_2 G_6$$

There are two forward paths.

$$\frac{C(s)}{R(s)} = \sum_{i=1}^{2} \frac{P_i \Delta_i}{\Delta} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

Loops-





$$L_1 = -G_2H_2$$
$$L_2 = -G_2G_3H_1$$
$$L_3 = G_5$$
$$L_4 = -G_2G_3G_4H_3$$
$$L_5 = -G_2G_6H_3$$
$$L_1, L_2, L_3, L_4, L_5$$
$$L_1L_2 \rightarrow Touching$$
$$L_1L_3 \rightarrow Non\ touching$$
$$L_2L_3 \rightarrow Touching$$
$$L_1L_4 \rightarrow Touching$$
$$L_1L_5 \rightarrow Touching$$
$$L_4L_5 \rightarrow Touching$$
$$L_3L_5 \rightarrow Non - touching$$

$$L_4 L_3 \rightarrow Touching$$
$$L_2 L_4 \rightarrow Touching$$

So $\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5) + (L_1 L_3 + L_3 L_5)$

$$\Delta = 1 - (-G_2 H_2 - G_2 G_3 H_1 + G_5 - G_2 G_3 G_4 H_3 - G_2 G_6 H_3)$$
$$+(-G_2 H_2 \times G_5 + G_5 \times -G_2 G_6 H_3)$$
$$\Delta = 1 + (G_2 H_2 + G_2 G_3 H_1 - G_5 + G_2 G_3 G_4 H_3 + G_2 G_6 H_3)$$
$$+(-G_2 H_2 G_5 - G_2 G_6 G_5 H_3)$$
$$1 + G_2 H_2 + G_2 G_3 H_1 - G_5 + G_2 G_3 G_4 H_3$$
$$-G_2 H_2 G_5 - G_2 G_6 G_5 H_3$$

determination of Δ_1

$$\Delta_1 = 1 - 0$$
$$= 1$$

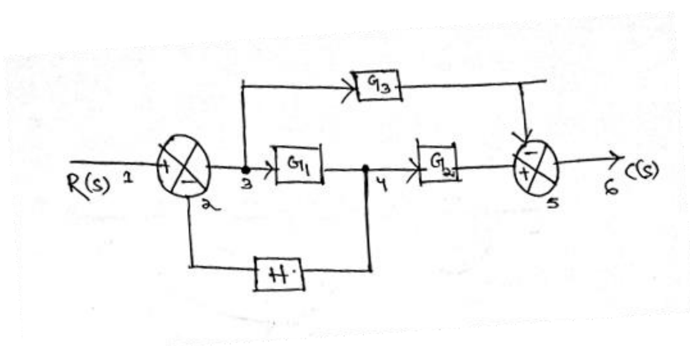
determination of Δ_2

$$\Delta_2 = 1 - G_5$$

$$T.F = \frac{C(s)}{R(s)} = \frac{(G_1 G_2 G_3 G_4).1 + G_1 G_2 G_6 (1 - G_5)}{1 + G_2 H_2 + G_2 G_3 H_1 - G_5 + G_2 G_3 G_4 H_3 - G_2 H_2 G_5 - G_2 G_6 G_5 H_3}$$

Problem.3

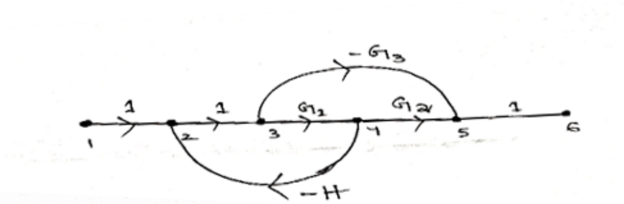
Convert to following block diagram to SFG and find transfer function.

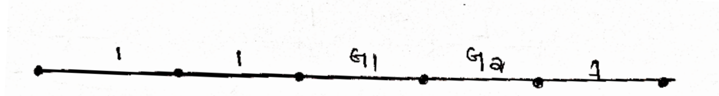


Solution.

Step-1

First forward path.

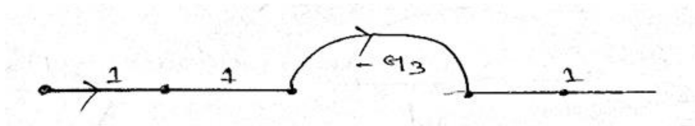




$$P_i = P_1 = 1.1.G_1.G_2.1$$
$$= G_1 G_2$$

Step-2

Second forward path.

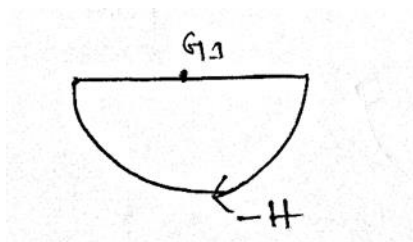


$$P_2 = -G_3$$

There are two forward path.

$$\frac{C(s)}{R(s)} = \sum_{i=1}^{2} \frac{P_i \Delta_i}{\Delta} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

Loops



$$L_1 = -G_1 H$$

So $\Delta = 1 + L_1 = 1 + (G_1 H)$

$$= 1 + G_1 H$$

Determination of Δ_1,

$$\Delta_1 = 1 - 0$$
$$= 1$$

Determination of Δ_2,

$$\Delta_2 = 1 - 0$$
$$= 1$$

$$T.F = \frac{C(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{G_1G_2 - G_3}{1 + G_1 H}$$

POSSIBLE SHORT TYPE QUESTIONS WITH ANSWERS

1. **What do you mean by block diagram?**

Ans. A block diagram is a diagram of a system in which the principal parts or functions are represented by blocks connected by lines that show the relationships of the blocks.

2. **What do you mean by summing point in feedback control system?**

Ans- the summing point is represented with a circle having cross inside it. It has two ormore input and single output. It produce the algebraic sum of inputs. It also perform the summation or subtraction or combination of summation and subtraction of inputsbased on the polarity of the inputs

3. **What is signal flow graph?**

Ans A graphical method of representing the control system using the linear algebraic equations is known as the signal flow graph. It is abbreviated as SFG. This graph basically signifies how the signal flows in a system.

4. **State mason's gain formula. [S-24]**

   Ans: It can be calculated by using Mason's gain formula.

   Δ = 1− (sum of all individual loop gains) + (sum of gain products of all possible two non-touching loops) − (sum of gain products of all possible three non-touching loops) + ...

   Δi is obtained from Δ by removing the loops which are touching the its forward path.

POSSIBLE LONG TYPE QUESTIONS

1.State the rules of block diagram reduction.

2.Write down procedure of reduction of block diagram.

3.write down the steps for solving signal flow graph.

4.write down the steps for finding transfer function of a system through mason gainformula

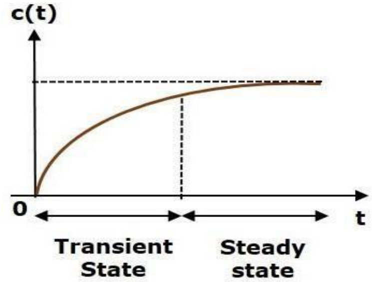
CHAPTER NO-05
TIME DOMAIN ANALYSIS OF CONTROL SYSTEMS

Learning Objectives:

5.1 Definition of Time, Stability, steady-state response, accuracy, transient accuracy, In-sensitivity and robustness.
5.2 System Time Response
5.3 Analysis of Steady State Error
5.4 Types of Input & Steady state Error(Step ,Ramp, Parabolic)
5.5 Parameters of first order system & second-order systems
5.6 Derivation of time response Specification (Delay time, Rise time, Peak time,Setting time,Peak over shoot)

5.1 Time Response of control system

- If the output of control system for an input varies with respect to time, then it is called the time response of the control system. The time response consists of two parts.
- Transient response
- Steady state response
  The response of control system in time domain is shown in the following figure



- Here, both the transient and the steady states are indicated in the figure. The response corresponding to these states are known as transient and steady state responses.
- Mathematically, we can write the time response c(t) as

$$c(t) = c_{tr}(t) + c_{ss}(t)$$

Where,

$c_{tr}(t)$ is the transient response
$c_{ss}(t)$ is the steady state response

Transient Response:

- Transient part is that part of response which becomes zero when t→ ∞

Mathematically, we can written as,

$$\lim_{t\to\infty} C_{tr}(t) = 0$$

steady state response:

- Steady state part is that part of response which remains even its t→ ∞

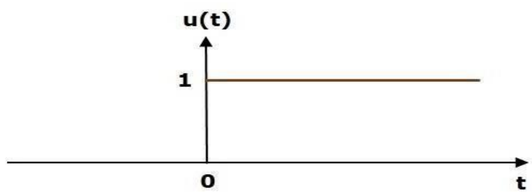
Mathematically, we can written *as*,

$$\lim_{t\to\infty} C_{ss}(t) \neq 0$$

- The poles lie in the left half of the s-plane are responsible for transient response.
- The poles lie on the imaginary axis of the s-plane are responsible for steady state response.

5.2 STANDARD TEST SIGNALS:

- The standard test signals are impulse, step, ramp and parabolic. These signals are used toknow the performance of the control systems using time response of the output.



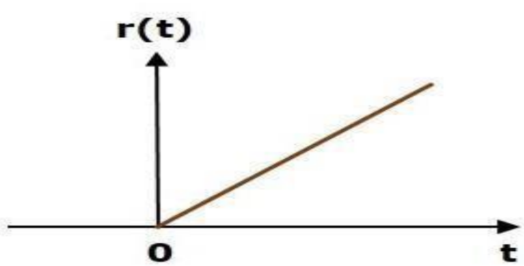
5.2. Step Signal

- A unit step signal, u(t) is defined as,$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$

- So, the unit step signal exists for all positive values of ‘t’ including zero. And its value is oneduring this interval. The value of the unit step signal is zero for all negative values of ‘t’.

$$L[u(t)] = \frac{1}{s}$$

5.2.2 Ramp Signal



A Unit ramp signal, r(t) is defined as,

$$r(t) = t\,u(t) = \begin{cases} t, & t > 0 \\ 0, & t < 0 \\ 0, & t = 0 \end{cases}$$

- So, the unit ramp signal exists for all positive values of ‘t’ including zero. And its value increases linearly with respect to ‘t’ during this interval. The value of unit ramp signal is zerofor all negative values of ‘t’.

$$L[r(t)] = \frac{1}{s^2}$$

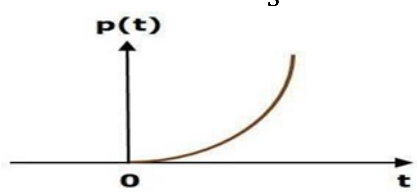
5.2.3 Parabolic Signal

A unit parabolic signal, p(t) is defined as,

$$x(t) = p(t) = \frac{t^2}{2}, t \geq 0$$

- So, the unit parabolic signal exists for all the positive values of **‘t’** including zero. And its value increases non-linearly with respect to ‘t’ during this interval. The value of the unit parabolic signal is zero for all the negative values of ‘t’.

$$L[p(t)] = \frac{1}{s^3}$$

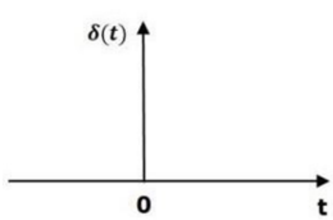


5.2.4 Impulse Signal

A unit impulse signal, δ(t) is defined as,

$$\delta(t) = 0, t \neq 0$$

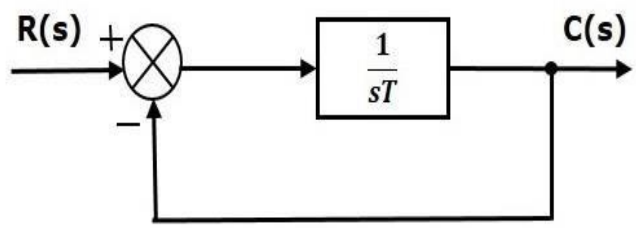
$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$



- So, the unit impulse signal exists only at ‘t’ is equal to zero. The area of this signal under small interval of time around ‘t’ is equal to zero is one. The value of unit impulse signal is zero for all other values of ‘t’.

$$L[\delta(t)] = 1$$

5.3 Time Response of the First Order System:



Where,

- C(s) is the Laplace transform of the output signal c(t),
- R(s) is the Laplace transform of the input signal r(t), and
- T is the time constant.

$$T.F = \frac{C(s)}{R(s)} = \frac{\frac{1}{sT}}{1 + \frac{1}{sT}.1} = \frac{1}{1 + sT}$$

$$C(s) = \frac{1}{1 + sT} \cdot R(s)$$

5.3.1 Unit step response

$$r(t) = u(t)$$

$$R(s) = \frac{1}{S}$$

$$\frac{C(s)}{R(s)} = \frac{1}{1 + sT}$$

$$C(S) = \frac{1}{1 + sT}.R(s)$$

$$= \frac{1}{1 + sT} \cdot \frac{1}{s}$$

$$= \frac{1}{s(1 + sT)}$$

$$= \frac{A}{s} + \frac{B}{1 + sT}(According\ to\ partial\ fraction)$$

$$C(s) = \frac{1}{s(s + sT)} = \frac{A}{s} + \frac{B}{1 + sT}$$

$$= \frac{A(1 + sT) + Bs}{s(1 + sT)}$$

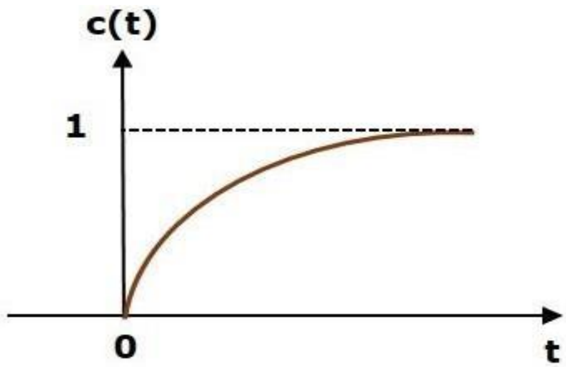
$$= \frac{A + AsT + Bs}{s(1 + sT)}$$

$$= \frac{A + s(AT + B)}{s(1 + sT)}$$

$$A = 1$$
$$AT + B = 0$$
$$1 \cdot T + B = 0$$
$$B = -T$$
$$C(s) = \frac{1 + s \cdot 0}{s(1+sT)} = \frac{A + s(AT + B)}{s(1+sT)}$$
$$so\ \ C(s)\ becomes$$
$$C(s) = \frac{1}{s} + \frac{-T}{1+sT}$$
$$C(s) = \frac{1}{s} \cdot -T.\frac{1}{1+sT}$$

Applying inverse Laplace transform on both side we gate-

$$ILT \downarrow C(t) = 1 - T \cdot \left[\frac{1}{T} \cdot e^{-t/T}\right]$$
$$C(t) = 1 - e^{-t/T}$$



5.3.2Unit impulse response

Impulse response means input r(t)=$\delta(t)$

$$\therefore R(s) = 1$$
$$\text{So}\ \ C(s) = \frac{1}{1+sT}.1$$
$$C(s) = \frac{1}{1+sT}$$

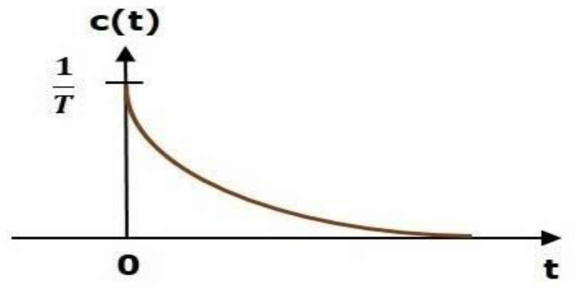
Dividing T in above equation we have

$$C(s) = \frac{1}{T}\ ,\frac{1}{\left(s + \frac{1}{T}\right)}$$
$$a = \frac{1}{T}$$

Taking inverse Laplace,

$$C(t) = \frac{1}{T} e^{-1/T \cdot t}$$

$$= \frac{1}{T} e^{-t/T}$$

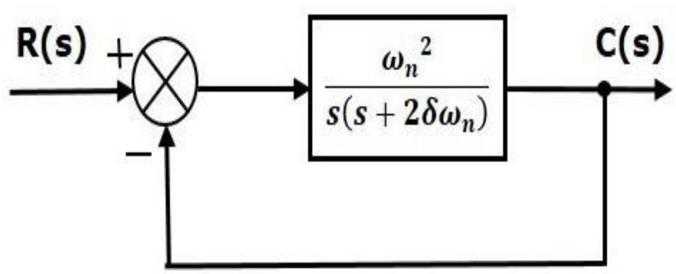


Note – Every system takes 5T Sec to reach at the steady state value**.**

- Impulse response of a system always gives transient term and transient curve consist of system parameter.
- For this reason, impulse is also called as system response.
- The system response is also called as natural response or zero forced response.

5.4 Time response of second order system to the unit step input

let us discuss the time response of second order system. Consider the following block diagram of closed loop control system is connected with a unity negative feedback.



- We know that the transfer function of the closed loop control system having unity negative feedback as

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

Substitute, $G(s) = \frac{\omega_n^2}{s(s+2\delta\omega_n)}$ in the above equation.

$$\frac{C(s)}{R(s)} = \frac{\left(\frac{\omega_n^2}{s(s+2\delta\omega_n)}\right)}{1+\left(\frac{\omega_n^2}{s(s+2\delta\omega_n)}\right)} = \frac{\omega_n^2}{s^2+2\delta\omega_n s+\omega_n^2}$$

- The power of ' s ' is two in the denominator term. Hence, the above transfer function is of the second order and the system is said to be the second order system.

The characteristic equation is –

$$s^2+2\delta\omega_n s+\omega_n^2 = 0$$

The roots of characteristic equation are -

$$\Rightarrow s = -\delta\omega_n \pm \omega_n\sqrt{\delta^2 - 1}$$

- The two roots are imaginary when $\delta = 0$.
- The two roots are real and equal when $\delta = 1$.
- The two roots are real but not equal when $\delta > 1$.
- The two roots are complex conjugate when $0 < \delta < 1$.

We can write $C(s)$ equation as,

$$C(s) = \left(\frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}\right) R(s)$$

Where,

$C(s)$ is the Laplace transform of the output signal, $c(t)$
$R(s)$ is the Laplace transform of the input signal, $r(t)$
ω_n is the natural frequency
δ is the damping ratio.

5.4.1-time response specification

Delay time(T_d)

- Delay time is the time taken by the response to reach at 50% of its final value. It is denoted by (T_d).

Rise Time (T_r)

- It is the time required for the response to rise from 0% to 100% of its final value. This is applicable for the under-damped systems. For the over-damped systems, consider the duration from 10% to 90% of the final value. Rise time is denoted by t_r.

Peak Time (T_P)

- Peak time is the time taken by the response to reach at its peak value for first time .it is denoted by (T_P).

Settling time (T_s)

- Settling time is time taken by the response to reach at its final value with some tolerance band error. It is denoted by (T_s).

Peak Overshoot(M_p)

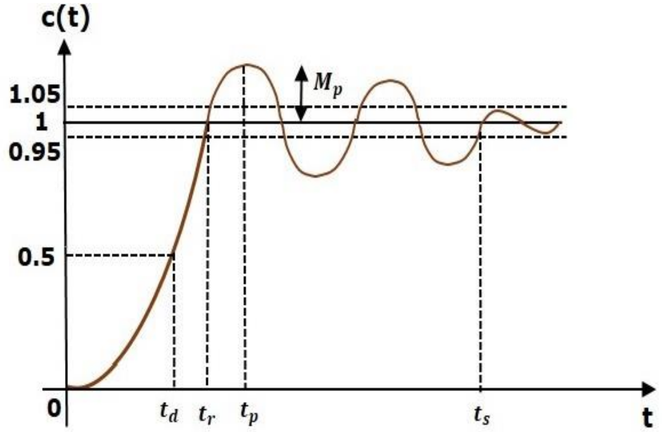
- Peak over shoot Mp is defined as the deviation of the response at peak time from the final value of response. It is also called the maximum over shoot.

Mathematically, we can write it as

$$M_p = c(t_p) - c(\infty)$$

- Where,
  $c(t_p)$ is the peak value of the response.
  $c(\infty)$ is the final (steady state) value of the response.

5.4.2: Derivation of expression for rise time, peak time, peak overshoot, settling time and steady state error.



- All the time domain specifications are represented in this figure. The response up to the settling time is known as transient response and the response after the settling time is known as steady state response.

Delay Time

- It is the time required for the response to reach half of its final value from the zero instant. It is denoted by t_d.
  Consider the step response of the second order system for $t \geq 0$, when ' δ ' lies between zero and one.

$$c(t) = 1 - \left(\frac{e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}}\right) sin(\omega_d t + \theta)$$

- The final value of the step response is one.
  Therefore, at $t = t_d$, the value of the step response will be 0.5. Substitute, these values in the above equation.

$$c(t_d) = 0.5 = 1 - \left(\frac{e^{-\delta\omega_n t_d}}{\sqrt{1-\delta^2}}\right) sin(\omega_d t_d + \theta)$$

$$\Rightarrow \left(\frac{e^{-\delta\omega_n t_d}}{\sqrt{1-\delta^2}}\right) sin(\omega_d t_d + \theta) = 0.5$$

$$\Rightarrow \left(\frac{e^{-\delta\omega_n t_d}}{\sqrt{1-\delta^2}}\right) sin(\omega_d t_2 + \theta) = 0$$

$$\Rightarrow sin(\omega_d t_2 + \theta) = 0$$

$$\Rightarrow \omega_d t_2 + \theta = \pi$$

$$\Rightarrow t_2 = \frac{\pi - \theta}{\omega_d}$$

Substitute t_1 and t_2 values in the following equation of rise time,

$$t_r \quad = t_2 - t_1$$

$$\therefore t_r \quad = \frac{\pi - \theta}{\omega_d}$$

By using linear approximation, you will get the delay time t_d as

$$t_d = \frac{1 + 0.7\delta}{\omega_n}$$

Rise Time

- It is the time required for the response to rise from 0% to 100% of its final value. This is applicable for the under-damped systems. For the over-damped systems, consider the duration from 10% to 90% of the final value. Rise time is denoted by t.
  At $t = t_1 = 0, c(t) = 0$.
- We know that the final value of the step response is one.
  Therefore, at $t = t_2$, the value of step response is one. Substitute, these values in the following equation.

$$c(t) = 1 - \left(\frac{e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}}\right) sin(\omega_d t + \theta)$$

$$(t_2) = 1 = 1 - \left(\frac{e^{-\delta\omega_n t_2}}{\sqrt{1-\delta^2}}\right) sin(\omega_d t_2 + \theta)$$

Peak time

- It is the time required for the response to reach the peak value for the first time. It is denoted by t_p.
  At $t = t_p$, the first derivate of the response is zero.
  We know the step response of second order system for under-damped case is
- $c(t) = 1 - \left(\frac{e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}}\right) sin(\omega_d t + \theta)$

Differentiate $c(t)$ with respect to ' t '.

$$\frac{dc(t)}{dt} = -\left(\frac{e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}}\right)\omega_d\, cos(\omega_d t + \theta) - \left(\frac{-\delta\omega_n e^{-\delta\omega_n t}}{\sqrt{1-\delta^2}}\right) sin(\omega_d t + \theta)$$

Substitute, $t = t_p$ and $\frac{dc(t)}{dt} = 0$ in the above equation.

$$0 = -\left(\frac{e^{-\delta\omega_n t_p}}{\sqrt{1-\delta^2}}\right)\left[\omega_d\, cos\left(\omega_d t_p + \theta\right) - \delta\omega_n\, sin\left(\omega_d t_p + \theta\right)\right]$$

$$\Rightarrow \omega_n\sqrt{1-\delta^2}\, cos\left(\omega_d t_p + \theta\right) - \delta\omega_n\, sin\left(\omega_d t_p + \theta\right) = 0$$

$$\Rightarrow \sqrt{1-\delta^2}\, cos\left(\omega_d t_p + \theta\right) - \delta\, sin\left(\omega_d t_p + \theta\right) = 0$$

$$\Rightarrow sin(\theta)\, cos\left(\omega_d t_p + \theta\right) - cos(\theta)\, sin\left(\omega_d t_p + \theta\right) = 0$$

$$\Rightarrow sin\left(\theta - \omega_d t_p - \theta\right) = 0$$

$$\Rightarrow sin\left(-\omega_d t_p\right) = 0 \Rightarrow - sin\left(\omega_d t_p\right) = 0 \Rightarrow sin\left(\omega_d t_p\right) = 0$$

$$\Rightarrow \omega_d t_p = \pi$$

$$\Rightarrow \frac{\pi}{\omega_d}$$

Peak Over shoot

- Peak over shoot Mp is defined as the deviation of the response at peak time from the final value of response. It is also called the maximum over shoot.
- Mathematically, we can write it as $M_p = c(t_p) - c(\infty)$
- Where,
  $c(t_p)$ is the peak value of the response.
  $c(\infty)$ is the final (steady state) value of the response.
  At $t = t_p$, the response $c(t)$ is -

$$c(t_p) = 1 - \left(\frac{e^{-\delta\omega_n t_p}}{\sqrt{1-\delta^2}}\right) sin(\omega_d t_p + \theta)$$

Substitute, $t_p = \frac{\pi}{\omega_d}$ in the right-hand side of the above equation.

$$c(t_P) = 1 - \left(\frac{e^{-\delta\omega_n\left(\frac{\pi}{\omega_d}\right)}}{\sqrt{1-\delta^2}}\right) sin\left(\omega_d\left(\frac{\pi}{\omega_d}\right) + \theta\right)$$

$$\Rightarrow c(t_p) = 1 - \left(\frac{e^{-\left(\frac{\delta_x}{\sqrt{1-\delta^2}}\right)}}{\sqrt{1-\delta^2}}\right)(-sin(\theta))$$

We know that, $sin(\theta) = \sqrt{1-\delta^2}$

So, we will get $c(t_p)$ as

$$C(t_p) = 1 + e^{-\left(\frac{s\pi}{\sqrt{1-\delta^2}}\right)}$$

Substitute the values of $c(t_p)$ and $c(\infty)$ in the peak overshoot equation.

$$M_p = 1 + e^{-\left(\frac{\delta_x}{\sqrt{1-\delta^2}}\right)} - 1$$

$$\Rightarrow M_p = e^{-\left(\frac{bx}{\sqrt{1-b^2}}\right)}$$

Settling time

- It is the time required for the response to reach the steady state and stay within the specified tolerance bands around the final value. In general, the tolerance bands are 2% and 5%. The settling time is denoted by t_s.

The settling time for 5% tolerance band is –

$$t_s = \frac{3}{\delta\omega_n} = 3\tau$$

The settling time for 2% tolerance band is -

$$t_s = \frac{4}{\delta\omega_n} = 4\tau$$

Where, τ is the time constant and is equal to $\frac{1}{\delta\omega_n}$.

Steady State Error

- The difference between input and output of the system is known as error. The error of the system as t→∞ is called as steady state error denoted by e_{ss}.

5.4.3 Steady state error and error constants.

- The difference between input and output of the system is known as error. The error of the system as t→∞ is called as steady state error denoted by e_{ss}

$$e_{ss} = \lim_{t\to\infty} e(t)$$

- Using the concept of final value theorem, the steady state error is also written as

$$e_{ss} = \lim_{s\to\infty} s.e(s)$$

Expression of steady state error.

Let us consider a unit negative feedback system.

E(s)=R(s)-C(s)

C(s)=E(s)G(s)

E(s)=R(s)-E(s)G(s)

E(s)+E(s)G(s)=R(s)

E(s)[1+G(s)]=R(s)

$$\frac{E(s)}{R(s)} = \frac{1}{1+G(s)} \rightarrow is\ called\ transfer\ function.$$

E(s) is called as the error signal.

$$E(s) = \frac{R(s)}{1+G(s)}$$

According to final value theorem the steady state error of the system is written as

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} S.E(s)$$

$$e_{ss} = \lim_{s\to 0} \frac{S.R(s)}{1+G(s)}$$

Error constants

There are 3 static error constants

positional error constant denoted by "K_p"

$$k_p = \lim_{s\to 0} G(s)$$

Velocity error constant denoted by "k_v"

$$k_v = \lim_{s\to 0} s.G(s)$$

Acceleration error constant denoted by "K_a"

$$k_a = \lim_{s\to 0} s^2 G(s)$$

5.5 Types of control system.[Steady state errors in Type-0, Type-1, Type-2 system]

Step input to type o system

we already know the steady state error 'ess' for a step input is $e_{ss} = \frac{A}{1+k_p}$.

Where $K_p = \lim_{s\to 0} = G(s)H(s)$

$$\lim_{s\to 0} = \frac{k(1+T_{z1}s)(1+T_{z2}s)\ldots\ldots\ldots}{s^0\left(1+T_{p1}s\right)\left(1+T_{p2}s\right)\ldots\ldots\ldots})$$

$$k_p = \frac{k(1)(1)\ldots}{s^0(1)(1)\ldots..}$$
$$= k$$
$$e_{ss} = \frac{A}{1+k_p} = \frac{A}{1+k}.$$

Hence when a type 0 system is subjected to a step input, we get a constant steady state error.

Step Input to a Type 1 System:

we already know' e_{ss} ' for a step input is $e_{ss} = \frac{A}{1+k_p}$

$$where\ k_p = G(s)H(s)$$

$$\lim_{s\to 0}\frac{k(1+T_{z1}s)(1+T_{z2}s)\cdots\cdots\cdots}{s^1(1+T_{P1}s)(1+T_{P2}s)\cdots\cdots\cdots}$$

$$k_p = \frac{k(1)(1)\ldots}{s^1(1)(1)\ldots} = \infty(\ infinity\ ).$$

$$e_{SS} = \frac{A}{1+K_p} = \frac{A}{1+K} = 0$$

- Hence when a type 1 system is subjected to a step input, we get steady state error i.e .. Hencewe can conclude that type 1 systems are excellent for step inputs as steady state error is 0

Step input to a type 2 system

- we already know the steady state error 'e_{ss}' for a step input is $e_{ss} = \frac{A}{1+k_p}$.

$$\text{where } k_p = \lim_{s\to 0} = G(s)H(s)$$
$$\lim_{s\to 0} = \frac{k(1+T_{z1}s)(1+T_{z2}s)\ldots\ldots\ldots}{s^0\left(1+T_{p1}s\right)\left(1+T_{p2}s\right)\ldots\ldots\ldots})$$
$$k_p = \frac{k(1)(1)\ldots}{s^2(1)(1)\ldots} = \infty\ \text{(infinity)}$$
$$e_{ss} = \frac{A}{1+k_p} = \frac{A}{1+k} = 0$$

- Hence when a type 2 system is subjected to a step input, we get steady state error i.e= 0. Hence, we can conclude that type 2 systems are excellent for step inputs as steady state error is 0

Ramp input to a Type 0 system

we already know the steady state error "e_{ss}" for a ramp input is $e_{ss} = \frac{A}{k_v}$

where, $$k_v = \lim_{s\to 0} = sG(s)H(s)$$

$$\lim_{s\to 0} = \frac{sk(1+T_{z1}s)(1+T_{z2}s)\ldots\ldots}{s^0(1+T_{p1}s)(1+T_{p2}s)\ldots\ldots}) = 0$$

$$k_v = \frac{sk(1)(1)\ldots}{s^0(1)(1)\ldots} = 0$$

$$e_{ss} = \frac{A}{k_v} = \frac{A}{0} = \infty \text{ (infinity)}$$

Hence when we subject a type 0 system to a ramp input, the steady state error increases continuously.

Ramp input to a Type 1 system

we already know' e_{ss} ' for a ramp input is

$$e_{ss} = \frac{A}{k_v}$$

where $,k_v = \lim_{s\to 0} = sG(s)H(s)$

$$\lim_{s\to 0} = \frac{sk(1+T_{z1}s)(1+T_{z2}s)\ldots\ldots\ldots}{s^1(1+T_{p1}s)(1+T_{p2}s)\ldots\ldots\ldots})$$

$$k_v = \frac{sk(1)(1)\ldots}{s^1(1)(1)\ldots} = k,$$

$$e_{ss} = \frac{A}{k_v} = \frac{A}{k} = constant$$

Hence when we subject a type 1 system to a ramp input, the steady state error is constant.

Ramp input to a Type 2 system

we already know' e_{ss} ' for a ramp input is $e_{ss} = \frac{A}{k_v}$

where ,

$$k_v = \lim_{s\to 0} = sG(s)H(s)$$

$$\lim_{s\to 0} = \frac{sk(1+T_{z1}s)(1+T_{z2}s)\ldots\ldots\ldots}{s^2(1+T_{p1}s)(1+T_{p2}s)\ldots\ldots\ldots})$$

$$k_v = \frac{sk(1)(1)\ldots}{s^2(1)(1)\ldots} = \infty,$$

$$e_{ss} = \frac{A}{k_v} = \frac{A}{k} = 0,$$

Hence when we subject a type 2 system to a ramp input, the steady state error is 0

Parabolic input to a Type 0 system

we already know' e_{ss} ' for a parabolic input is $e_{ss} = \frac{A}{k_a}$

where

$$k_v = \lim_{s\to 0} = sG(s)H(s)$$

$$\lim_{s\to 0} \frac{s^2k(1+T_{z1}s)(1+T_{z2}s)\ldots\ldots\ldots}{s^2(1+T_{p1}s)(1+T_{p2}s)\ldots\ldots\ldots})$$

$$k_a = \frac{s^2k(1)(1)\ldots}{s^2(1)(1)\ldots} = k$$

$$e_{ss} = \frac{A}{k_a} = \frac{A}{k} = \text{ const}$$

- Hence when we subject a type 0 system to a parabolic input, the steady state error increases continuously. hence type 0 system are not suitable when the input is parabolic in nature.
  ow we shall shift our focus to type 1 systems:
  'Type 1 ' system is given by

$$(s)H(s) = \lim_{s\to 0} \frac{k(1+T_{z1}s)(1+T_{z2}s)\ldots\ldots\ldots}{s^1(1+T_{p1}s)(1+T_{p2}s)\ldots\ldots\ldots..} = 0\ (one\ pole\ at\ origin)$$

Parabolic Input to a Type 1 System:

we already know' e_{ss} ' for a parabolic input is $e_{ss} = \frac{A}{k_a}$

where

$$k_a = \lim_{s\to 0} = sG(s)H(s)$$

$$= \lim_{s\to 0} \frac{s^2k(1+T_{z1}s)(1+T_{z2}s)\ldots\ldots\ldots}{s^1(1+T_{p1}s)(1+T_{p2}s)\ldots\ldots\ldots.} = 0$$

$$k_a = \frac{s^2k(1)(1)\ldots}{s^1(1)(1)\ldots} = 0$$

$$e_{ss} = \frac{A}{k_a} = \frac{A}{k} = \infty \text{ (infinity) },$$

- Hence when we subject a type 1 system to a parabolic input, the steady state error increases continuously. Hence type 1 system are not suitable when the input is parabolic in nature.

Parabolic Input to a Type 2 System

we already know' e_{ss} ' for a parabolic input is $e_{ss} = \frac{A}{k_a}$

where

$$k_a = \lim_{s\to 0} = sG(s)H(s)$$

$$= \lim_{s\to 0} \frac{s^2k(1+T_{z1}s)(1+T_{z2}s)\ldots\ldots\ldots}{s^2(1+T_{p1}s)(1+T_{p2}s)\ldots\ldots\ldots.} = 0$$

$$k_a = \frac{s^2k(1)(1)\ldots}{s^1(1)(1)\ldots} = k$$

$$e_{ss} = \frac{A}{k_a} = \frac{A}{k} = constant$$

- Hence when we subject a type 2 system to a parabolic input, the steady state error is constant. Hence wecan

conclude that Type 2 systems are excellent for step and ramp signals and gives constant error for parabolic inputs.

5.6: Effect of adding poles and zero to transfer function:

- When a zero is added to a system, the relative stability of a system increases.
- When a pole is added to a system, the relative stability of a system decreases.
- Zeros and poles are added only on the left-hand side of s-plane. No addition of the pole or zeros on the right-hand side of s-plane of any system.

- To understand over damped, under damped and Critical damped in control system, let we take the closed loop transfer function in generic form and analysis that to find out different condition Over damped, under damped andCritical damped in control system.

$$T(s) = \frac{\omega_n 2}{s^2 + 2\delta\omega_n s + \omega_n 2}$$

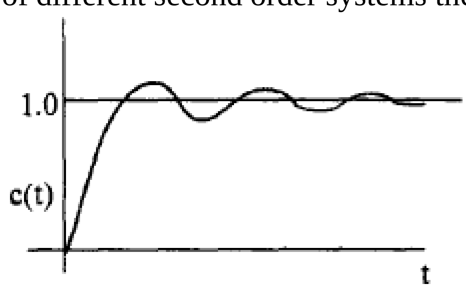
- Now we know that the transient response of any system depends on the poles of the transfer function T(s). And aswe know that the roots of the denominator polynomial in s of T(s) are the poles of the transfer function. So in our case the denominator polynomial of T(s), is known as the
  $D(s) = s^2 + 2\delta\omega_n s + \omega_n^2$
  The characteristic polynomial of the system and D(s) = 0 is known as the characteristic equation of the system

  So The poles of T(s), or, the roots of the characteristic equation we can get by

  $s^2 + 2\delta\omega_n s + \omega_n^2 = 0$

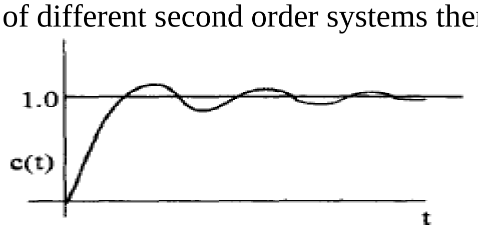
$$
\begin{aligned}
s_{1,2} &= \frac{-2\delta\omega_n \pm \sqrt{4\delta^2\omega_n^2 - 4\omega_n^2}}{2} \\
&= -\delta\omega_n \pm j\omega_n\sqrt{1-\delta^2} \\
&= -\delta\omega_n \pm j\omega_d
\end{aligned}
$$

- Where is known as the damped natural frequency of the system.
- Now If δ > 1, the two roots s1 and s2 are real and we have an over damped system.
- If δ = 1, the system is known as a critically damped system
- The more common case of 0 < 1 is known as the under damped system.
- Now if we go for step responds of different second order systems then we can see

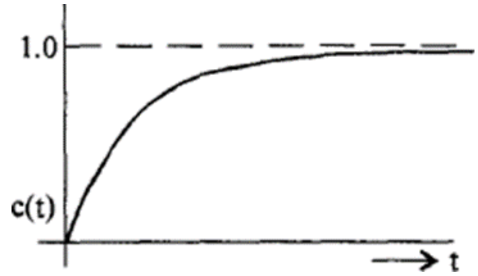


- Step response of an under damped second order system.

- Where is known as the damped natural frequency of the system.
- Now If δ > 1, the two roots s1 and s2 are real and we have an over damped system.
- If δ = 1, the system is known as a critically damped system
- The more common case of 0 < 1 is known as the under damped system.
- Now if we go for step responds of different second order systems then we can see

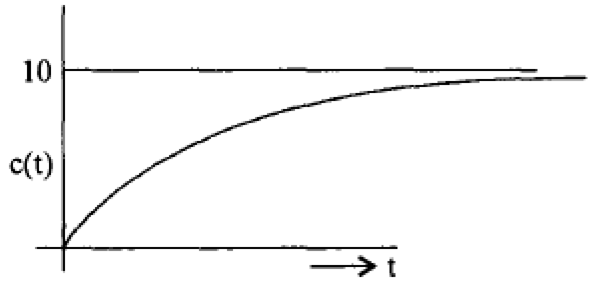


Step response of an under damped second order system.



Step response of a critically damped second order system.

Step response of an over damped second order system.



5.7 Response with P, PI, PD and PID controller.

- Process controls are necessary for designing safe and productive plants. A variety of process controls are used to manipulate processes, however the most simple and often most effective is the PID controller.
- The controller attempts to correct the error between a measured process variable and desired setpoint by calculating the difference and then performing a corrective action to adjust the process accordingly. A PID controller controls a process through three parameters: Proportional (P), Integral (I), and Derivative (D).

Proportional (P) Control

- One type of action used in PID controllers is the proportional control. Proportional control is a form of feedback control. It is the simplest form of continuous control that can be used in a closed-looped system.
- P-only control minimizes the fluctuation in the process variable, but it does not always bring the system to the desired set point. It provides a faster response than most other controllers, initially allowing the P-only controller to respond a few seconds faster.
- However, as the system becomes more complex (i.e. more complex algorithm) the response time difference could accumulate, allowing the P-controller to possibly respond even a few minutes faster. Although the P-only controller does offer the advantage of faster response time, it produces deviation from the set point. This deviation is known as the offset, and it is usually not desired in a process
- P-control linearly correlates the controller output (actuating signal) to the error (difference between measured signal and set point). This P-control behavior is mathematically illustrated in Equation

$$c(t)=Kc\ e(t)+b$$

where

c(t) = controller output
Kc = controller gain
e(t) = error
b = bias

Proportional-Integral (PI) Control

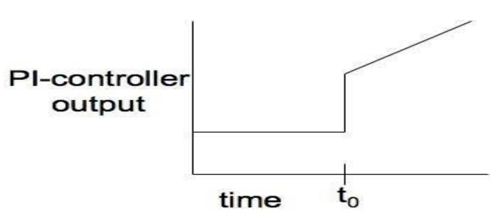
- One combination is the PI-control, which lacks the D-control of the PID system. PI control is a form of feedback control. It provides a faster response time than I-only control due to the addition of the proportional action.
- PI control stops the system from fluctuating, and it is also able to return the system to its set point. Although the response time for PI-control is faster than I-only control, it is still up to 50% slower than P-only control. Therefore, in order to increase response time, PI control is often combined with D-only control.
- PI-control correlates the controller output to the error and the integral of the error. This PI-control behavior is mathematically illustrated in Equation

$$c(t)=Kc(e(t)+\frac{1}{T_i}\int e(t)dt)+C$$

where

c(t) is the controller output,
Kc is the controller gain,
Ti is the integral time,
e(t) is the error, and
C is the initial value of controller

- In this equation, the integral time is the time required for the I-only portion of the controller to match the control provided by the P-only part of the controller.
- The equation indicates that the PI-controller operates like a simplified PID-controller with a zero derivative term. Alternatively, the PI-controller can also be seen as a combination of the P-only and I-only control equations. The bias term in the P-only control is equal to the integral action of the I-only control.
- The P-only control is only in action when the system is not at the set point. When the system is at the set point, the error is equal to zero, and the first term drops out of the equation. The system is then being controlled only by the I-only portion of the controller. Should the system deviate from the set point again, P-only control will be enacted.
- A graphical representation of the PI-controller output for a step increase in input at time t0 is shown below in Figure . As expected, this graph resembles the qualitative combination of the P-only and I-only graphs.



PI-controller output for step input

Proportional-Derivative (PD) Control

- Another combination of controls is the PD-control, which lacks the I-control of the PID system. PD-control is combination of feedforward and feedback control, because it operates on both the current process conditions and predicted process conditions. In PD-control, the control output is a linear combination of the error signal andits derivative. PD-control contains the proportional control's damping of the fluctuation and the derivative control's prediction of process error.
- As mentioned, PD-control correlates the controller output to the error and the derivative of the error. This PD-control behavior is mathematically illustrated in Equation .

$$c(t)=Kc(e(t)+T_d \frac{d_e}{dt}) + C$$

where
c(t) = controller output
K_C = proportional gain
e = error
C = initial value of controller

- The equation indicates that the PD-controller operates like a simplified PID-controller with a zero integral term. Alternatively, the PD-controller can also be seen as a combination of the P-only and D-only control equations.
- In this control, the purpose of the D-only control is to predict the error in order to increase stability of the closed loop system. P-D control is not commonly used because of the lack of the integral term. Without the integral term, the error in steady state operation is not minimized. P-D control is usually used in batch pH control loops,where error in steady state operation does not need to be minimized.
- In this application, the error is related to the actuating signal both through the proportional and derivative term. A graphical representation of the PD- controller output for a step increase in input at time t0 is shown below in Figure Again, this graph is a combination of the P-only and D-only graphs, as expected.

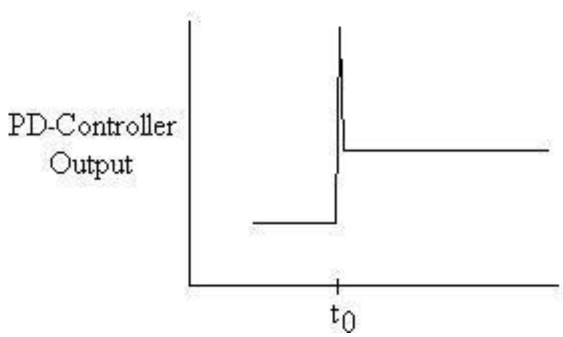


Figure. PD-controller output for step input

Proportional-Integral-Derivative (PID) Control

- Proportional-integral-derivative control is a combination of all three types of control methods. PID-control is mostcommonly used because it combines the advantages of each type of control. This includes a quicker response time because of the P-only control, along with the decreased/zero offset from the combined derivative and integral controllers.
- This offset was removed by additionally using the I-control. The addition of D-control greatly increases the controller's response when used in combination because it predicts disturbances to the

system by measuring the change in error. On the contrary, as mentioned previously, when used individually, it has a slower response time compared to the quicker P-only control.

- However, although the PID controller seems to be the most adequate controller, it is also the most expensive controller. Therefore, it is not used unless the process requires the accuracy and stability provided by the. PID controller.
- PID-control correlates the controller output to the error, integral of the error, and derivative of the error. This PID-control behavior is mathematically illustrated in Equation

$$c(t)=Kc(e(t)+Td\,\frac{d_e}{dt}+c)\frac{1}{T_i}\int e(t)dt+T_d\,\frac{d_e}{dt})$$

where

c(t) = controller output

K_c = controller gain

e(t) = error

T_i = integral time

T_d = derivative time constant

C = initial value of controller

- As shown in the above equation, PID control is the combination of all three types of control. In this equation, the gain is multiplied with the integral and derivative terms, along with the proportional term, because in PID combination control, the gain affects the I and D actions as well.
- Because of the use of derivative control, PID control cannot be used in processes where there is a lot of noise, since the noise would interfere with the predictive, feedforward aspect. However, PID control is used when the process requires no offset and a fast response time.
- A graphical representation of the PID-controller output for a step increase in input at time t0 is shown below in Figure. This graph resembles the qualitative combination of the P-only, I-only, and D-only graphs.

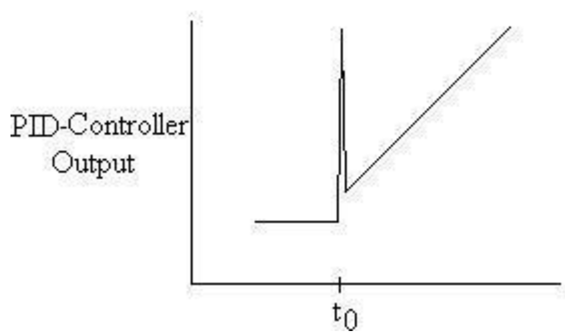


Figure PID-controller output for step input

POSSIBLE SHORT TYPE QUESTIONS WITH ANSWERS

1. **What is an order of a system?**

Ans- The order of a system is the order of the differential equation governing the system. The order of the systemcan be obtained from the transfer function of the given system.

2. **What is step signal?**

Ans- The step signal is a signal whose value changes from zero to A at t= 0 and remains constant at A for t>0.

3. **What is ramp signal?**

Ans- The ramp signal is a signal whose value increases linearly with time from an initial value of zero at t=0.theramp signal resembles a constant velocity.

4. **What is a parabolic signal?**

Ans- The parabolic signal is a signal whose value varies as a square of time from an initial value of zero at t=0.Thisparabolic signal represents constant acceleration input to the signal.

5. **What is transient response?**

Ans-The transient response is the response of the system when the system changes from one state to another.

6. **What is steady state response?**

Ans-The steady state response is the response of the system when it approaches infinity.

7. **Define Damping ratio.**

Ans- Damping ratio is defined as the ratio of actual damping to critical Damping.

8. **List the time domain specifications.**

Ans-The time domain specifications are

1. Delay time
2. Rise time
3. Peak time
4. Peak overshoot

9. **What is damped frequency of oscillation?**

Ans-In under damped system the response is damped oscillatory. The frequency of damped oscillation is given by $\omega_d = \omega_n \sqrt{(1-\zeta^2)}$

10. **What will be the nature of response of second order system with different types of damping?**

Ans-

- For undamped system the response is oscillatory.
- For under damped system the response is damped oscillatory.
- For critically damped system the response is exponentially rising.

- For over damped system the response is exponentially rising but the rise time will be very large.

11. Define Delay time.

Ans- The time taken for response to reach 50% of final value for the very first time is delay time.

12. Define Rise time.

Ans- The time taken for response to raise from 0% to 100% for the very first time is rise time.

13. Define peak time [S-23,24]

Ans- The time taken for the response to reach the peak value for the first time is peak time.

14. Define peak overshoot.

Ans-Peak overshoot is defined as the ratio of maximum peak value measured from the Maximum value to final value.

15. Define settling time. [S-24]

Ans: The settling time of a dynamic system is defined as the time required for the output to reach and steady within a given tolerance band

POSSIBLE LONG TYPE QUESTIONS:

1. Derive time response of first order system with unit step response.
2. Explain different types of controllers.
3. Derive expression for rise time, peak time, peak overshoot.
4. Derive expression for rise time, peak time, settling time for under damped second order system with unit step input. [S-23,24]
5. Explain details of PD controller used in control system. [S-24]

CHAPTER NO.-06
FEEDBACK CHARACTERISTICS OF CONTROL SYSTEMS

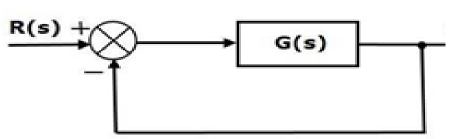
Learning objectives:

6.1 Effect of parameter variation in Open loop System & Closed loop Systems
6.2 Introduction to Basic control Action& Basic modes of feedback control: proportional, integral and derivative
6.3 Effect of feedback on overall gain, Stability
6.4 Realisation of Controllers(P, PI,PD,PID) with OPAMP

6.1 Root Locus concept:

When order of the characteristics equation increases, it becomes very difficult to analyze the system and its stability to avoid this difficult we are using root locus technique.

When system gain K varies from 0 to ∞,the roots of the characteristics equation follow a path for graph known as root locus.



K=0 of the characteristic's equation represents open loop poles and
k=∞ represent open loop zeros.

Explanation

k=0 open loop poles
k=∞ open loop zeros

let us consider,

$$G(s)H(s) = \text{Open Loop Transfer Function}$$
$$= k\,\frac{N(S)}{D(s)}$$

D(s)=0 open loop poles
N(s)=0 open loop zeros

Characteristics equation (close loop case)

Char.equation$=1 + G(s)H(s) = 0$

$$1 + \frac{KN(s)}{D(s)} = 0$$
$$K = \frac{-D(s)}{N(s)}$$

Taking mode on both sides,$|K| = \left|\frac{-D(s)}{N(s)}\right|$

Case-1

When k=0, D(s), open loop poles i.e., k=0 represents open loop poles.

Case-2

When k=∞,$\infty = \frac{\mathrm{D(s)}}{\mathrm{N(s)}}$

$\mathrm{N}(s) = 0 \rightarrow$ open loop zeros.

i.e. k=∞ represent open loop zeros.

- Root locus starts from open loop poles i.e. k=0 and terminate at open loop zeros i.e k=∞
- A point will lie on the root locus branch provided that at the point two condition must be satisfied.

I. Magnitude condition

II. Angle condition or Phase condition

Explanation: -

Let us consider the characteristics equation i.e

$$1 + G(s)H(s) = 0$$

$$G(s)H(s) = -1$$

$$|G(s)H(s)| = |-1| = 1 \text{ this is magnitude condition}$$

$$-1 = -1 + j0$$

$$\text{angle } \emptyset = \tan^{-1}\left(\frac{0}{-1}\right) = -\tan^{-1}(0) \text{this is angle condition}$$

Advantages of Root Locus Technique

1. Root locus technique in control system is easy to implement as compared to other methods.
2. With the help of root locus, we can easily predict the performance of the whole system.
3. Root locus provides the better way to indicate the parameters.

6.2 Construction of root loci

1.Root locus is symmetrical about real axis.

2.The root locus branch start from k=0 i.e. open loop poles and terminate at open loop zero's i.e $k = \infty$.If there are n no. of poles and m no. of zeros and n>m then their exist n-m no. of zeros at infinity therefore n-m no. of roots locus branches terminate at infinity.

3**.** Existence of root locus branch on real axis **:-**A point on real axis will lie on root locus if no. of poles and zeros right to that point is a odd number.

4.The n-m no. of root locus branches move to infinite or goes to infinite along lines as asymptotes.

Asymptotical Angle($\boldsymbol{\varphi_A}$): $-$

$$\emptyset_{\boldsymbol{A}} = \pm \frac{(2q+1)180}{n-m}$$

Where q=0,1,2…………(n-m)

5.Centroid(σ):-

Asymptotes meet at a point on real axis called as centroid denoted by 'σ'

$$\sigma = \frac{\sum(real\ part\ of\ poles) - \sum(real\ part\ of\ zeros)}{n-m}$$

Where-

n=no. of poles

m=no. of zeros

6**.intersection of RL with imaginary axis: -**

The intersection point of root locus with imaginary axis is evaluated by considering marginal stable condition in Routh array.

7.**Break-away/Break -in point:-**

Break-away point:-

Suppose two root loci which start from pole and moves in opposite direction collide with each other such that after collision they start moving in different directions in the symmetrical way. Or the breakaway points at which multiple roots of the characteristic equation 1 + G(s)H(s) = 0 occur. The value of K is maximum at the points where the branches of root loci break away. Break away points may be real, imaginary or complex.

Break-in point: -

Condition of break in to be there on the plot is written below: Root locus must be present between two adjacent zeros on the real axis.

8.Angle of departure and angle of arrival: -

We calculate angle of departure when there exist complex poles in the system. Angle of departure can be calculated as 180-{(sum of angles to a complex pole from the other poles)-(sum of angle to a complex pole from the zeros)}.

6.3 Rules for construction of the root locus

Keeping all these points in mind we are able to draw the **root locus plot** for any kind of system. Now let us discuss the procedure of making a root locus.

1. Find out all the roots and poles from the open loop transfer function and then plot them on the complex plane.
2. All the root loci start from the poles where k = 0 and terminates at the zeros where K tends to infinity. The number of branches terminating at infinity equals to the difference between the number of poles & number of zeros of G(s)H(s).
3. Find the region of existence of the root loci from the method described above after finding the values of M and N.
4. Calculate break away points and break in points if any.
5. Plot the asymptotes and centroid point on the complex plane for the root loci by calculating the slope of the asymptotes.
6. Now calculate angle of departure and the intersection of root loci with imaginary axis.
7. Now determine the value of K by using any one method that I have described above.
8. By following above procedure you can easily draw the root locus plot for any open loop transfer function.
9. (a)Calculate the gain margin.
   (b)Calculate the phase margin.
10. You can easily comment on the stability of the system by using Routh Array

6.4 Effects of Adding Open Loop Poles and Zeros on Root Locus

The root locus can be shifted in 's' plane by adding the open loop poles and the open loop zeros.

If we include a pole in the open loop transfer function, then some of root locus branches will move towards right half of 's' plane. Because of this, the damping ratio δ decreases. Which implies, damped frequency ω_d increases and the time domain specifications like delay time t_d, rise time tr and peak time t_p decrease. But it effects the system stability.

If we include a zero in the open loop transfer function, then some of root locus branches will move towards left half of 's' plane. So, it will increase the control system stability. In this case, the damping ratio δ increases. Which implies, damped frequency ω_d decreases and the time domain specifications like delay time t_d, rise time tr and peak time t_p increase.

So, based on the requirement, we can include (add) the open loop poles or zeros to the transfer function.

POSSIBLE SHORT TYPE QUESTIONS WITH ANSWERS

1. **Consider the loop transfer function K(s+6)/(s+3)(s+5) find out- In the root locus diagram the centroid willbe located at___?**

Ans: Centroid =Sum of real part of open loop pole-sum of real part of open loop zeros/P-Z.

2. **What is the number of the root locus segments which do not terminate on zeroes?**

Ans: The number of the root locus segments which do not lie on the root locus is the difference between thenumber of the poles and zeroes.

3. **If the gain of the system is reduced to a zero value, the roots of the system in the s-plane**,

Ans: The roots of the system in s plane coincides with the poles if the gain of the system is reduced to a valuezero.

4. **When the number of poles is equal to the number of zeroes, how many branches of root locus tends towardsinfinity?**

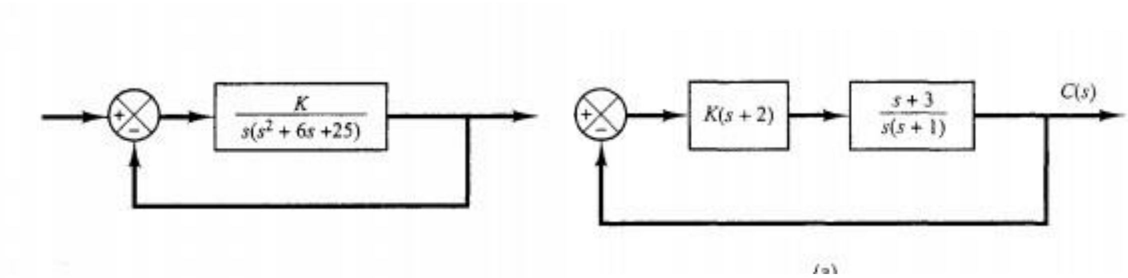
Ans: Branches of the root locus is equal to the number of poles or zeroes which ever is greater and tends towardinfinity when poles or zeroes are unequal.

5. **What is the main objective of root locus analysis technique? [S-23,24]**
   Ans: The main objective of drawing root locus plot is to obtain a clear picture about the transient response of feedback system for various values of open loop gain K and to determine sufficient condition for the value of 'K' that will make the feedback system unstable.
6. **How do you define relative stability? [S-23,24]**
   Ans: The relative stability is the measure of how close the system is to instability. It is usually defined using gain margin and phase margin. The Routh Hurwitz criteria and Bode plots can be utilized for finding the gain margin of a stable system. When you are designing a control system, stability is of prime importance.
7. **Write effect of adding poles to closed loop control system. [S-23,24]**
   Ans: Effect of addition of pole to closed loop transfer function:
   The addition of left half pole tends to slow down the system response. The effect of addition of pole becomes more pronounced as pole location drifts away from imaginary axis. Addition of right half pole will make overall system response to be less stable.

POSSIBLE LONG TYPE QUESTIONS

1. Sketch the root loci for the system shown in Figure). (The gain K is assumed to be positive.) Observe that forsmall or large values of K the system is overdamped and for medium values of K it is under damped.

2. Sketch the root loci of the control system shown in Figure



3. sketch the root locus of the system whose T.F is given by [S-23]

$$G(S)H(S) = \frac{K}{S(S+2)(S+4)}$$

4. Sketch the root locus of the system whose transfer function is given by [S-24]

$$G(S)H(S) = \frac{K}{S(S+3)(S+5)}$$

CHAPTER NO.-07

STABILITY CONCEPT & ROOT LOCUS METHOD

Learning Objectives:

7.1 Effect of location of poles on stability
7.2 RouthHurwitz stability criterion.
7.3 Steps for Root locus method
7.4 Root locus method of design(Simple problem)

7.1 Correlation between time response and frequency response.

Time Response

$$M_P = e^{-\pi\varepsilon/\sqrt{1-\varepsilon^2}}$$
$$W_d = \omega_n\sqrt{1-\varepsilon^2}$$

Frequency Response

$$M_p = \frac{1}{2\varepsilon\sqrt{1-\varepsilon^2}}$$
$$W_r = \omega_{n\sqrt{1-2\varepsilon^2}}$$

7.2 Polar Plot

- Polar plot contains both magnitude and phase in a single graph. In polar plot ω varies from $0\ to\ \infty$.
- The advantages of using polar plot over bode plot is that since it is drawn in a single graph it is easier to find the values of magnitude and phase.

Construction of Polar Plot

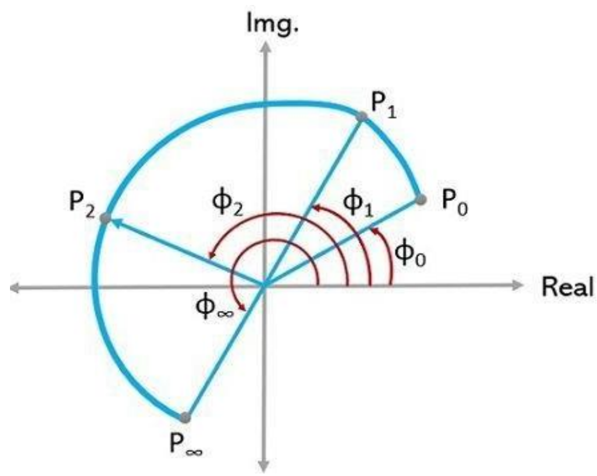
- It is known to us that plotting frequency response signifies sketching the variations in themagnitude and phase angle with respect to the input frequency. These plots are known as magnitude plot (gain plot) and phase plot respectively.
- In the Bode plot, the frequency response is sketched using a logarithmic scale.
- So, in a polar plot, a sketch between the magnitude and phase angle of the transferfunction G(jω) is formed for different values of ω.
- Suppose M represents the magnitude and φ denotes the phase angle, then for thetransfer

$$M = |G(j\omega)H(j\omega)|$$
$$\varphi =< G(j\omega)H(j\omega)$$

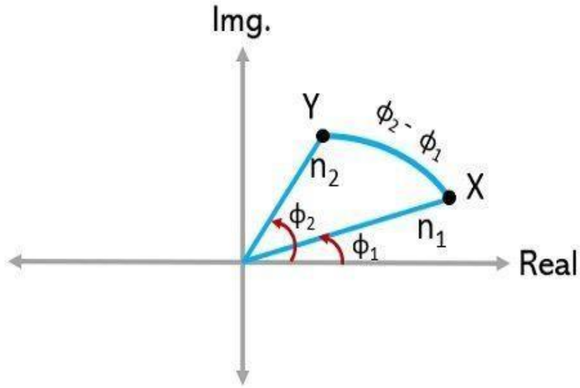
- So, with the variation in ω from 0 to ∞, the values of M and φ can be determined. As we have already discussed in the beginning that polar plot is magnitude versus phaseangle graph plotted for various values of ω.
- So, to construct a polar plot, the different values of magnitude and phase angle istabulated and further, the sketch is formed. The table is given below-

Frequency	Magnitude	Phase Angle
0	M_0	$\emptyset_0$
ω_1	M_1	$\emptyset_1$
ω_2	M_2	$\emptyset_2$
1	1	1
1	1	1
∞	M_∞	$\emptyset_\infty$

- Basically, here each point on the polar plot is significantly plotted for each specific valueof magnitude and phase angle for particular frequency ω.
- Like from the above table, for $\omega = \omega_1$, $M = M_1$ and $\varphi = \emptyset_1$ a point in the polar co-ordinate system is decided that represents $M_1\angle\emptyset_1$, hence, the point on the plot corresponds to thetip of the phasor of magnitudeM_1 plotted at an angle $\emptyset_1$
- So, by using the tabulated data, the polar plot can be formed. Thus, in this way, the magnitude vs phase angle plot is can be constructed for various values of frequency.
- It is to be noted here that conversion of magnitude into dB or logarithm values is not necessary. Also, the anticlockwise direction represents positive phase angles, while theclockwise direction shows the negative phase angles
- The figure below represents the polar plot for ω between 0 to ∞:



- Thus, from the above discussion, we can conclude that polar plot is started from a pointspecifying magnitude and angle for ω = 0 and is terminated at a point specifying magnitude and angle for ω = ∞.
- Another method is used to roughly sketch a polar plot in which magnitudeand angles for the various values of ω are not calculated.
- Basically, in a polar coordinate system, suppose we have two points $n_{1\angle}\emptyset_1$ and $n_{2\angle}\varphi_2$ as indicated below.



- Here, it is clear from the above figure that movement of point X from Y, causes an anglerotation, $\varphi_{2-}\varphi_1$. And if the difference is negative, the rotation will be in the clockwise direction. While, if the difference is positive, the rotation will be in the anti-clockwise direction.
- In a similar way, the variation in ω from 0 to ∞, two points can be considered. One at ω =0, with magnitude M_0 and angle φ_0 while the other at ω = ∞ with magnitude M∞ and angle φ_∞. Then there will be a rotation from φ_∞ to $\emptyset_0$
- More simply,
- ω = 0 gives $M_0\angle\varphi_0$ is the starting point,
- ω = ∞ gives $M_\infty\angle\varphi_\infty$ is the terminating point and
- $\varphi_\infty - \varphi_0$ corresponds to the rotation
- Hence, in this way, the polar plot can be constructed

Example of Polar Plot

- Till now, we have discussed what basically a polar plot is and how it is constructed let us now consider an example to understand the construction of polar plot in a better way.
- Suppose we have a Type 0 system whose transfer function is given as$G(s) = \frac{1}{1+s}$:We have to sketch the polar plot for it.
- The first step is to convert the given transfer function into the frequency

$$G(j\omega) = \frac{1 + j0}{1 + j\omega}$$

$$G(j\omega) \quad = \frac{1}{1 + j\omega}$$

$$G(j\omega) \quad = \frac{1 + j0}{1 + j\omega}$$

- Now, further calculating the magnitude,

$$|G(j\omega)H(j\omega)| = M = \frac{1}{\sqrt{1 + \omega^2}}$$

Also, the phase angle condition

$$\angle G(j\omega)H(j\omega) = \phi = \frac{\tan^{-1}\left(\frac{0}{1}\right)}{\tan^{-1}\left(\frac{\omega}{1}\right)}$$

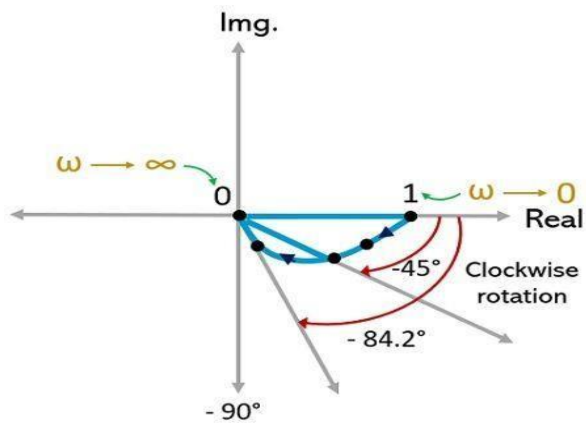
$$\angle G(j\omega)H(j\omega) = \phi = \frac{0°}{(\tan^{-1}\omega)}$$

$$\angle G(j\omega)H(j\omega) = \phi = -\tan^{-1}(\omega)$$

- Now, we have to calculate magnitude and angle by substituting different values of ω between 0 and ∞. Thus, the tabular representation will be

Frequency	**Magnitude**	**Phase angle**
0	1	0^0
1	$1/\sqrt{2}$	-45^0
10	$1/\sqrt{101}$	-84.2^0
1	1	1
1	1	1
∞	0	-90^0

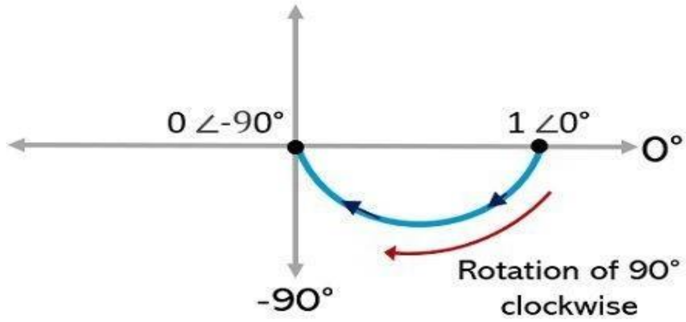
- Hence, the tabulated data shows that the starting point is 1 ∠0° and terminating point is0 ∠-90°. Thus, the plot will terminate at the origin, tangential to the axis of angle -90°.Thus, the plot is represented as:



- Now, let us apply the alternative method to sketch the polar plot. As we have discussed earlier that in this method only the starting and terminating pointsare of major significance. Thus, frequency is needed for 0 and ∞. From the above tabular representation, it is clear that, For, ω = 0 magnitude and angle=1 ∠0° For, **ω** = ∞ magnitude and angle = 0 ∠-90° Therefore,

$$\varphi_\infty - \varphi_0 = 90^0 - 0^0 = -90^0$$

- As the difference of the two is negative, thus, the rotation from starting to the terminating point will be in the clockwise direction. Thus, the starting point, 1 ∠0° is rotated 90° in the clockwise direction, in order to getterminated at 0 ∠-90°. Hence, the rough sketch of the polar plot is given below:



- It is to be noted here that mostly this approximate method is used for sketching the polar plot.

7.3. Bode Plot:

- Bode plot is a graphical way to study the frequency response of a system. The Bode plot or the Bode diagram consists of two plots –
- Magnitude plot
- Phase plot
- In both the plots, x-axis represents angular frequency (logarithmic scale). Whereas, y-axisrepresents the magnitude (linear scale) of open loop transfer function in the magnitude plot andthe phase angle (linear scale) of the open loop transfer function in the phase plot.
- The magnitude of the open loop transfer function in dB is –
- $M = 20\log|G(j\omega)H(j\omega)|$
- The phase angle of the open loop transfer function in degrees is –
- $\emptyset = \angle G(j\omega)H(j\omega)$
- Note − The base of logarithm is 10.

Basic of Bode Plots:

- The following table shows the slope, magnitude and the phase angle values of the terms presentin the open loop transfer function. This data is useful while drawing the Bode plots.

Typeof term	G(jω) H(jω)	Slope (dB /dec)	Magnitude (dB)	Phase angle(degrees)
Constant	K	0	20 log K	0
Zero at origin	jω	20	20 log ω	90
‘n’ zeros at origin	$(j\omega^n)$	20n	20 n log ω	90 n
Poleat origin	$\frac{1}{j\omega}$	−20	−20 log ω	−90 or270

n poles at origin	$\frac{1}{(j\omega)^n}$	−20n	−20 n log ω	−90 n or270 n
Simple zero	1+jωr	20	0 for ω <$\frac{1}{r}$ 20 log ω_r for ω< $\frac{1}{r}$	0 for ω <$\frac{1}{r}$ 0 for ω 90 For ω >$\frac{1}{r}$
Simple pole	$\frac{1}{1+j\omega r}$	−20	0 for ω < $\frac{1}{r}$	0 for ω < $\frac{1}{r}$ 0 for ω <$\frac{1}{r}$
Second order derivative term	$\omega_n{}^2(1-\frac{\omega^2}{\omega_n{}^2}+\frac{2j\delta\omega}{\omega n})$	40	$40log\omega_n$ for $\omega < \omega_n$ 20 log $(2\delta\omega_n{}^2)$	$0\ for\ \omega < \omega_n$ $90\ for\ \omega$
Second order integral term	$\frac{1}{\omega_n{}^2(1-\frac{\omega^2}{\omega_n{}^2}+\frac{2j\delta\omega}{\omega n})}$	−40	$-40\log\omega_n\ for\ \omega < \omega_n$ $-20\log(2\delta\omega_n{}^2)for\ \omega = \omega_n$	$-0\ for\ \omega < \omega_n$ $-90\ for\ \omega = \omega_n$

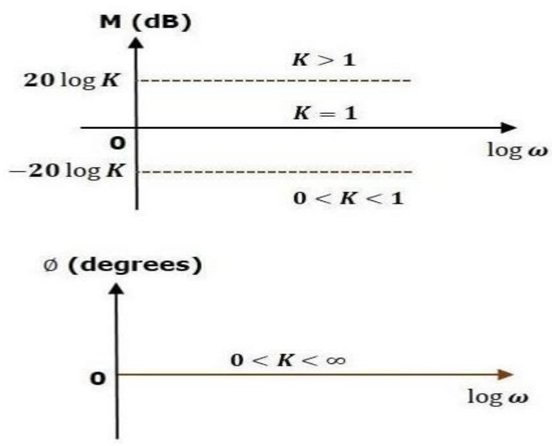
- Consider the open loop transfer function G(s)H(s)=K

Magnitude M=20 log dB

Phase angle ϕ=0 degree

- ➢ If K=1, then magnitude is 0 db.
- ➢ If K>1, then magnitude will be positive.
- ➢ If K<1, then magnitude will be negative.

The following figure shows the corresponding Bode plot

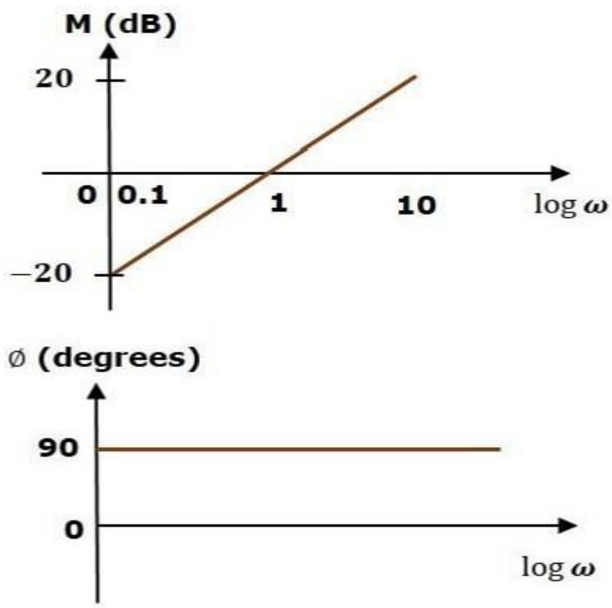


- The magnitude plot is a horizontal line, which is independent of frequency. The 0 dB line itself isthe magnitude plot when the value of K is one. For the positive values of K, the horizontal line will shift 20 log k dB above the 0 dB line. For the negative values of K, the horizontal line will shift 20 log dB below the 0 dB line. The Zero degrees line itself is the phase plot forall the positive values of K.
- Consider the open loop transfer function G(s)H(s)=s

Magnitude M=20logω

Phase angle ϕ=90

  - At ω=0.1 rad/sec, the magnitude is -20
  - dB. At ω=1 rad/sec, the magnitude is 0 dB.
  - At ω=10 rad/sec, the magnitude is 20 dB.



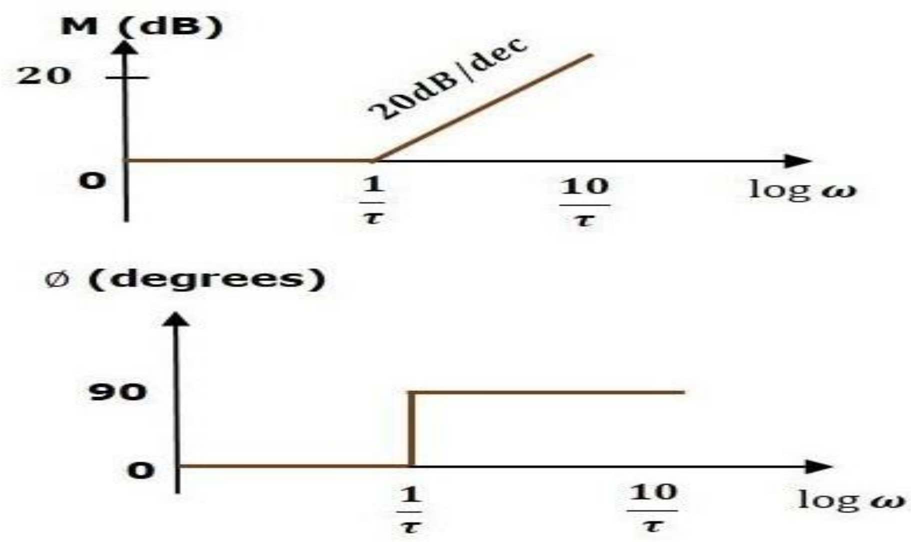
- The magnitude plot is a line, which is having a slope of 20 dB/dec. This line started at ω=0.1 rad/sec having a magnitude of -20 dB and it continues on the same slope. It is touching 0 dB line at ω=1 rad/sec. In this case, the phase plot is 90^0 lines.

- Consider the open loop transfer function G(s)H(s)=1+sT

Magnitude $M = 20\log\sqrt{1+\omega^2T^2}$ dB

Phase angle $\emptyset = \tan^{-1}\omega T$ degrees

  - For ω<$\frac{1}{T}$, the magnitude is 0 dB and phase angle is 0 degrees.
  - For ω>$\frac{1}{T}$, the magnitude is 20logωT dB and phase angle is 90.
- The following figure shows the corresponding Bode plot.



- The magnitude plot is having magnitude of 0 dB up to ω=$\frac{1}{T}$ rad/sec. From ω=$\frac{1}{T}$ rad/sec, it is having a slope of 20 dB/dec. In this case, the phase plot is having phase angle of 0 degrees up to ω=$\frac{1}{T}$ rad/sec and from here, it is having phase angle of 90^0. This Bode plot is called the**a**symptotic Bode plot.
- As the magnitude and the phase plots are represented with straight lines, the Exact Bode plots resemble the asymptotic Bode plots. The only difference is that the Exact Bode plots will have simple curves instead of straight lines. Similarly, you can draw the Bode plots for other terms of the open loop transfer function whichare given in the table.

<u>Rules for Construction of Bode Plots</u>

Follow these rules while constructing a Bode plot.

- Represent the open loop transfer function in the standard time constant form.
- Substitute, s=jω in the above equation.
- Find the corner frequencies and arrange them in ascending order.
- Consider the starting frequency of the Bode plot as $1/10^{th}$ of the minimum corner frequency or 0.1 rad/sec whichever issmaller value and draw the Bode plot upto 10 times maximum corner frequency
- Draw the magnitude plots for each term and combine these plots properly
- Draw the phase plots for each term and combine these plots properly

Note − The corner frequency is the frequency at which there is a change in the slope of the magnitude plot

<u>7.4. All Pass and Minimum phase system</u>

- A transfer function G(s) is minimum phase if both G(s) and $\frac{1}{G(s)}$ are causal and stable.
- Roughly speaking it means that the system does not have zeros or poles on the right half plane. Moreover it, does not have delay.
- Bode discovered that the phase can be uniquely derived from the slope of the magnitude of minimum phase system.

Basic factor	Mag slop (low freq.)	Phase (low freq.)	Mag slop (high freq.)	Phase (high freq.)
K	0	0	0	0
S^N	20N	90N	20N	90N
1/(Ts+1)	0	0	-20	-90
$1/((\frac{s}{\omega_n})^2 + 2\tau\left(s/\omega_n\right) + 1)$	0	0	-40	-180

7.5 Computation of Gain Margin and phase margin

Gain margin

Let us consider the open loop T. F of a system is G(s). Gain margin is defined as

$$GM = \frac{1}{|G(j\omega)|}\ at\ \omega = \omega_{pc}$$

Where, ω_{pc} is the phase cross over frequency.

Phase cross over frequency

The frequency at which phase of open loop T.F becomes 180^0 is called phase cross over frequency.

Phase Margin

Let us consider the open loop T. F of a system is G(s). phase margin is defined as

$$PM = 180 + < G(j\omega)\ at\ \omega = \omega_{gc}$$

Where, ω_{gc}= Gain cross over frequency.

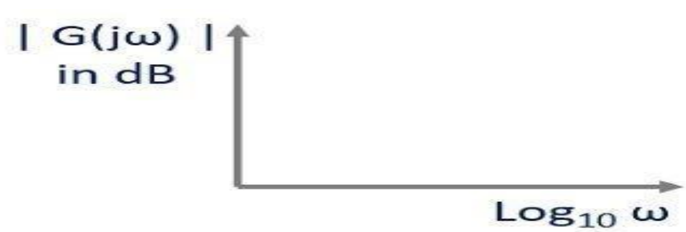
Gain cross over frequency

The frequency at which gain of the open loop transfer function becomes 1 or 0 dB is called so.The stability of the control system based on the relation between gain margin and phasemargin is listed below

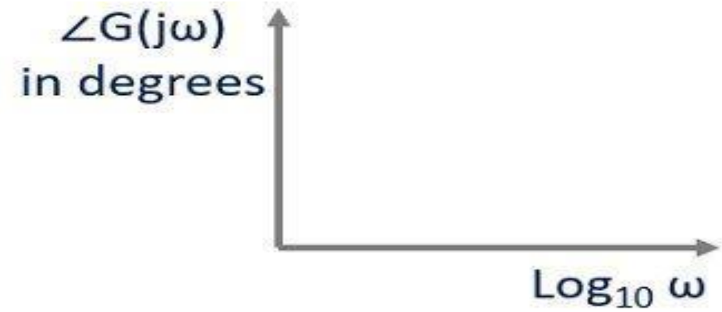
- If both the gain margin GM and the phase margin PM are positive, then the control system is stable.
- If both the gain margin GM and the phase margin PM are equal to zero, then the control systemis marginally stable.
- If the gain margin GM and / or the phase margin PM are/is negative, then the control system is unstable

7.6 Log magnitude versus phase plot

- Magnitude Plot: In this plot, magnitude is represented in logarithmic values against logarithmic values of frequency.
- For the transfer function G(jω) H(jω), in order to express the magnitude in logarithmicvalues, we need to find, $\left|G_{j\omega}\right| = 20\ log_{10}|G(j\omega)|dB$
- And this magnitude in dB is plotted for log10 ω. This is represented in the general



- Phase Angle Plot: Here, the phase angle in degrees is sketched against logarithmic values of frequency. Here, the angular value of G(jω) in degrees is sketched against log_{10} ω. The figure hererepresents the general representation of phase angle plot:



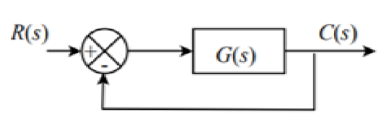
Bode Plot is also known as the logarithmic plot as it is sketched on the logarithmic scaleand represents a wide range of variation in magnitude and phase angle with respect to frequency, separately. Thus, the bode plots are sketched on semi-log graph paper.

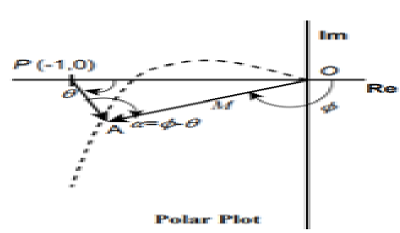
- Also, as we can see that in both the plots the logarithmic value of frequency is scaled onthe x-axis, so, x-axis can be kept common and both magnitude and phase angle plots can be drawn on the same log paper.
- It is to be noted here that, suppose, we are having open-loop transfer function of the system G(jω) H(jω) and we have to determine the closed-loop stability by making use of frequency response of the open-loop system. Then, not simply G(jω) but magnitude andphase angle of G(jω) H(jω) is to be plotted against log 10 ω

7.7 Closed Loop Frequency Response

- The Bode plot is generally constructed for an open loop transfer function of a system. In order to draw the Bodeplot for a closed loop system, the transfer function has to be developed, and then decomposed into its poles andzeros. This process is tedious and cannot be carried out without the ais of a very powerful calculator or a computer. With reference to the unit feedback system the block diagram and its polar plot, the transfer function is given by

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)}$$





- The dashed line in the polar plot is the trace of tip of the vector OA which represent the system the length of vector measures the magnitude of the system at a given frequency ω and angle$\emptyset$ represents the phase shift.
- It is possible to obtain a frequency response test with good accuracy and it is very useful when it is very difficult to obtain transfer function by an analytical technique.
- It is very easy to design open-loop transfer function for specified closed -loop performance in frequency domain compared to time domain.
- In frequency domain it is very easy to visualize the effects of noise disturbance and parameter variation.

POSSIBLE SHORT TYPE QUESTIONS WITH ANSWERS

1. **What do you mean by frequency response?**

Ans It is defined as the steady state response of a system due to sinusoidal input.

2. **Define bode plot?**

Ans A Bode plot is generally used in electrical engineering and control theory and is represented by a graph depictingthe frequency responses of a particular system. It is an important tool used in linear time invariant systems (LTI systems) for showing its gain or the magnitude and the phase response with respect to different operating frequencies.

3. **Define gain margin?**

Ans greater the Gain Margin (GM), the greater the stability of the system. The gain margin refers to the amount ofgain, which can be increased or decreased without making the system unstable. It is usually expressed as a magnitude in dB.
We can usually read the gain margin directly from the Bode plot (as shown in the diagram above). This is done bycalculating the vertical distance between the magnitude curve (on the Bode magnitude plot) and the x-axis at thefrequency where the Bode phase plot = 180°. This point is known as the phase crossover frequency.

4. **Define Phase margin?**

The greater the Phase Margin (PM), the greater will be the stability of the system. The phase margin refers tothe amount of phase, which can be increased or decreased without making the system unstable. It is usually expressed as a phase in degrees.

5. **what are the advantages of polar plot? [S-22]**
   Ans: Polar plot or Nyquist plot at a particular value of ω. In the polar plot, it is important to show the frequency graduation of the locus. An advantage in using a polar plot is that it depicts the frequency-response characteristics of a system over the entire frequency range in a single plot.

POSSIBLE LONG TYPE QUESTIONS

1. Explain close loop frequency response analysis.
2. Explain polar plot and its rule to construct polar plot.
3. Explain all pass minimum phase equation.

4. The open loop T.F of the plant is

   $$G(S)H(S) = \frac{80(S+5)}{S^2(S+50)}$$

   Use bode plot, find gain margin & phase margin. [S-23]

5. The open loop T.F of the plant is

   $$G(S)H(S) = \frac{10(S+2)}{S^2(S+10)}$$

   Use bode plot, find gain margin & phase margin. [S-24]

CHAPTER NO.-08

FREQUENCY-RESPONSE ANALYSIS & BODE PLOT

Learning objectives:

8.1 Frequencyresponse,Relationship between time & frequency response
8.2 Methods of Frequency response
8.3 Polar plots & steps for polar plot
8.4 Bodes plot & steps for Bode plots
8.5 Stability in frequency domain, Gain Margin& Phase margin
8.6 Nyquist plots. Nyquiststability criterion.
8.7 Simple problems as above

CONCEPT OF NYQUIST PLOT:

- Nyquist plot is similar to polar plot accept that in Nyquist plot ω varies -∞ to ∞.
- Nyquist plot is symmetrical about real axis.
- Nyquist plot is based on the principle is called principle of argument.

8.1 Principle of argument:

According to this principle if any closed path in s-plane encloses P no. of pole's and Z no. of zero's then in the G(S)H(S) plane it must encircles of origin N times.
Where N= P-Z
If N becomes +ve the encircles occurs in anticlockwise direction and if N is -ve the encircle occur in clockwise direction.

8.2 Nyquist stability criterion:

According to Nyquist no. of encirclement about the critical point
N=P-Z
Where N=no of encirclement about (-1+j0) point.
P=no of poles of characteristic equation present in the right half s plane or no of poles of open loop T.F present right half s-plane.
Z= no of zeros of characteristic equation present in right hand side of s- plane or no of poles of close loop T.F present in right half.

8.3 Nyquist stability criterion applied to inverse polar plot.

(The Nyquist stability criterion can be applied equally well to inverse polar plots. The mathematical derivation ofthe Nyquist stability criterion for inverse polar plots is the same as that for direct polar plots.)

The inverse polar plot of G(jω)H(jω) is a graph of 1/[G(jω)H(jω)] as a function of ω. For example, if G(jω)H(jω) is

$$G(j\omega)H(j\omega) = \frac{j\omega T}{1+j\omega T}$$

Then $\frac{1}{G(j\omega)H(j\omega)} = \frac{1}{j\omega T} + 1$

The inverse polar plot for $\omega \geq 0$ is the lower half of the vertical line starting at the point (1,0) on the real axis.
The Nyquist stability criterion applied to inverse plots may be stated as follows: For a closed-loop system to be stable, the encirclement, if any, of the -1 j0 point by the [G(s)H(s)] locus (as s moves along the Nyquist path) mustbe counter clockwise, and the number of such encirclements must be equal to the number of poles of 1/[G(s)H(s)] [that is, the zeros of G(s)H(s)] that lie in the right-half s plane. [The number of zeros of G(s)H(s) in the right-half s plane may be determined by use of the Routh stability criterion.]

If the open-loop transfer function G(s)H(s) has no zeros in the right-half s plane, then for a closed-loop system tobe stable the number of encirclements of the -1 j0 point by the 1/[G(s)H(s)] locus must be zero.

Note that although the Nyquist stability criterion can be applied to inverse polar plots, if experimental frequency-response data are incorporated, counting the number of encirclements of the 1/[G(s)H(s)] locus may be difficult because the phase shift corresponding to the infinite semi-circular path in the s plane is difficult to measure. For example, if the open-loop transfer function G(s)H(s) involves transport lag such that
G(S)H(S)=$\frac{Ke^{-j\omega L}}{S(T_S+1)}$
then the number of encirclements of the -1 j0 point by the 1/[G(s)H(s)] locus becomes infinite, and the Nyquiststability criterion cannot be applied to the inverse polar plot of such an open-loop transfer function.

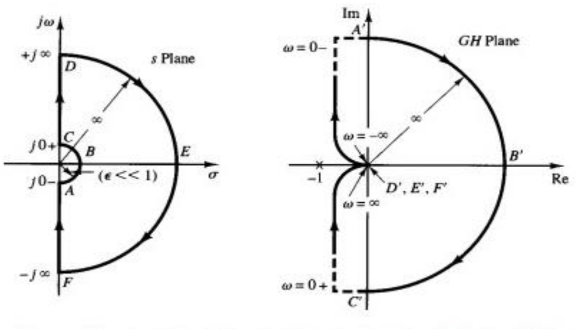
In general, if experimental frequency-response data cannot be put into analytical form, both the G(jω)H(jω) and 1/[G(jω)H(jω)] loci must be plotted. In addition, the number of right-half plane zeros of G(s)H(s) must be determined. It is more difficult to determine the right-half plane zeros of G(s)H(s) (in other words, to determinewhether a given component is minimum phase) than it is to determine the right-half plane poles of G(s)H(s) (in other words, to determine whether the component is stable).

Depending on whether the data are graphical or analytical and whether non-minimum-phase components are included, an appropriate stability test must be used for multiple-loop systems. If the data are given in analyticalform or if mathematical expressions for all the components are known, the application of the Nyquist stability criterion to inverse polar plots causes no difficulty, and multiple-loop systems may be analysed and designed inthe inverse GH plane

8.4 Effect of addition of poles and zeros to G(S)H(S) on the shape of Nyquist plot:

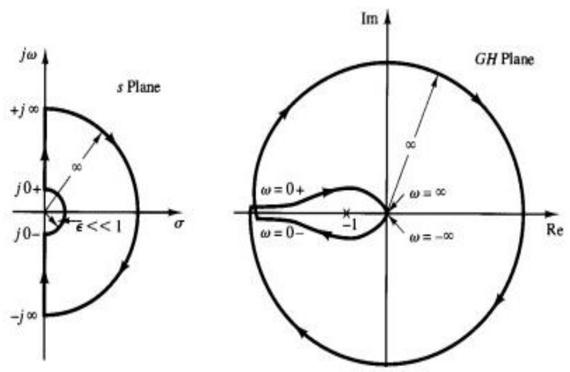
- Typically, the Nyquist path should not go through any pole or zero. Hence, the Nyquist path should be slightly modified to avoid this situation.
- Nyquist path is altered by allowing a semi-circle detour with an infinitesimal radius around this origin.
- The small semi-circle is presented using magnitude and phase $\epsilon e^{j\theta}$
- Note that for type-1 system, $\lim_{s \to \epsilon e^{j\theta}} GH(s) = \frac{1}{\epsilon^2} e^{-j\theta}$
- Note that for type-1 system, $\lim_{s \to \epsilon e^{j\theta}} GH(s) = \frac{1}{\epsilon^2} e^{-j2\theta}$

  Example G(s)=$\frac{K}{[S(1+T_S)]}$



- P=0
- No encirclement from contour mapping N=0
- Z=P+N=0=>the system is stable.

Example: G(s) =$\frac{K}{[S^2(1+T_S)]}$



- P=0 For positive T
- Two clockwise encirclements N=2
- Z=P+N=2=>there exist two zeros for the characteristic's equations in the RHP.
- Hence, the system is unstable.

8.5 Assessment of relative stability:

We have introduced the Nyquist criterion for the absolute stability analysis of the system. Using the Nyquist criterion, it is also possible to find the relative stability of the system .by relative stability we mean how close the system is to instability, and we can improve the stability of the system .the degree or extent of the system is called relative stability. If the Nyquist polar plot is close to -1+j0 point, the system is on the verge of the instability .the proximity to -1+J0point is specified in terms of the following two quantise.

(i)Gain margin.
(ii) phase margin

(i)Gain margin:

The gain margin is defined as the reciprocal of the open -loop transfer function evaluated at the frequency (ω_c) at which the phase angle is -180^0.

Gain margin =$\frac{1}{|G(j\omega)H(j\omega)|}$

$$\varphi =< G(j\omega)H(j\omega).$$

(ii)phase margin:

A rigorous definition of the phase margin ($\emptyset_{PM}$) is the angle between the negative real axis and the radious vector joining the origin to the gain crossover frequency (ω_{gc}). The radius vector is $|G(j\omega)H(j\omega)|$.The gain crossover frequency is the frequency at which $|G(j\omega)H(j\omega)| = 1$,i.e the point of intersection of the polar plot and the (-1,j0) circle.

Phase margin=$180 + \emptyset$

$\emptyset =< G(j\omega)H(j\omega) and\ |G(j\omega)H(j\omega)| = 1$

8.6 Constant M&N circle:

- Constant magnitude loci that are M-circles and constant phase angle loci that are N-circles are the fundamental components

- The constant M and constant N circles in G (jω) plane can be used for the analysis and design of control systems.
- However the constant M and constant N circles in gain phase plane are prepared for system design and analysis as these plots supply information with fewer manipulations.

- Gain phase plane is the graph having gain in decibel along the ordinate (vertical axis) and phase angle along the abscissa (horizontal axis).

- The M and N circles of G (jω) in the gain phase plane are transformed into M and N contours in rectangular co- ordinates.

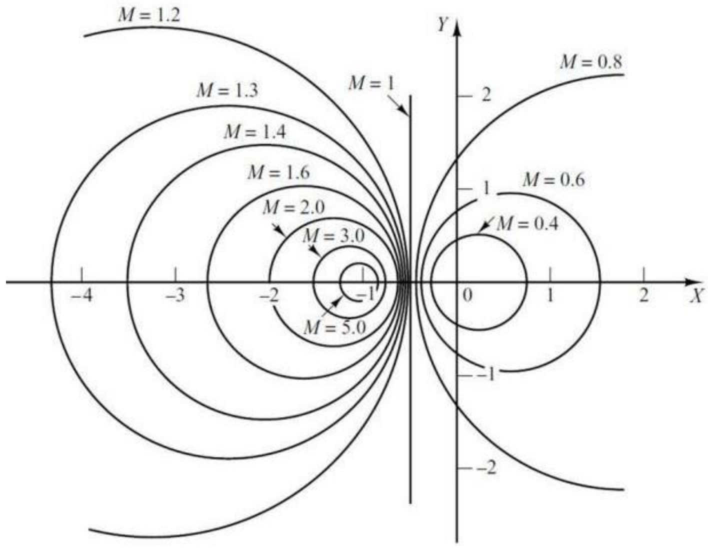
- A point on the constant M loci in G (jω) plane is transferred to gain phase plane by drawing the vector directed from the origin of G (jω) plane to a particular point on M circle and then measuring the length in db and angle in degree.

- The critical point in G (jω), plane corresponds to the point of zero decibel and -180o in the gain phase plane. Plot of M and N circles in gain phase plane is known as Nichols chart /plot.

- The Nichols plot is named after the American engineer N.B Nichols who formulated this plot. Compensators can be designed using Nichols plot.
- Nichols plot technique is however also used in designing of dc motor. This is used in signal processing and control design.

- Nyquist plot in complex plane shows how phase of transfer function and frequency variation of magnitude are related.

- Angle of positive real axis determines the phase and distance from origin of complex plane determines the gain.



8.7 Nicholas chart:

- Nichols chart represents the conversion of M- circle and N- circle from GH plane to gain phase plane.
- (-1, 0) in GH plane is represent by (0 dB, -180 dB) point in gain phase plane and this point is considered as origin of gain phase plane.
- On N-Circle and M-Circle in GH Plane ,Take no. of points and calculate the corresponding gain Db and phase. After calculation of gain and phase ,locate the points in the gain phase plane .By joining these point we can generate the Nichols chart.

Advantages of Nichols chart:

- It is used to find the closed loop frequency response from the open loop frequency response.
- The gain of the system can be adjusted to satisfy the given specifications.
- The frequency domain specifications can be determined from Nicholas chart.

POSSIBLE SHORT TYPE QUESTIONS WITH ANSWERS:

1**.Define encircled?**

Ans-if point is found to be inside the path. The point is said to be encircled by the close path.

2.Define analytic function?

Ans-A function is said to be analytic at a point in a plane. if its value and derivative have finite existence at that point.

3.what do you mean by Nyquist criterion?

Ans-It focus on relative stability of the system. It is possible to determine the stability of a close loop pole from an open loop pole without knowing the roots of close loop system. A Nyquist plot is based on a polar plot.

POSSIBLE LONG TYPE QUESTIONS

1. For G(s)H(s)=1/s(s+2) draw the Nyquist plot and decide stability?
2. For G(s)H(s)=1/s^2(s+2) sketch the Nyquist plot and determine the stability of the system?
3. Explain about the Nichola's chart used in frequency response analysis briefly. [S-22]

CHAPTER NO.-09
STATE VARIABLE ANALYSIS

Learning objectives:

9.1 Concepts of state, state variable, state model,
9.2 state models for linear continuous time functions (Simple)

9.1. CONCEPTS OF STATE, STATE VARIABLE,STATE MODEL

The concepts of state, state variable, and state model are central in fields like systems theory, control theory, and computer science. These concepts are used to describe the dynamic behavior of systems over time.

1. State

- A **state** represents the condition or configuration of a system at a particular point in time. It encapsulates all the necessary information required to describe the system's behavior at that instant.
- The state of a system determines its future behavior, assuming no external influences. If you know the state of the system, you can predict its future states (depending on the rules governing the system).
- **Example**: For a moving car, the state could include its position, speed, and direction at any given time.

2. State Variable

- A **state variable** is a variable that describes one aspect of the state of a system. These are the components or measurable quantities that define the state.
- State variables change over time according to the dynamics of the system (e.g., the input/output behavior or the interactions within the system).
- The collection of state variables represents the entire state of the system.
- **Example**: In a simple model of a car, the state variables might include **position** (how far the car has traveled) and **velocity** (how fast the car is moving).

3. State Model

- A **state model** is a mathematical or conceptual framework that describes how the state of a system evolves over time based on its current state and inputs.
- This model typically consists of a set of **state equations** that describe how state variables change over time. These equations depend on the current state and the inputs (or controls) applied to the system.
- A state model can be represented in different forms, like **continuous** or **discrete** models, depending on the nature of the system (e.g., continuous-time systems or digital systems).
- **Example**: In control systems, a state-space representation often includes a set of differential equations for continuous systems that describe how the state evolves.

Example (in the context of a simple mechanical system):

- **State**: The system is a moving car, and its state is defined by its position and velocity.
- **State Variables**: Position (x) and velocity (v) are the state variables.

- **State Model**: The state model could include the equations of motion (e.g., Newton's laws) that describe how the position and velocity evolve over time based on the forces acting on the car (such as acceleration).

State Variable

A **state variable** is one of the set of variables that are used to describe the mathematical "state" of a dynamical system. Intuitively, the state of a system describes enough about the system to determine its future behaviour in the absence of any external forces affecting the system. Models that consist of coupled first-order differential equations are said to be in state-variable form

- In mechanical systems, the position coordinates and velocities of mechanical parts are typical state variables; knowing these, it is possible to determine the future state of the objects in the system.

- In thermodynamics, a state variable is an independent variable of a state function like internal energy, enthalpy, and entropy. Examples include temperature, pressure, and volume. Heat and work are not state functions, but process functions.

- In electronic/electrical circuits, the voltages of the nodes and the currents through components in the circuit are usually the state variables. In any electrical circuit, the number of state variables are equal to the number of storage elements, which are inductors and capacitors. The state variable for an inductor is the current through the inductor, while that for a capacitor is the voltage across the capacitor.

9.2. STATE MODELS FOR LINEAR CONTINUOUS TIME FUNCTIONS

In the context of **linear continuous-time systems**, state models are used to describe how the system's state evolves over time. These models are essential for understanding and analyzing the behavior of dynamic systems, especially in control theory and signal processing.

The general form of a **state model for linear continuous-time systems** is often expressed in **state-space representation**. In this representation, the system's dynamics are described using a set of **first-order linear differential equations**.

1. State-Space Representation for Linear Continuous-Time Systems

The state-space representation consists of two main equations:

- **State equation** (describes the evolution of the system state)
- **Output equation** (describes how the state relates to the output)

Conclusion

State models for linear continuous-time systems in state-space form provide a powerful and general framework for analysing and controlling dynamic systems. The state equation describes how the system's state evolves, while the output equation relates the system's state to the outputs. Linear systems allow for powerful analysis techniques such as stability analysis, controllability, and observability.

SHORT QUESTIONS WITH ANSWER

Q1. What do you mean by state variable?
Ans : A state variable is one of the set of variables that are used to describe the mathematical "state" of a dynamical system. Intuitively, the state of a system describes enough about the system to determine its future behaviour in the absence of any external forces affecting the system. Models that consist of coupled first-order differential equations are said to be in state-variable form.

Q2. How the state variable electrical circuit is represented?
Ans:In electronic/electrical circuits, the voltages of the nodes and the currents through components in the circuit are usually the state variables. In any electrical circuit, the number of state variables are equal to the number of storage elements, which are inductors and capacitors. The state variable for an inductor is the current through the inductor, while that for a capacitor is the voltage across the capacitor.

Q3. Write down the state space representation of a LTI system ?
Ans: The state space model of Linear Time-Invariant (LTI) system can be represented as,
$\dot{X}=AX(t)+BU(t)$
$Y=CX(t)+DU(t)$

LONG QUESTION

Q1. Derive the state space representation of a series RLC circuit.