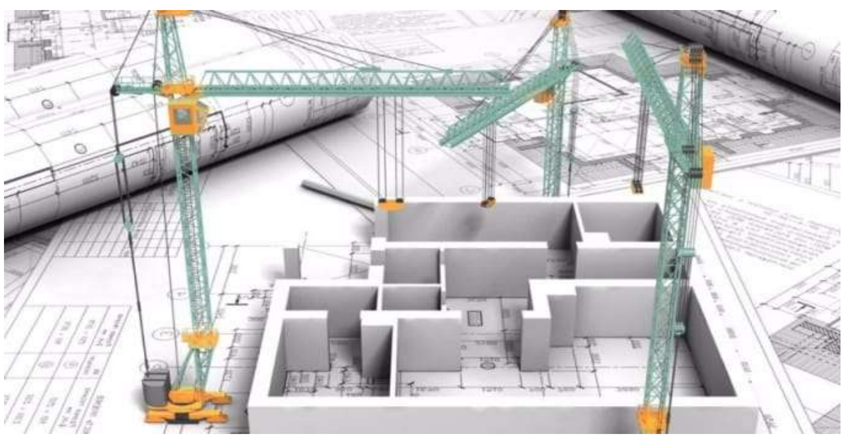
# STRUCTURAL DESIGN –I

# (Th-01)

(As per the 2019-20 syllabus of the SCTE&VT, Bhubaneswar, Odisha)



## Fourth Semester

## Civil Engg.

**Prepared By :** *Er. A.K.MALLICK*

**TOPIC WISE DISTRIBUTION PERIODS**

| Sl. No. | Topic | Periods as per syllabus | Expected marks |
|---|---|---|---|
| 01 | **Working stress method** | 05 | 15 |
| 02 | **Philosophy Limit state method** | 03 | 05 |
| 03 | **Analysis and design of singly reinforced & doubly reinforced sections (LSM ).** | 15 | 10 |
| 04 | **Shear, Bond and Development Length (LSM)** | 04 | 15 |
| 05 | **Analysis and Design of T-Beam (LSM )** | 15 | 15 |
| 06 | **Analysis Design of Slab and Stair case (LSM)** | 15 | 15 |
| 07 | **Design of Axially loaded columns and Footings (LSM )** | 18 | 15 |
| | **Total** | **75** | **100** |

# CHAPTER NO-1

# WORKING STRESS METHOD

## Learning objective

*1.1 Objectives of design and detailing. State the different methods of design of concrete structures.*

*1.2 Introduction to reinforced concrete, R.C. sections their behavior, grades of concrete and steel. Permissible stresses, assumption in W.S.M.*

*1.3 Flexural design and analysis of single reinforced sections from first principles.*

*1.4 Concept of under reinforced, over reinforced and balanced sections*

*1.5 Advantages and disadvantages of WSM, reasons for its obsolescence.*

## 1.1OBJECTIVES OF DESIGN AND DETAILING. STATE THE DIFFERENT METHODS OF DESIGN OF CONCRETE STRUCTURES

### Objectives of design and detailing.

Everystructuremustbe**designed**tosatisfythreebasicrequirements;

**1.Stability**topreventoverturning,slidingorbucklingofthestructure,orpartsofit,undertheactionofloads;

**2.Strengths**toresistsafelythestressesinducedbytheloads inthevariousstructuralmembers;

**3.Serviceability** to ensure satisfactory performance under service load conditions – which impliesproviding adequate stiffness to contain deflections , crack widths and vibrations within acceptablelimits,andalsoprovidingimpermeability,durabilityetc.Therearetwootherconsiderationsthatasensibledesigneroughttobearinmind,viz.economyandaesthetics.

## DifferentMethodsofDesign

Over the years, various design philosophies have evolved in different parts of the world, with regard toreinforcedconcretedesign.Adesignphilosophyisbuiltuponafewfundamentalassumptionsandisreflectiveof awayof thinking.

The following design methods are used for the design of RCC Structures.

a) The working stress method (WSM)

b) The ultimate load method (ULM)

c) The limit state method (LSM)

## Working Stress Method (WSM)

- This method is based on linear elastic theory or the classical elastic theory. This method ensured adequate safety by suitably restricting the stress in the materials (i.e. concrete and steel) induced by the expected working leads on the structures.
  The assumption of linear elastic behaviour considered justifiable since the specified permissible stresses are kept well below the ultimate strength of the material.
- The ratio of yield stress of the steel reinforcement or the cube strength of the concrete to the corresponding permissible or working stress is usually called factor of safety.
- The WSM uses a factor of safety of about 3 with respect to the cube strength of concrete and a factor of safety of about 1.8 with respect to the yield strength of steel.

## Ultimate load method (ULM)

- The method is based on the ultimate strength of reinforced concrete at ultimate load is obtained by enhancing the service load by some factor called as load factor for giving a desired margin of safety .
- Hence the method is also referred to as the load factor method or the ultimate strength method. In the ULM, stress condition at the state of in pending collapse of the structure is analysed, thus using, the non-linear stress – strain curves of concrete and steel.
- The safely measure in the design is obtained by the use of proper load factor. The satisfactory strength performance at ultimate loads does not guarantee satisfactory strength performance at ultimate loads does not guarantee satisfactory serviceability performance at normal service loads.

## Limit state method (LSM)

- Limit states are the acceptable limits for the safety and serviceability requirements of the structure before failure occurs.
- The design of structures by this method will thus ensure that they will not reach limit states and will not become unfit for the use for which they are intended.
- It is worth mentioning that structures will not just fail or collapse by violating (exceeding) the limit states. Failure, therefore, implies that clearly defined limit states of structural usefulness has been exceeded..

# 1.2 INTRODUCTION TO REINFORCED CONCRETE, R.C. SECTIONS

## THEIR BEHAVIOR, GRADES OF CONCRETE AND STEEL. PERMISSIBLE STRESSES, ASSUMPTIONIN W.S.M.

### Reinforced cement concrete:

Plain cement concrete has very low tensile strength. To improve the tensile strength of concrete some reinforcement is needed which can take up the tensile stresses developed in the structure. The most common type of reinforcement is in the form of steel bars which are quite strong in tension. The reinforcing steel is placed in the forms and fresh concrete is poured around it. This solidified composite mass is called reinforced cement concrete.

### RC sections and their behaviour;

**Slabs :**Slabs are the plate elements and carry the lodes primarily by flexures. . they usually carry vertical loads.

Under the action of horizontal loads ,due to a large moment of inertia, they can carry quite large wind and earthquake forces and transfer them to beam

**Beam:** Beams carry loads from slab and also direct loads such as masonry walls and their self weight.

**Columns:** Columns are the vertical members carrying loads from beams and upper columns. The loads may be axial or eccentric . The importance of column is greater than beam and column.

**Foundation**: these are the load transmitting members. The loads from super structure are transmitted to the solid ground through foundation.

### AdvantagesOf ReinforcedConcrete

Thefollowingaremajoradvantagesofreinforcedcementconcrete(RCC)

- ReinforcedCementConcretehasgoodcompressivestress(becauseofconcrete).
- RCCalsohashightensilestress(becauseofsteel).
- Ithasgoodresistancetodamagebyfireandweathering(becauseofconcrete).
- RCCprotectssteelbarsfrombucklingandtwistingatthehightemperature.
- RCCpreventssteelfromrusting.
- ReinforcedConcreteisdurable.
- Themonolithiccharacterofreinforcedconcretegivesitmorerigidity.
- MaintenancecostofRCCispracticallynil.

### Grades of concrete:

Concrete grades are expressed by letter **M** followed by a number. The letter ‘M’ refers to the mix and the number represents the

Characteristics compressive strength of concrete in N/mm2

**TABLE 2, IS456:2000**

| Group | Designation | Characteristics compressive strength $f_{ck}$(N/MM2) |
|---|---|---|
| Ordinary concrete | M10<br>M15<br>M20 | 10<br>15<br>20 |
| Standard concrete | M25<br>M30<br>M35<br>M40<br>M45<br>M50<br>M55 | 25<br>30<br>35<br>40<br>45<br>50<br>55 |
| High strength concrete | M60<br>M65<br>M70<br>M75<br>M80 | 60<br>65<br>70<br>75<br>80 |

## Grade of steel

| Type of steel | Grade | Characteristics strength(N/MM2) |
|---|---|---|
| Mild steel | Fe 250 | 250 |
| High strength deformed steel | Fe 415<br>Fe 500<br>Fe 550 | 415<br>500<br>550 |
| Thermo mechanically treated bars (TMT)or corrosion resistant steel(CRS) | Fe 500 | 500 |
| Steel wired fabric | - | 480 |

## Permissible stress:

The working stress method is based on the concept of permissible stresses. Permissible stresses are obtained by dividing ultimate strength of concrete or yield strength of steel by appropriate factor of safety. The factor of safety used in working stress method are-

**Material** **FactorofSafety**

| | |
|---|---|
| Forconcrete | 3.0 |
| ForSteel | 1.78 |

The permissible stress as per codal provision are given in IS-456:2000, TABLE-21,22

**AssumptionsofWSM**

TheanalysisanddesignofaRCCmemberarebasedonthefollowingassumptions.

(i) Concreteisassumedtobehomogeneous.

(ii) Atanycrosssection,planesectionsbeforebendingremainplaneafterbending.

(iii) Thestress-strainrelationshipforconcreteisastraightline,underworkingloads.

(iv) Thestress-strainrelationshipforsteelisastraightline,underworkingloads.

(v) Concreteareaontensionsideisassumedtobeineffective.

(vi) Alltensilestressesaretakenupbyreinforcementsandnonebyconcreteexceptwhenspeciallypermitted.

(vii) Thesteelareaisassumedtobeconcentratedatthecentroidofthesteel.

(viii) Themodularratiohasthevalue280/3$\sigma_{cbc}$where$\sigma_{cbc}$ispermissiblestressincompressionduetobendinginconcreteinN/mm$^2$asspecifiedincode(IS:456-2000)

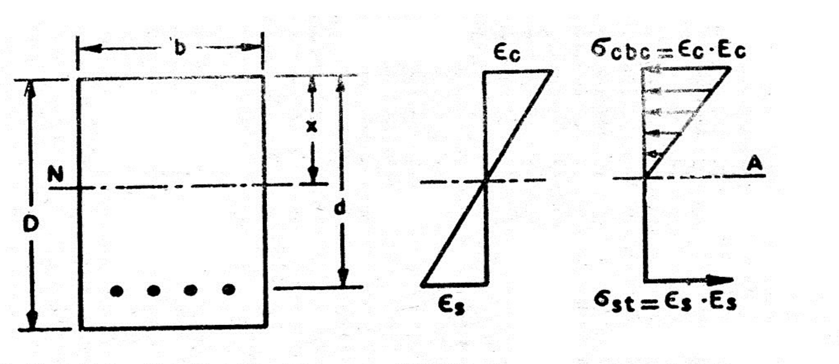
## 1.3 FLEXULAR DESIGN AND ANALYSIS OF SINGLY REINFORCED RECTANGULAR SECTION FROM FIRST PRINCIPLE:

Here ,$b$=widthofsection

$A_s$ =area of steel reinforcement

$D$=overalldepthofsection

$d$=effectivedepthofsection(distancefromextremecompressionfibertothe centroid ofsteel




(a) Rectangular Section with Reinforcement (b) Strain Distribution (c) Stress Distribution

$\epsilon_c$=Maximumstraininconcrete,

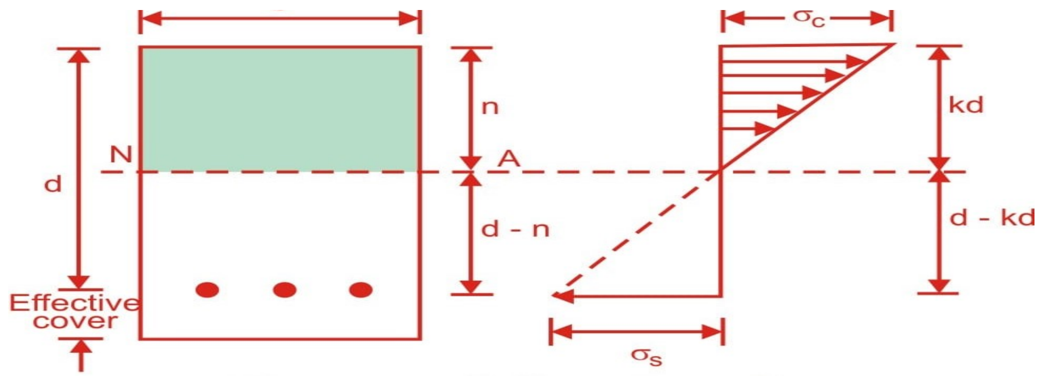
$\epsilon_s$=maximumstrainatthecentroidofthesteel,

$\sigma_{cbc}$=maximumcompressivestressinconcreteinbending

$\sigma_{st}$=permissible stress ininsteel in tension

$E_s/E_c$=ratioofYong'smodulusofelasticityofsteeltoconcrete

=modularratio'$m$'=$\frac{280}{3\times\sigma_{cbc}}$
X=depth of neutral axis
Sincethestrainsinconcreteandsteelareproportionaltotheirdistancesfromtheneutralaxis,



(a) (b) Stress diagram for given loads

1)to find neutral axis: from the strain diagram

$$\frac{x_{bal}}{d-x_{bal}} = \frac{\sigma_{cbc/E_c}}{\sigma_{st/E_s}} = m\frac{\sigma_{cbc}}{\sigma_{s'm'=\frac{280}{3\times\sigma_{cbc}}t}}$$

$x_{bal} = \frac{m\sigma_{cbc}}{m\sigma_{cbc}+\sigma_{st}} \times d = \frac{1}{1+\frac{\sigma_{st}}{m\sigma_{cbc}}} \times d$..................(1)

$$x_{bal} = k \times d$$

Where the constant $k = \frac{1}{1+\frac{\sigma_{st}}{m\sigma_{cbc}}}$

and is known as neutral axis constant

Substitute the value of '$m$'=$\frac{280}{3\times'm'=\frac{280}{3\times\sigma_{cbc}}}$, the value of

$$k = \frac{1}{1+0.0107\sigma st}$$

2)To find lever arm:

From the stress diagram

$$Z = d - \frac{x_{bal}}{3} = d - \frac{kd}{3}$$

$$= d\left(1 - \frac{k}{3}\right) = jd$$

$$=> Z = dj$$

Where the constant j=$d\left(1 - \frac{k}{3}\right)$is known as lever arm constant

3) To find total force:

Totalcompressiveforce =C=$\frac{1}{2} \times \sigma_{cbc} \times b \times x_{bal} = \frac{b \times x_{bal} \times \sigma_{cbc}}{2}$

Totaltensileforces=T=$\sigma_{st}$×$A_{stbal}$=

4) to fond the moment of resistance:

Capacity of a section to resist the moment is know as moment of resistance (M.R).this is equal to total compressive

Force or tensile force ×lever arm.

Considering the compressive force

Momentofresistance

=*MR*=total compression

×lever arm

$$M_{bal} = \frac{1}{2} \times \sigma_{cbc} \times b \times x_{bal} \times dj$$

$$= \frac{1}{2} \times \sigma_{cbc} \times b \times kd \times dj$$

$$= \frac{1}{2} \times \sigma_{cbc} \times b \times kj \times d^2$$

$$M_{bal} = Q_{bal} \times bd^2$$

Where the constant $Q_{bal} = \frac{1}{2} \times \sigma_{cbc} \times k \times j$ is known as moment of resistance factor for balanced rectangular section.

Considering the tensile force

M.R=total tension × lever arm

$M_{bal}$=$\sigma_{St}$×$A_{stbal}$× d ×j

Steel area,

$$A_{st} = \frac{M_{bal}}{\sigma_{st} \times dj}$$

Percentage steel Pt=$100 \times \frac{A_{st}}{bd}$

For a balanced section $p_{tbal} = 100 \times \frac{A_{stbal}}{bd}$

6) To design balanced section

For a given moment M, consider M= $M_{bal}$ if width ‘b’ of the beam is assumed.

$$\sqrt{\frac{M_{bal}}{Q_{bal} \times d}}$$

Area of steel $A_{st} = A_{stbal} = M / \sigma_{st} . d . j$

Design constant for balanced section

| Concrete grade | Steel grade | $\sigma_{cbc}$ | $\sigma_{st}$ | k | j | $Q_{bal}$ | $P_{tbal}$ |
|---|---|---|---|---|---|---|---|
| $M_{20}$ | Fe 250 | 7 | 140 | 0.4 | 0.87 | 1.21 | 1.0 |
| $M_{20}$ | Fe 415 | 7 | 230 | 0.29 | 0.9 | 0.91 | 0.44 |

**Example1: For a rectangular beam of size 250mm wide and 520mm effective depth, find out the balanceddepth of neutral axis, balanced lever arm, balanced moment of resistance and balanced steel area. The materials are M20 grade of concrete and HYSD reinforcement of grade Fe415.**

**Solution:**

Given b=250mm

d=520mm

from table $\sigma_{cbc}$ =7N/mm$^2$

$\sigma_{st}$=230 N/MM$^2$

K= 0.29

$Q_{bal}$=0.91

$p_{tbal}$ =0.44

j=0.90

depth of neutral axis, $X_{bal}$ =kd=0.29×520=150.8mm

lever arm Z=j×d=0.29×520=468mm

moment of resistance of balanced section ,M.R=$Q_{bal} \times bd^2$=0.91×250×520$^2$×10$^{-6}$=61.52KN.m

$$A_{stbal} = 100 \times \frac{\text{ptbal}}{bd} = \frac{0.44 \times 250 \times 520}{100} = 572mm^2$$

**Example2: calculate the design constants for the following materials considering the balanced design for singly reinforced section. The materials are M20 concrete and mild steel reinforcement.**

**Solution :**For M20 mix $\sigma_{cbc}$ =7N/mm$^2$

$\sigma_{st}$=230 N/MM$^2$

$m = \frac{280}{3\times7} = 13.33$

neutral axis constant $k = \frac{1}{1+\frac{\sigma_{st}}{m\sigma_{cbc}}}$

$$\frac{1}{1+\frac{140}{13.33\times7}} = 0.4$$

Lever arm constant , $j = \left(1-\frac{k}{3}\right) = \left(1-\frac{0.4}{3}\right) = .866 \cong 0.87$

$$\text{moment of resistance factor} Q_{bal} = \frac{1}{2} \times \sigma_{cbc} \times k \times j = \frac{1}{2} \times 7 \times 0.4 \times 866 = 1.21$$

$$p_{tbal} = \frac{50\times\sigma_{cbc}\times k}{\sigma_{st}} = \frac{50\times7\times0.4}{140} = 1.0$$

# 1.4 CONCEPT OF UNDER REINFORCED,OVER REINFORCED AND BALANCED SECTION

## Underreinforcedsection

When the percentage of steel in a section is less than that required for a balanced section, thesection is called 'Under-reinforced section.' In this case (Fig.2.2) concrete stress does not reach itsmaximum allowable value while the stress in steel reaches its maximum permissible value. The positionof the neutral axis will shift upwards, i.e., the neutral axis depth will be smaller than that in the balancedsection as shown in Figure2.2.

The moment of resistance of such a section will be governed by allowable tensilestressinsteel.

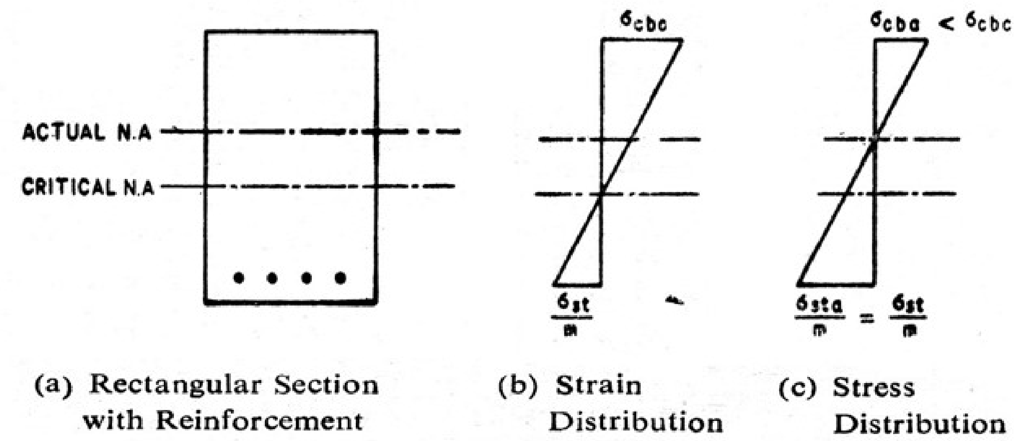


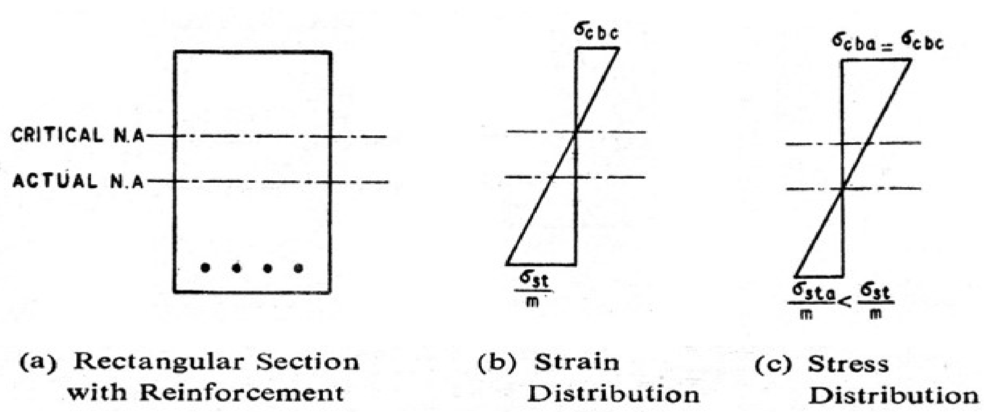

Fig.2.2(a-c)

## **Overreinforcedsection**:

When the percentage of steel in a section is more than that required for a balanced section, thesectioniscalled ‘Over-reinforcedsection’.Inthiscase(Fig.2.3)thestressinconcretereachesitsmaximum allowable value earlier than that in steel. As the percentage steel is more, the position of theneutral axis will shift towards steel from the critical or balanced neutral axis position. Thus the neutralaxisdepthwillbegreaterthanthatincaseofbalancedsection.

## **Balanced section:**

When the maximum stresses in steel and concrete simultaneously reach theirallowable values, the section is said to be a ‘Balanced Section’. The moment of resistance shall beprovided by the couple developed by compressive force acting at the centroid of stress diagram on thearea of concrete in compression and tensile forces acting at the centroid of reinforcement multiplied bythedistancebetweentheseforces.Thisdistanceisknownas‘lever




**Fig(2.3)**

## 1.5 ADVANTAGES AND DISADVANTAGES OF WSM:-

### Advantages :

1. It is a simple method.
2. Due to it simplicity it is still used for design for some complex structure such as over head water tank

### Disadvantages:

1. The assumption of linear elastic behaviour and control of stresses within specially defined permissible stresses are unrealistic due to several reason viz, creep, shrinkage and other long term effect, stress concentration and other secondary effects.
2. Different type of load acting simultaneously have different degree of uncertainties.
3. The actual factor of safety is not known in this method of design.

## POSSIBLE SORT TYPE QUESTIONS WITH ANSWER

**1.what do you mean by working stress method? [2010]**

Ans : In WSM it is assumed that structural material e.g. concrete and steel behave in linearly elasticmanner and adequate safety can be ensured by restricting the stresses in the material induced by working loads(service loads)on thestructure.

**2.What is factor of safety?[2011]**

Ans : Theratioofthestrengthofthematerial to thepermissible stressisoften referredto as thefactorofsafety.

In working stress methods factor of safety for concrete and steel are taken as 3 and 1.78 respectively.

**3.Define modular ratio.[2011, W-2015 ,W-2019]**
*Ans :R*atioofYong'smodulusofelasticityofsteeltoconcrete

$E_s/E_c$=modular ratio

**4.what do you mean by under reinforced section?[W-2018]**

Ans : When the percentage of steel in a section is less than that required for a balanced section, thesection is called 'Under-reinforced section.' In this case concrete stress does not reach itsmaximum allowable value while the stress in steel reaches its maximum permissible value.

**5.what do you mean by balanced section?**

Ans: When the maximum stresses in steel and concrete simultaneously reach theirallowable values, the section is said to be a 'Balanced Section .

## POSSIBLE LONG TYPE QUESTIONS

1. What are the advantages and disadvantages of WSM?[2019]

2.Write the advantages of RCC.[2015,S-2017,2019BP]

3.What are the assumptions for WSM?

4. A RCC beam 250mm wide and 400mm effective depth is reinforced with 4-12mm dia bars in tension . find out the depth of N.A and state the type of beam. The materials are M20 grade conc. and Fe415 grade steel using WSM. [W-2019]

# CHAPTER NO.-2

# PHILOSOPHY OF LIMITSTATEMETHOD

## Learning objective

*2.1 Definition, Advantages of LSM over WSM, IS code suggestions regarding design philosophy.*

*2.2 Types of limit states, partial safety factors for materials strength, characteristic strength, characteristic load, design load, loading on structure as per I.S. 875*

*2.3 Study of I.S specification regarding spacing of reinforcement in slab, cover to reinforcement in slab, beam column & footing, minimum reinforcement in slab, beam & column, lapping, anchorage, effective span for beam & slab.*

## 2.1 DEFINITION, ADVANTAGES OF LSM OVER WSM, IS CODE SUGGESTIONS REGARDING DESIGN PHILOSOPHY

### Definition of LSM

Forensuringthedesignobjectives,thedesignshouldbebasedoncharacteristicvaluesformaterialstrengths and applied loads (actions), which take into account the probability of variations in the materialstrengths and in the loads to be supported. The characteristic values should be based on statistical data, ifavailable. Where such data is not available, they should be based on experience. The design values are derivedfromthecharacteristicvaluesthroughtheuseofpartialsafetyfactors,bothformaterialstrengthsandforload s.Intheabsenceofspecialconsiderations,thesefactorsshouldhavethevaluesgiveninthissectionaccordingtothe material,the type of load and the limitstatebeing considered.

### Advantages of working stress method and limit state method

| Working Stress Method | Limit State Method |
|---|---|
| This method is based on the elastic theory which assumes that concrete and steel are elastic and the stress strain curve is linear for both | This method is based on the actual stress-strain curves of steel and concrete. For concrete the stress-strain curve is non-linear. |
| In this method the factor of safety are applied to the yield stresses to get permissible stresses. | In this method, partial safety factors are applied to get design values of stresses |

| | |
|---|---|
| No factor of safety is used for loads. | Design loads are obtained by multiplying partial safety factors of load to the working loads |
| Exact margin of safety is not known | Exact margin of safety is known |
| This method gives thicker, sections, so less economical | This method is more economical as it gives thinner sections |
| This method assumes that the actual loads, permissible stresses and factors of safety are known. So it is called as deterministic method | This method is based upon the probabilistic approach which depends upon the actual data or experience, hence it is called as non-deterministic method. |

### IS code suggestions regarding design philosophy

**ACCORDING TO IS-456:2000**

Cl no.18.2.1Structureand structural elements shall normally be designed by Limit State Method. Account should be taken of accepted theories, experiment and experience and the need to design for durability .Calculations alone do not produce safe, serviceable and durable structures. Suitable materials, quality control. adequate detailing and good supervision are equally important.

Cl no.18.2.2 Where the Limit State Method can not be conveniently adopted, Working Stress Method *(see* Annex B) may be used.

## 2.2TYPES OF LIMIT STATES, PARTIAL SAFETY FACTORS FOR MATERIALS STRENGTH,CHARACTERISTIC STRENGTH, CHARACTERISTIC LOAD, DESIGN LOAD, LOADING ON STRUCTURE AS PER I.S. 875

### Types of limit state

Limitstatesarethestatesbeyondwhichthestructurenolongersatisfiestheperformancerequirementsspecified.The limitstates areclassified as

i)Limitstateofstrength
ii)Limitstateofserviceability

i)**The limit state of strength** are those associated with failures (or imminent failure),under the action ofprobableandmostunfavourablecombinationofloadsonthestructureusingtheappropriatepartialsafetyfactors, whichmayendangerthesafetyoflifeandproperty.Thelimitstateofstrengthincludes:

a) Lossofequilibriumofthestructureasawholeoranyofitspartsorcomponents.

b) Lossofstabilityofthestructure(includingtheeffectofswaywhereappropriateandoverturning )or anyofitspartsincludingsupportsandfoundations.

c) Failurebyexcessivedeformation,ruptureofthestructureoranyofitspartsorcomponents.

d) Fractureduetofatigue.

e) Brittlefracture.

ii)Thelimitstateofserviceabilityinclude

f) Deformation and deflections, which may adversely affect the appearance or, effective, use of thestructure or may cause improper functioning of equipment or services or may cause damages tofinishesandnon-structuralmembers.

g) Vibrations in the structure or any of its components causing discomfort to people, damages to thestructure,its contents orwhich may limit its functional effectiveness.Special consideration shallbe given to floor vibration systems susceptible to vibration, such as large open floor areas free ofpartitionstoensurethatsuchvibrationsisacceptablefortheintendeduseandoccupancy.

h) Repairabledamageduetofatigue.

i) Corrosionanddurability.

## LimitStatesofServiceability

Tosatisfythelimitstateofserviceabilitythedeflectionandcrackinginthestructureshallnotbeexcessive. Thislimitstatecorrespondstodeflectionandcracking.

**Deflection**

Thedeflectionofastructureorpartshallnotadversely affecttheappearanceorefficiency ofthestructureor finishes orpartitions.

**Cracking**

Cracking of concrete should not adversely affect the appearance or durability of the structure; theacceptablelimitsofcrackingwouldvarywiththetypeofstructureandenvironment.Theactualwidthofcracks will vary between the wide limits and predictions of absolute maximum width are not possible. Thesurface widthof crackshouldnotexceed0.3mm.

In members where cracking in the tensile zone is harmful either because they are exposed to the effects of theweather or continuously exposed to moisture or in contact soil or ground water, an upper limit of 0.2 mm issuggested for the maximum width of cracks. For particularly aggressive environment, such as the ‘severe’category,theassessedsurfacewidthofcracksshouldnotingeneral,exceed0.1mm.

## PartialSafetyFactors:

### 1. PartialSafetyFactor$\gamma_f$forLoads

| Sr.No. | LoadCombination | UltimateLimitState | ServiceabilityLimitState |
|---|---|---|---|
| 1 | DL+ LL | 1.5(DL+LL) | DL+ LL |
| 2 | DL+WL i) DLcontributeto stability ii) DL assistsoverturning | 0.9DL+1.5WL 1.5(DL +WL) | DL + WLDL+ WL |
| 3 | DL+LL+WL | 1.2(DL +LL+WL) | DL+0.8LL+0.8WL |

| Sr.No. | Material | UltimateLimitState | ServiceabilityLimit State |
|---|---|---|---|
| 1 | Concrete | 1.50 | $E_c$=5000$\sqrt{f_{ck}}$MPa |
| 2 | Steel | 1.15 | $E_s$=2x$10^5$MPa |

## 2.PartialSafetyFactor$\gamma_m$forMaterialStrength

When assessing the strength of a structureorstructuralmemberfor the limitstate of collapse, the values ofpartialsafetyfactor,shouldbetakenas1.5forconcreteand1.15forsteel.

## CharacteristicStrengthofMaterials

Characteristic strength means that value of the strength of the material below which not more than 5percentofthetestresultsareexpectedtofallandisdenotedby$f$.Thecharacteristicstrengthofconcrete($f_{ck}$) is as per the mix of concrete.The characteristicstrength of steel ($f_y$) is the minimum stress or 0.2 percent ofproofstress.

## CharacteristicLoads

Characteristic load means that value of load which has a 95 percent probability of not being exceededduringthelifeofthestructure.Sincedataarenotavailabletoexpressloadsinstatisticalterms,forthepurposeofthisstandard,deadloadsgiveninIS875(Partl),imposedloadsgiveninIS875(Part2),windloads giveninIS875(Part3),snowloadasgiveninIS875(Part**4)**andseismicforcesgiveninIS1893-2002(part-I)shallbe assumed asthe characteristicloads

## DesignValueMaterials

Thedesignstrengthofthematerials$f_d$isgivenby

$$f_d = \frac{f}{\gamma_m}$$

where

f=characteristicstrengthofthematerial

$\gamma_m$=partialsafetyfactorappropriatetothematerialandthelimitstatebeingconsidered.

**Load**

Thedesignload,F,isgivenby

$$f_d = \frac{F}{\gamma_f}$$

Where,F=characteristicload

and$\gamma_f$=partialsafetyfactorappropriatetothenatureofloadingandthelimitstatebeingconsidered.

## ConsequencesofAttainingLimitState(IS456:2000)

Wheretheconsequencesofastructureattainingalimitstateareofaseriousnaturesuchashugelossoflife anddisruptionoftheeconomy,highervaluesfor$\gamma_f$and$\gamma_m$applied.

## loading on structure as per I.S. 875

IS : 875 is divided in to the following five parts :

Part 1 Dead loads

Part 2 Imposed loads

Part3 Wind loads

Part 4 Snow loads Part

Part 5 Special loads and load combinations

TABLE 1 UNIT WEIGHT OF BUILDING MATERIALS — *Contd*

| MATERIAL | NOMINAL SIZE OR THICKNESS mm | WEIGHT/MASS kN | kg | per |
|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) |
| 21. *Cement Concrete, Prestressed* ( conforming to IS : 1343-1980* ) | — | 23·50 | 2 400 | $m^3$ |
| 22. *Cement Concrete, Reinforced* | | | | |
| With sand and gravel or crushed natural stone aggregate: | | | | |
| With 1 percent steel | — | 22·75 to 24·20 | 2 310 to 2 470 | ,, |
| With 2 percent steel | — | 23·25 to 24·80 | 2 370 to 2 530 | ,, |
| With 5 percent steel | — | 24·80 to 26·50 | 2 530 to 2 700 | ,, |
| 23. *Cement Concrete Pipes* ( *see* under 41 'Pipes' in this table ) | | | | |
| 24. *Cement Mortar* | — | 20·40 | 2 080 | ,, |
| 25. *Cement Plaster* | — | 20·40 | 2 080 | ,, |

TABLE 1 IMPOSED FLOOR LOADS FOR DIFFERENT OCCUPANCIES

(Clauses 3.1, 3.1.1 and4.1.1 )

| SL NO. (1) | OCCUPANCY CLASSIFICATION (2) | UNIFORMLY DISTRIBUTED LOAD ( UDL ) (3) $Kn/m^2$ | CONCENTRATED LOAD (4) kN |
|---|---|---|---|
| **1** | i ) RESIDENTIAL BUILDINS<br>a) Dwelling houses:<br>1) All rooms and kitchens<br>2) Toilet and bath rooms<br>3) Corridors, passages, staircases including tire escapes and store rooms<br>4)Balconies | 2.0<br>2.0<br>3.0<br>3.0 | 1.8<br>----<br>4.5<br>1.5 per metre run concentrated at the outer edge |
| 2 | b) Dwelling units planned and executed in accordance with IS : 888S1979* only:<br>1) Habitable rooms, kitchens, toilet and bathrooms<br>2) Corridors, passages and staircases including fire escapes<br>3) Balconies | 1.5<br>1.5<br>3.0 | 1.4<br>1.4<br>1.5 per metre run concentrated at the outer edge |

| | | | |
|---|---|---|---|
| 3 | C) Hotels, hostels, boarding houses, lodging houses, dormitories, residential clubs:<br>1) Living rooms, bed rooms and dormitories<br>2) Kitchens and laundries<br>3)Billiards room and public lounges<br>4)Store rooms<br>5)Dining rooms, cafeterias and restaurants<br>6)Office rooms<br>7)Rooms for indoor games<br>8)Baths Lind toilets<br>9)Corridors, passages, staircases including fire escapes, lobbies -- as per the floor serviced ( excluding stores and the like ) but not less than<br>10)Balconies | 2.0<br>3.0<br>3.0<br>5.5<br>4.0<br>2.5<br>3.1<br>2.0<br>3.0<br>Same as rooms to which they give access but with a minimum of 4’0 | 1.8<br>4.5<br>2.7<br>4.5<br>2.7<br>2.7<br>1.8<br>4.5<br>1.5 per metre run concentrated at the outer edge |
| 4 | d) Boiler rooms and plant rooms - to be calculated but not less than | 5.0 | 6.7 |

## 2.3 STUDY OF I.S SPECIFICATION REGARDING SPACING OF REINFORCEMENT IN SLAB, COVER TO REINFORCEMENT IN SLAB, BEAM COLUMN & FOOTING, MINIMUM REINFORCEMENT IN SLAB, BEAM & COLUMN, LAPPING, ANCHORAGE, EFFECTIVE SPAN FOR BEAM & SLAB

SPACING AND REINFORCEMENT FOR DIFFERENT MEMBER

(IS 456:2000,CL NO:26.3-26.5.3) page-45

Cover to reinforcement-table16 IS 456:2000

ANCHORAGE(IS456:2000,CL NO-26.2.2-26.2.2.4)page 43

EFFECTIVE SPAN(IS456:2000,CL NO-22.3) Page-34

# POSSIBLE SHORT TYPE QUESTIONS WITH ANSWER

**1. What is limit state?[W-2014]**

Ans: Limit state is defined as the acceptable limit of safety and serviceability requirements before failure.

**2.what is limit state of collapse?**

Ans: The limit state corresponding to maximum load carrying capacity is known as limit state of collapse.

**3.How can we calculate the design load?**

Ans: The design loads are obtained by multiplying characteristic loads and the appropriate partial safety factor.

**4. Define limit state of serviceability.[W-2017]**

Ans: Tosatisfythelimitstateofserviceabilitythedeflectionandcrackinginthestructureshallnotbeexcessive. Thislimitstatecorrespondstodeflectionandcracking

**5. What is characteristic strength of materials?[W-2015,W-2018,W-2019]**

Ans: characteristic strength of materials means that the value of strength of materials below which not more than 5% of the test result are expected to fall.

**6.What is characteristic load?**

Ans: Characteristic load means that value of load which has a 95 percent probability of not being exceededduringthelifeofthestructure.

# POSSIBLE LONG TYPE QUESTIONS:

1.What is the difference between WSM and LSM?[2015,W-2019]

2.What are the different types of limit state of design?

3. Write the IS specifications regarding spacing of reinforcement in slab and beam.

4. Write the assumption made for flexure in LSM design.[W-2014,W-2015,W-2017]

# CAPTER NO.-3

# ANALYSIS AND DESIGN OF SINGLY REINFORCED SECTION AND DOUBLY REINFORCER SECTION(LSM)

## Learning objectives

*3.1 Limit state of collapse (flexure), Assumptions, Stress-Strain relationship for concrete and steel, neutral axis, stress block diagram and strain diagram for singly reinforced section.*
*3.2 Concept of under- reinforced, over-reinforced and limiting section, neutral axis co-efficient, limiting value of moment of resistance and limiting percentage of steel required for limiting singly R.C. section.*
*3.3 Analysis and design: determination of design constants, moment of resistance and area of steel for rectangular sections.*
*3.4 Necessity of doubly reinforced section, design of doubly reinforced rectangular section.*

## 3.1 LIMIT STATE OF COLLAPSE (FLEXURE), ASSUMPTIONS, STRESS-STRAIN RELATIONSHIP FOR CONCRETE AND STEEL, NEUTRAL AXIS, STRESS BLOCK DIAGRAM AND STRAIN DIAGRAM FOR SINGLY REINFORCED SECTION.

### Limitstateofcollapse:flexure

This limit state refers to the strength of the structure. A structure or its parts should be strong enough to resist the design applied loads. This is called limit state of collapse.

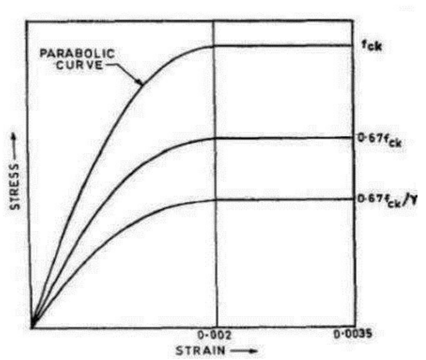
**AssumptionsforLimitStateofCollapse(Flexure):**

The basic assumptions involved in the analysis at the ultimate limit state of flexure (Cl. 38.1 of the Code) are listed here.

a) Plane sections normal to the beam axis remain plane after bending, i.e., in an initially straight beam, strain varies linearly over the depth of the section.

b) The maximum compressive strain in concrete (at the outermost fibre) shall be taken as 0.0035 in bending.

c) The relationship between the compressive stress distribution in concrete and the strain in concrete may be assumed to be rectangle, trapezoid, parabola or any other shape which results in prediction of strength in substantial agreement with




STRESS-STRAIN CURVE FOR CONCRETE

the results of test. An acceptable stress-strain curve is given in figure .

For design purposes, the compressive strength of concrete in the structure shall be assumed to be 0.67 times the characteristic strength. The partial safety factor y, = 1.5 shall be applied in addition to this.

d) The tensile strength of the concrete is ignored.

e) The stresses in the reinforcement are derived from representative stress-strain curve for the type of steel used. Typical curves are given in figure 1.3. For design purposes the partial safety factor $\gamma_m$ equal to 1.15 shall be applied.

f) The maximum strain in the tension reinforcement in the section at failure shall not be less than: $\frac{f_y}{1.15 \times E_s} + 0.002$

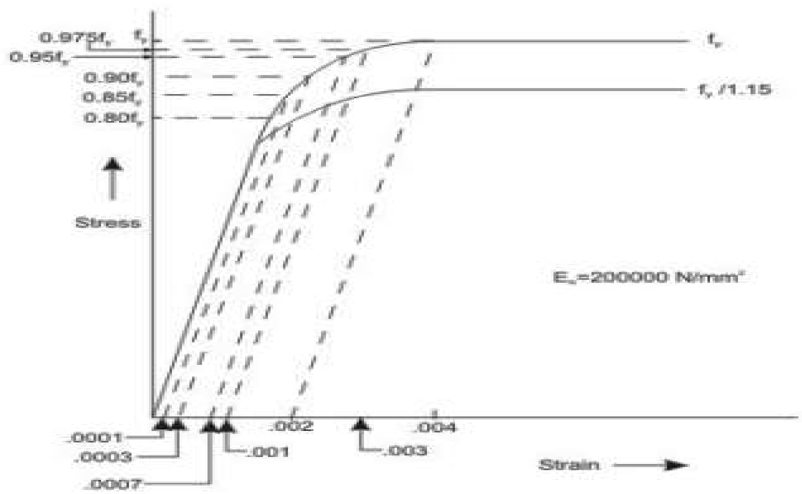
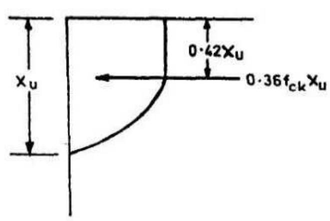
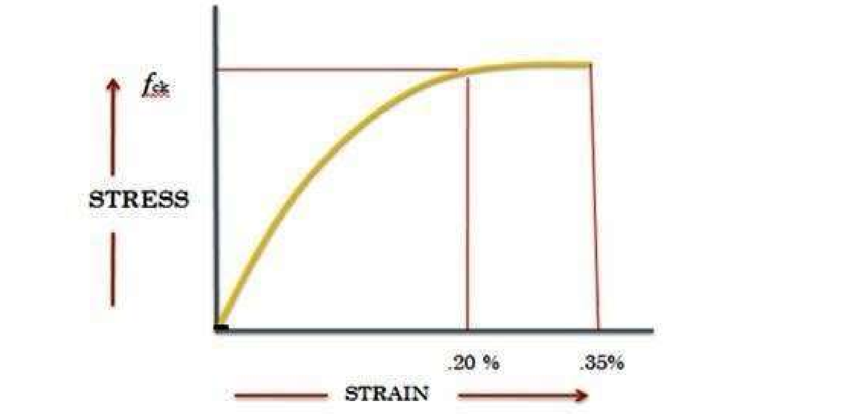



Figure1.3: Stress-strain curve for cold worked deform bar




STRESS BLOCK PARAMETERS




STRESS – STARIN CURVE FOR STEEL

Fordesignpurposes,thecompressivestrengthofconcreteinthestructureshallbeassumedtobe0.67timesthecharacteristicstrength.Thepartialsafetyfactor$\gamma_m$=1.5shallbeappliedinadditiontothis.

**NOTE-**Fortheabovestress-

straincurvethedesignstressblockparametersareasfollows:Areaof stressblock=0.36.$f_{ck}$.$x_u$

Depthofcentreofcompressiveforce=0.42$x_u$fromtheextremefibreincompressionWhere

$f_{ck}$=characteristiccompressivestrengthofconcret

e,and$x_u$=depth ofneutral axis.

4) Thetensilestrengthoftheconcreteisignored.

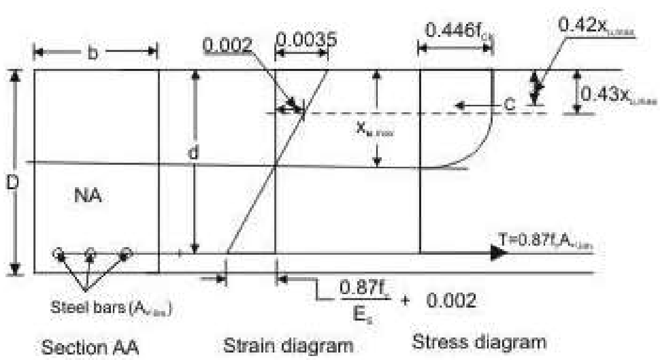
5) Thestressesinthereinforcementarederivedfromrepresentativestress–straincurveforthetypeof steelused.

.

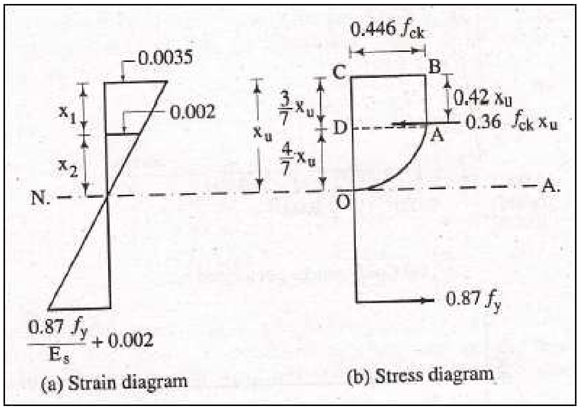
**Neutral axis :**

Neutral axis is the axis at which the stresses are zero and it is situated at the centre of gravity of the section. Depth of neutral axis for singly reinforced beam is calculated by taking equilibrium of tension and compression.

**Stress strain block diagram for singly reinforced section:**




**(fig 1.8)**




Based on the assumption given above, an expression for the depth of the neutral axis at the ultimate limit state, $x_u$, can be easily obtained from the strain diagram in Fig.
Considering similar triangles,

$$\frac{x_u}{d} = \frac{0.0035}{0.0035 + \frac{0.87 f_y}{E_s} + 0.002}$$

According to IS 456: 2000cl no 38.1 (f), when the maximum strain in tension reinforcement is equal to $\frac{0.87 f_y}{E_s} + 0.002$, then the value of neutral axis will be $x_{u,\max}$.

Therefore,$\frac{x_{u,\max}}{d} = \frac{0.0035}{0.0035+\frac{0.87 f_y}{E_s}+0.002}$……………………..(2)

The values of $x_{u,\max}$ for different grades of steel, obtained by applying Eq. (2), are listed in table.

| $f_y$ | $\frac{X_{umax}}{d}$ |
|---|---|
| 250 | 0.53 |
| 415 | 0.48 |
| 500 | 0.46 |

The limiting depth of neutral axis $x_{u,\max}$corresponds to the so-called balanced section, i.e., a section that is expected to result in a _balanced' failure at the ultimate limit state in flexure. If the neutral axis depth u x is less than $x_{u,\max}$, then the section is under-reinforced (resulting in a _tension' failure); whereas if u x exceeds $x_{u,\max}$, it is over-reinforced (resulting in a _compression' failure)

Analysis of Singly Reinforced Rectangular Sections Analysis of a given reinforced concrete section at the ultimate limit state of flexure implies the determination of the ultimate moment Mu of resistance of the section. This is easily obtained from the couple resulting from the flexural stresses (Fig 1.9)

$M_{u=}C_u \cdot z = T_u \cdot z$ ………………….(3)

where $C_u$ and $T_u$ are the resultant (ultimate) forces in compression and tension respectively, and $z$ is the lever arm.

$T_u = f_{st} \cdot A_{st}$ …………………………..(4)

Where $f_{st} = 0.87 f_y$ for $x_u \leq x_{u,\max}$ and the line of action of $T_u$ corresponds to the level of the centroid of the tension steel.

**Concrete Stress Block in Compression**

In order to determine the magnitude of $C_u$ and its line of action, it is necessary to analyse the concrete stress block in compression. As ultimate failure of a reinforced concrete beam in flexure occurs by the crushing of concrete, for both under-and over-reinforced beams, the shape of the compressive stress distribution ('stress block') at failure will be, in both cases, as shown in Fig. 1.9. The value of $C_u$ can be computed knowing that the compressive stress in concrete is uniform at $0.447 f_{ck}$ for a depth of $3x_u/7$, and below this it varies parabolically over a depth of $4x_u/7$ to zero at the neutral axis [Fig. 1.9].

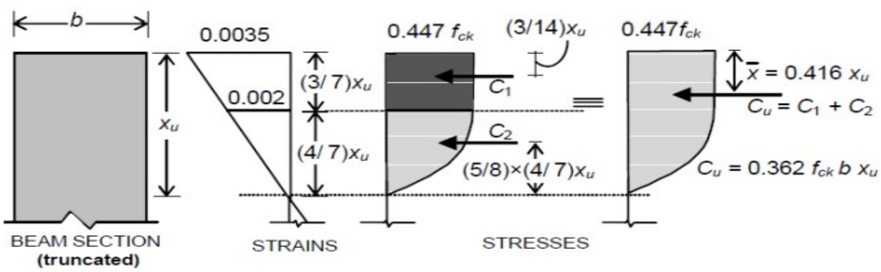



Fig. 1.9 Concrete stress-block parameters in compression

For a rectangular section of width $b$,

$$C_u = 0.447 f_{ck} \mathrm{b} \left[\frac{3x_u}{7} + \left(\frac{2}{3} \times \frac{4x_u}{7}\right)\right]$$

Therefore, $C_u = 0.361 f_{ck} b x_u$………………………………(5)

Also, the line of action of $C_u$ is determined by the centroid of the stress block, located at a distance $\bar{\boldsymbol{I}}$ from the concrete fibres subjected to the maximum compressive strain. Accondingly, considering moments of compressive forces $C_2, C_1$ and $C_2 [Fig - 1.9]$ about the maximum compressive strain location,

$$(0.362 f_{ck} b x_u) x.\pi = (0.447 f_{ck} b x_u) \left[\left(\frac{3}{7}\right)\left(\frac{1.5x_u}{7}\right) + \left(\frac{2}{3} \times \frac{4}{7}\right)\left(x_u - \frac{5}{8} \times \frac{4x_u}{7}\right)\right]$$

Solving $\bar{x} = 0.416 x_u$ ………………………..(6)

**Depth of Neutral Axis**

For any given section, the depth of the neutral axis should be such that $C_u = T_u$, satisfying cquilibrium of forces. Equating $C_u = T_u$, with expressions for $C_u$ and $T_u$ given by Eq. (5) and Fq. (4) respectively.

$x_u = \frac{0.87 f_y A_{st}}{0.361 f_{ck} b}$…………………………………(7)

valid only if resulting $x_u \leq x_{\text{umax}}$

## 3.2 CONCEPT OF UNDER- REINFORCED, OVER-REINFORCED AND LIMITING SECTION, NEUTRAL AXIS CO-EFFICIENT, LIMITING VALUE OF MOMENT OF RESISTANCE AND LIMITING PERCENTAGE OF STEEL REQUIRED FOR LIMITING SINGLY R.C. SECTION.

### Concept of under reinforced, over reinforced, limiting section

- Sectioninwhich,tensionsteelalsoreachesyieldstrainsimultaneouslyastheconcretereachesthefailure straininbending arecalled,**'Balanced Section' or limiting section**
- Sectioninwhich,tensionsteelalsoreachesyieldstrainatloadslowerthantheloadatwhichconcretereacheshesthefailure strainin bendingarecalled,**'UnderReinforcedSection'.**
- Sectioninwhich,tensionsteelalsoreachesyieldstrainatloadshigherthantheloadatwhichconcretereachesthefailure strainin bendingarecalled,**'OverReinforcedSection'.**

### Depth of neutral axis($x_u$ )

Depth of neutral axis($x_u$ )can be calculated by equating total tension to total compression

Total compression=total tension

$0.36f_{ck}bx_u=0.87f_yA_{st}$

$$x_u = \frac{0.87 \times f_y \times A_{st}}{0.36 \times f_{ck} \times b}$$

Note that the value of neutral axis as obtained above should not exceed$X_{umax}$ for a given section.

Modes of failure: Types of section
A reinforced concrete member is considered to have failed when the strain of concrete in extreme compression fibre reaches its ultimate value of 0.0035. At this stage, the actual strain in steel can have the following values:
(a) Equal to failure strain of steel $\left(\frac{0.87\ y}{E_x} + 0.002\right)$ corresponding to balanced section
(b) More than failure strain, corresponding to under reinforced section.
(c) Less than failure strain corresponding to over reinforced section.
Thus for a given section, the actual value of $\frac{x_u}{d}$ can be determined from Eq. (7). Three cases may arise.
Case 1: $\frac{x_u}{d}$ equal to the lmiting value $\frac{x_{a,\max}}{d}$ : Balanced section.
Case-2: $\frac{x_u}{d}$ less than limiting value: under-reinforced section.
Case-3: $\frac{x_u}{d}$ more than limiting value: over-reinforced section.

### limiting moment of Resistance

The ultimate moment of resistance $M^2$ of a given beam section is obtainable from Eq. (3). The lever arm z, for the case of the singly reinforced rectangular section [Fig. 1.8. Fig. 1.9] is given by

$$z = \mathrm{d} - 0.42x_u \ldots\ldots\ldots\ldots\ldots\ldots(8)$$

Accordingly, in terms of the concrete compressive strength,

$M_{\mathrm{u}} = 0.361 f_{\mathrm{ck}} b x_u (\mathrm{d} - 0.42\mathrm{x}_u)$ for all $x_u$………………..(9)

Alternatively, in terms of the steel tensile stress

$M_{\text{u}} = 0.361 f_{ck} \left(\frac{x_{v\max}}{d}\right)\left(1 - \frac{0.416\ \ u_{max}}{d}\right) bd^2$……………..(10)

## Limiting Percentage Tensile Steel

Corresponding to the limiting moment of resistance $M_{\mu.\text{lim}}$ , there is a limiting percentage tensile steel $P_{1,\text{lan}} = 100\pi/_{at,\text{tan}}/bd$. An expression for $P_{r,\text{lin}}$ is obtainable from Eq. (7) with: $x_u = X_{umax}$

$$\frac{x_{u,\text{lim}}}{d} = \frac{0.87 f_y}{0.361 f_{ck}} \times \frac{p_{t\text{lim}}}{100}$$

$$\Rightarrow P_{tim} = 41.61 \left(\frac{f_d}{f_y}\right)\left(\frac{x_{uli\,\text{m}}}{d}\right)$$

The values of $p_{t,lim}$ and $\frac{M_u}{bd^2}$ (in MPa units) for, different combinations of steel and concrete grades are listed in Table 2. These values correspond to the so-called "balanced' section for a singly reinforced rectangular section.

Limiting values of $p_{\text{tlim}}$ and $\frac{M_{\text{u}}}{bd^2}$ for singly reinforced rectangular beam sections for various grades of steel and concrete.
(a) $P_{tlim}$ = values
(b) $\frac{M_{ulim}}{bd^2}$ values (MPa)**Table-2**

| | M20 | M25 | M30 | M35 | M40 |
|---|---|---|---|---|---|
| (a)$P_{tlim}$ *value* | | | | | |
| Fe250 | 1.769 | 2.211 | 2.653 | 3.095 | 3.537 |
| Fe415 | 0.961 | 1.201 | 1.441 | 1.681 | 1.921 |
| Fe500 | 0.759 | 0.949 | 1.138 | 1.328 | 1.518 |
| (b) $\frac{\text{M}_{u.1im}}{bd^2}$ | values (MPa) | | | | |
| Fe 250 | 2.996 | 3.746 | 4.495 | 5.244 | 5.993 |
| Fe 415 | 2.777 | 3.472 | 4.166 | 4.860 | 5.555 |
| Fe 500 | 2.675 | 3.444 | 4.013 | 4.682 | 5.350 |

If $p_t < p_{tlim}$ ,The section is under reinforced section.

If $p_t = p_{tlim}$ ,The section is under balanced section.

If $p_t > p_{tlim}$ ,The section is over reinforced section.

## 3.3 ANALYSIS AND DESIGN: DETERMINATION OF DESIGN CONSTANTS, MOMENT OF RESISTANCE AND AREA OF STEEL FOR RECTANGULAR SECTIONS

From stress diagram

Lever arm $Z=d-0.42x_u$

Moment of resistance=(tension or compression)× Lever arm

Consider compression forces

$M_u=0.36f_{ck}x_ub(d-.42x_u)$

$=0.36f_{ck}x_ubd(1-0.42x_{u/}d)$

$=0.36fckXu(1-0.42x_{u/}d)bd^2$

for a limiting value substitute $X_{umax}$ for $X_u$ and $M_{ulim}$ for $M_u$

$M_{ulim}$=0.36fckXumax$(1-0.42x_{umax/}d)bd^2$

$M_{ulim}=Q_{lim}\times bd^2$

$Q_{lim}=M_{ulim}/bd^2$

$Q_{lim}=0.36fck\times X_{umax}(1-.42x_{umax}d)$

$A_{st}=Pt\times bd/100$

**Limiting moment of resistance**

| Grade of Concrete | Grade of Steel | | |
|---|---|---|---|
| | Fe 250 | Fe 415 | Fe 500 |
| **General MoR** | $0.148f_{ck}bd^2$ | $0.138f_{ck}bd^2$ | $0.133f_{ck}bd^2$ |
| **For M20** | $2.96bd^2$ | $2.76bd^2$ | $2.66bd^2$ |
| **For M25** | $3.70bd^2$ | $3.45bd^2$ | $3.33bd^2$ |
| **For M30** | $4.44bd^2$ | $4.14bd^2$ | $3.99bd^2$ |

**Limiting moment of resistance factor**

| $f_{ck}$(N/MM$^2$) | $f_y$(N/MM$^2$) | | | |
|---|---|---|---|---|
| | 250 | 415 | 500 | 550 |
| 15 | 2.22 | 2.07 | 2.0 | 1.94 |
| 20 | 2.96 | 2.76 | 2.66 | 2.58 |
| 25 | 3.70 | 3.45 | 3.33 | 3.23 |
| 30 | 4.44 | 4.14 | 3.99 | 3.87 |

## Limiting percentage of reinforcement

| $f_{ck}$(N/MM$^2$) | $f_y$(N/MM$^2$) | | | |
|---|---|---|---|---|
| | 250 | 415 | 500 | 550 |
| 15 | 1.32 | 0.72 | 0.57 | 0.50 |
| 20 | 1.75 | 0.96 | 0.76 | 0.66 |
| 25 | 2.19 | 1.20 | 0.95 | 0.83 |
| 30 | 2.63 | 1.44 | 1.14 | 0.99 |

**Problem 01:Determine the moment of resistance of the beam having dimension as 300×550mm(effective). The beam is reinforced with 1963$mm^2$ of steel in the tension zone. Use $M_{20}$ concrete and Fe415 steel. Also comment on the design of beam.**

**Solution:** Given: b=300mm

D=550mm

$$A_{st} = 1963mm^2$$

$$f_{ck} = 20N/mm^2$$

$$f_y = 415N/mm^2$$

Depth of neutral axis($x_u$)

$$x_u = \frac{0.87 \times f_y \times A_{st}}{0.36 \times f_{ck} \times b}$$

$$x_u = \frac{0.87 \times 415 \times 1963}{0.36 \times 20 \times 350}$$

$$x_u = 326.3mm$$

Limiting depth of neutral axis, $x_u$

$$x_{umax} = 0.48d$$

=0.48×550=263.5mm <$x_u$

$$x_{umax} < x_u$$

hence, the section is over reinforced and the moment of resistance of such section is equal to $M_{ulim}$.

Moment of resistance ($M_u$)

$$X_u = X_{umax} = 263.5mm$$

$$M_U = 0.36 \times f_{ck} \times b \times X_{umax}(d - 0.42 \times X_{umax})$$

$$=0.36 \times 20 \times 300 \times 263.5(550 - 0.42 \times 263.5)$$

$$=250.47 \times 10^6 Nmm$$

**Comment:** the beam is over reinforced and hence it should be redesigned because the failure of such beam is sudden and withstand wearing.

**Problem 02: An RCC beam 200mm×400mm(effective), is reinforced with 3-16mm diameter bars of 415 steel. Find the ultimate uniformly distributed load which the beam can carry safely over a span of 5m. take M20 concrete.**

**Solution:** Given: b=300mm

$$d=400mm$$

$$A_{st} = 3 \times \frac{\pi}{4} \times 16^2 = 603.19mm^2$$

$$f_{ck} = 20N/mm^2$$

$$f_y = 415N/mm^2$$

$$L=5m$$

Depth of neutral axis($x_u$)

$$x_u = \frac{0.87 \times f_y \times A_{st}}{0.36 \times f_{ck} \times b}$$

$$x_u = \frac{0.87 \times 415 \times 603.19}{0.36 \times 20 \times 200}$$

$$x_u = 151.2mm$$

Limiting depth of neutral axis, $x_u$

$$x_{umax} = 0.48d$$

$$=0.48 \times 400=192mm > x_u$$

$$x_{umax} > x_u$$

hence, the section is under reinforced

Moment of resistance ($M_u$)

$$M_U = 0.87 \times f_y \times A_{st}(d - 0.42 \times X_u)$$

$$=0.87 \times 415 \times 603.19(400 - 0.42 \times 151.2)$$

$$=73.28 \times 10^6 Nmm$$

**Ultimate load($w_u$)**

Equating maximum factored bending moment and the ultimate moment of resistance.

Maximum moment $M=\frac{w_u \times l^2}{8}$

$M=M_u$

$\frac{w_u \times l^2}{8}=73.28$

$w_u = 23.45k$N/m

**Problem 03: A rectangular beam is 20cm wide and 40cm deep upto the centre of reinforcement. Find the area of reinforcement required if it has to resist a moment of 25kNm.use M20 concrete and Fe 415 steel.**

**Solution:** Given: b = 20cm = 200mm

d=40cm = 400mm

$A_{st} = 1963mm^2$

$f_{ck} = 20N/mm^2$

$f_y = 415N/mm^2$

Moment , M= 25kNm=25× $10^6 Nmm$

Factored moment $M_u = 1.5 \times 25 \times 10^6 = 37.5 \times 10^6 Nmm$

Factored moment = moment of resistance

$$M_u = 0.87 \times f_y \times A_{st} \times d\left(1 - \frac{f_y \times A_{st}}{f_{ck} \times bd}\right)$$

$$37.5 \times 10^6 = 0.87 \times 415 \times A_{st} \times 400\left(1 - \frac{415 \times A_{st}}{20 \times 200 \times 400}\right)$$

$$A_{st}(400 - 0.10375A_{st}) = 103863.73$$

$$A_{st}{}^2 - 3855.4A_{st} + 1001096.192 = 0$$

$$A_{st} = \frac{3855.4 \pm \sqrt{3855.4^2 - 4 \times 1001096.192}}{2}$$

$$A_{st} = 280mm^2$$

$$depth\ of\ neutral\ axis, \qquad x_u = \frac{0.87 \times f_y \times A_{st}}{0.36 \times f_{ck} \times b}$$

$$x_u = \frac{0.87 \times 415 \times 280}{0.36 \times 20 \times 200}$$

$$x_u = 70.2mm$$

Limiting depth of neutral axis ($X_{umax}$)

$$x_{umax} = 0.48d$$

=0.48×400=192mm >$x_u$

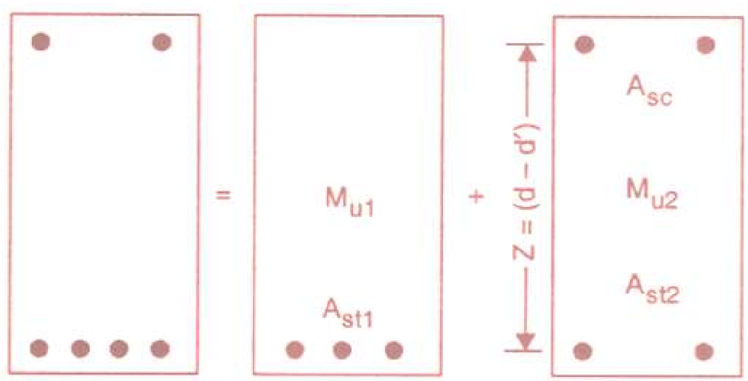
$\because x_{umax} > x_u$, hence the section is under reinforced and design is ok.

## 3.4 NECESSITY OF DOUBLY REINFORCED SECTION, DESIGN OF DOUBLY REINFORCED RECTANGULAR SECTION.

### Necessity of DoublyReinforcedSection

DoublyReinforcedSectionsectionsareadoptedwhenthedimensionsofthebeamhavebeenpredetermined from other considerations and the design moment exceeds the moment of resistance of a singlyreinforced section. The additional moment of resistance is carried by providing compression reinforcement andadditional reinforcementin tension zone.

### Design of doubly reinforced section:



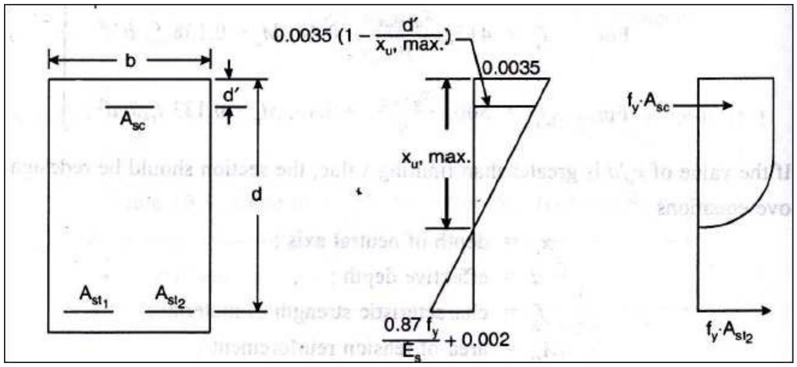

Section-1 section-2

Section-1 consists of a singly reinforced balanced section having area of steel $A_{st1}$ and moment of resistance $M_{ulim}$..

Section-2 consists of compression steel $A_{sc}$.The moment of resistance of the section is $M_{u2}$ such that, The moment of resistanceof adoubly reinforcedsection isthesumofthelimitingmomentofresistance$M_{ulim}$ofasinglereinforcedsectionandtheadditionalmomentofresistance$M_{u2}$.

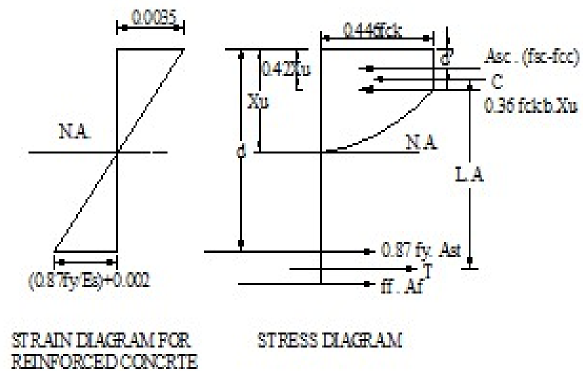
Mu2=Mu–$M_{ulim}$

Theleverarmfortheadditionalmomentofresistanceisequaltothedistancebetweenthecentroidsoftension and compressionreinforcement,(d–$d'$).




$M_{u2}$=0.87$f_y$.$A_{st2}$(d− $d'$)=$A_{sc}(f_{sc}-f_{cc})$(d−$d'$)

Where:$A_{st2}$=Areaofadditionaltensilereinforcement




b = width of beam

$X_u$=depth of neutral axis

d= depth of beam

$A_{st}$= Area of tension steel

$A_{sc}$=Areaofcompressionreinforcement

$f_{sc}$=Stressincompressionreinforcement

$f_{cc}$=Compressivestressinconcreteatthelevelofcompression reinforcement

**Depth of neutral axis:**

Total compression= $C_1$+ $C_2$

$C_1$=force carried by concrete area

$C_2$=compressive stress carried by compression steel $A_{sc}$

$C_1$= $0.36 \times f_{ck} \times b \times X_u$

$C_2$= $A_{sc}f_{sc} - f_{cc}A_{sc}$

Total compression=$0.36 f_{ck} b X_u + A_{sc}f_{sc} - f_{cc}A_{sc}$

=$0.36 f_{ck} b X_u + A_{sc}(f_{sc} - f_{cc})$

Total tension=$0.87 f_y . A_{st}$

Equating total tension to total compression we get

$0.36 f_{ck} b X_u + A_{sc}(f_{sc} - f_{cc}) = 0.87 f_y . A_{st}$

$$X_u = \frac{0.87f_y.\text{Ast} - A_{sc}(f_{sc} - f_{cc})}{0.36f_{ck}b}$$

Since $f_{cc}$ is very small as compare to $f_{sc}$ ,it can be neglected.

$$X_u = \frac{0.87f_y.\text{Ast} - A_{sc}f_{sc}}{0.36f_{ck}b}$$

**Moment of resistance :**

$M_{ulim}$ corresponding to section-2

$$M_{ulim} = C_1 \times lever\ arm$$

$$M_{ulim} = 0.361f_{\text{ck}}bx_u(\text{d} - 0.42\text{x}_u)$$

Stress diagram gives the additional moment of resistance $M_{u2}$ corresponding to section-2

$$M_{u2} = C_2 \times Lever\ arm$$

$$M_{u2} = A_{sc}(f_{sc} - f_{cc})(d - d')$$

$$M_u = M_{ulim} + M_{u2}$$

$$M_u = 0.361f_{\text{ck}}bx_u(\text{d} - 0.42\text{x}_u) + A_{sc}(f_{sc} - f_{cc})(d - d')$$

If loss of concrete area is neglected then

$$M_{u2} = 0.36f_{\text{ck}}bx_u(\text{d} - 0.42\text{x}_u) + A_{sc}f_{sc}(d - d')$$

Stress compression steel ($f_{sc}$) $N/mm^2$

| Grade of steel $f_y(N/mm^2)$ | $\frac{d'}{d}$ | | | |
|---|---|---|---|---|
| | 0.05 | 0.10 | 0.15 | 0.20 |
| 250 | 217 | 217 | 217 | 217 |
| 415 | 355 | 353 | 342 | 329 |
| 500 | 424 | 412 | 395 | 370 |
| 550 | 458 | 441 | 419 | 380 |

Sincetheadditionalreinforcementisbalancedby theadditionalcompressiveforce.

$A_{sc}(f_{sc} - f_{cc}) = 0.87f_y.A_{st2}$

$M_{u2} = 0.87f_y.A_{st2}(d - d')$

$A_{st2} = \frac{M_{u2}}{0.87fy.(d - d')}$

Thestrainatlevelofcompressionreinforcementis $.0035\left(1-\frac{d'}{X_{umax}}\right)$Totalareaofreinforcementshallbe obtainedby

Ast=Ast1+Ast2

$A_{st1}$=Areaofreinforcementforasinglyreinforcedsectionfor$M_{ulim}$

$= \frac{M_{ulim}}{0.87 f_y (d-0.42X_u)}$

**Problem : determine the factored moment of resistance of a beam 230mm×460mm(effective). The beam is reinforced with 2-16mm diameter bars on compression side and 4-20mm diameter bars on tension side. The compression bars are placed at a distance of 40mm from top. Use M20 concrete and Fe415steel.**

**Solution** b =230mm

d=460mm

$$d' = 40mm$$

$A_{st}$=$4\times\frac{\pi}{4}\times 20^2 = 1256mm^2$

$A_{sc}$=$4\times\frac{\pi}{4}\times 16^2 = 402mm^2$

$$f_{ck} = 20N/mm^2$$

$$f_y = 415N/mm^2$$

$$\frac{d'}{d} = \frac{40}{460} = 0.087 \cong 0.1$$

From table

$$f_{sc}\text{=}353N/mm^2$$

**Depth of neutral axis**

$$X_u = \frac{0.87f_y.\text{Ast} - A_{sc}f_{sc}}{0.36f_{ck}b}$$

$$X_u = \frac{(0.87\ \times 415 \times 1256) - (353 \times 402)}{0.36 \times 20 \times 230}$$

$$X_u = 188.15mm$$

$$X_{umax} = 0.88 \times X_u$$

$$= 220mm$$

$X_u > X_{umax}$ Hence, the section is under reinforced

Factored moment of resistance

$M_u = 0.36 f_{ck} b x_u (d - 0.42 x_u) + A_{sc} f_{sc} (d - d')$

$M_u = 0.36 \times 20 \times 230 \times 188.15(460 - 0.42 \times 188.15) + 353 \times 402(460 - 40)$

$M_u = 178.3 kNm$

**Problem-2 A doubly reinforced beam of size 230 mm x 500 mm effective is subjected to a factored moment of 200 KNm. Use M20 concrete and Fe 415 steel.**

Solution;

Given:

Breadth ( b )= 230 mm

Depth( d )= 500 mm

$M_u$ = 200 KNm = 200 x $10^6$ mm

$F_{ck}$ = 20 N/mm$^2$

$f_y$ = 415 N/mm$^2$

**Step-1**

$\frac{d'}{d} = \frac{50}{500} = 0.1$ (Assuming 50 mm effective cover for compression steel)

*From table F of code sp* 16;

Stress in compression steel ($f_{sc}$) = 353 N/mm$^2$

**Step-**2

**Limiting moment of resistance ($M_{ulim}$)**

$$M_{ulim} = 0.36\, f_{ck} b . x_{umax}(d - 0.42 x_{umax})$$

From clause 38.1 of code IS 456:200 ;

For Fe 415 steel

$\frac{Xumax}{d} = 0.48$

= 0.48 x 500 = 240 mm

$M_{u\,lim}$ = 0.36x 20x 230x 240 (500- 0.42 x 240 )

= 158658048 Nmm

$M_{u2} = M_u - M_{u\,lim}$

$= 200 \times 10^6 - 158658048$

$M_{u2} = 41341952$ Nmm

**Step-**3

**Area of tension steel ($A_{st}$)**

$A_{st} = A_{st1} + A_{st2}$

Where, $A_{st1}$ = Area of reinforcement for a singly reinforced section for $M_{ulim}$

$$= \frac{M_{ulim}}{0.87 f_y (d - 0.42 X_u)} = \frac{158658048}{0.87 \times 415 (500 - 0.42 \times 240)} = 1100.7 mm^2$$

$$A_{st2} = \frac{\mathrm{Mu2}}{0.87\mathrm{fy}.(\mathrm{d} - d')} = A_{st2} = \frac{41341952}{0.87 \times 415 (500 - \quad)} = 254.2 mm^2$$

Total area of tension steel = $A_{st1} + A_{st2}$

$= 1100.7 + 254.2 = 1354.9$ mm$^2$

Area of one 20mm dia bar$= \frac{\pi}{4} \times 20^2 = 314$mm$^2$

No. of 16mm dia bars required$= \frac{1354.9}{314} = 4.3\ say\ 5$

**provide 5 – 20 mm diameter bars as tension steel.**

**Step-**3 Area of compression steel($A_{sc}$)

$$A_{sc} = \frac{\mathrm{Mu2}}{f_{sc}(\mathrm{d} - d')} = \frac{41341952}{353 \times (500 - 50)} = 260.2 \text{ mm}^2$$

∴ Area of one 16mm dia bar$= \frac{\pi}{4} \times 16^2 = 201$mm$^2$

No. of 16mm dia bars required$= \frac{260.2}{201} = 1.3\ say\ 2$

∴ Provide 2-16 mm dia bars as compression steel

## POSSIBLE SHORT TYPE QUESTIONS WITH ANSWERS:

**1.The area of stress block is how much?**

**Ans:** The area of stress block is 0.36fckXu

**2.what do you mean by double reinforced section?[W-2018]**

**Ans:** When the moment of resistance is greater than the limiting moment of resistance and the size of the section is restricted,then we provide reinforcement in compression zone to give additional strength to compression such sections are known as doubly reinforced section.

**3.what is cracking moment?**

Ans:The cracking moment or nominal flexural strength of a beam is defined as the moment which causes the tensile stress in concrete equal to the tensile strength of concrete is given by its modulus of rupture.

( $f_{cr}$= 0.7√fck)

**4.what is design strength of materials?**

Ans: The reduced value of strength which is obtained by applying partial safety factor to that characteristics strength is called as design strength of materials.

Design strength=characteristics strength/partial safety factor

**5.What is design load?**

Ans:The design loads are obtained by multiplying characteristic load and the appropriate partial safety factor.

Design load=characteristics load×partial safety factor.

**6.Where we use doubly reinforced section**.[W-2015,W-2018]

Ans:1) where beam dimension is restricted.

2) when the beam is continuous.

3) when the load are eccentric.

## POSSIBLE LONG TYPE QUESTIONS:

1.Write the assumptions of design for structure in collapse by LSM.[W-2017,W-2018,W-2019]

2.Determine the ultimate moment of resistance of a rectangular beam 300×600mm(effective) reinforced with 5-25mm dia bars in tension zone and 2-25mm dia bars in compression zone.Use M20 concrete and

Fe 415 steel take d'=50mm.

3.Determine the depth of neutral axis for a beam section 250×400mm (effective). The beam is reinforced with 3-20mm dia bars.Use M20 concrete and Fe 415 steel.

4. Differentiates between balanced,over reinforced and under reinforced section.

5.Determine the ultimate moment of resistance of abeam250×500mm effective depth reinforced with 4-20 mm dia bars .Use M20 concrete and Fe 415 steel take d'=60mm.[W-2015]

6. Design a rectangular beam to resist a bending moment equal to 75kN-m using M25 concrete and Fe415 steel.[W-2017]

7. Write down the codal provision for beam ,slab, column.[w-2018]

# CHAPTER. 04

# SEAR BOND AND DEVELOPMENT LENGTH(LSM)

## Learning objective:

*4.1 Nominal shear stress in R.C. section, design shear strength of concrete, maximum shear stress, design of shear reinforcement, minimum shear reinforcement, forms of shear reinforcement.*

*4.2 Bond and types of bond, bond stress, check for bond stress, development length in tension and compression, anchorage value for hooks 900 bend and 450 bend standards lapping of bars, check for development length.*

*4.3 Numerical problems on deciding whether shear reinforcement is required or not, check for adequacy of the section in shear. Design of shear reinforcement; Minimum shear reinforcement in beams (Explain through examples only)*

## 4.1 NOMINAL SHEAR STRESS IN R.C. SECTION, DESIGN SHEAR

# STRENGTH OF CONCRETE, MAXIMUM SHEAR STRESS, DESIGN OF SHEAR REINFORCEMENT, MINIMUM SHEAR REINFORCEMENT, FORMS OF SHEAR REINFORCEMENT.

## Nominal shear stress:

The nominal shear will basically be the shear capacity of the member based on its material and geometric properties and composition.

Sear stress is found out in limit state theory bt a simple formula similar to that of elastic theory were $\tau_v$ defined as the nominal shear stress

$\tau_v$.=$V_u$/bd

$\tau_v$= nominal shear stress

$V_u$=design shear force

b=width of section

d=dept of section

## Designshearstrength(IS 456:2000)

Themagnitudeofdesignshearstrength($\tau_C$)dependsbasicallyonthegradeofconcrete( $f_{ck}$)andthepercentageoftensionsteel($P_t$).AsperIS456:2000thedesignshearstrengthofconcreteinbeamswithout shearreinforcement shallbegivenintable 5.1.(table 19 IS 456:2000)

The design shear strength of concrete in beams without shear reinforcement is liven in Table 19.

40.11.1 For solid slabs, the design shear strength for concrete shall be $\tau_C$ k where $k$ has the values given below:

| Overall depth(mm) | 300 OR more | 275 | 250 | 225 | 200 | 175 | 150 OR less |
|---|---|---|---|---|---|---|---|
| k | 1.00 | 1.05 | 1.10 | 1.15 | 1.20 | 1.25 | 1.30 |

TABLE 5.1 DESIGN SHEAR STRENGTH ($\tau_c$) OF CONCRETE. ( $\mathrm{N/mm^2}$,

| $100\frac{A_{sf}}{bd}$ | Grade of concrete | | | | | |
|---|---|---|---|---|---|---|
| | $M15$ | $M20$ | $M25$ | $M30$ | $M35$ | $M40$ and above |
| $\leq 0.15$ | 0.28 | 0.28 | 0.29 | 0.29 | 0.29 | 0.30 |
| 0.25 | 0.35 | 0.36 | 0.36 | 0.37 | 0.37 | 0.38 |
| 0.50 | 0.46 | 0.48 | 0.49 | 0.50 | 0.50 | 0.51 |
| 0.75 | 0.54 | 0.56 | 0.57 | 0.59 | 0.59 | 0.60 |
| 1.00 | 0.60 | 0.62 | 0.64 | 0.66 | 0.67 | 0.68 |
| 1.25 | 0.64 | 0.67 | 0.70 | 0.71 | 0.73 | 0.74 |
| 1.50 | 0.68 | 0.72 | 0.74 | 0.76 | 0.78 | 0.79 |
| 1.75 | 0.71 | 0.75 | 0.78 | 0.80 | 0.82 | 0.84 |
| 2.00 | 0.71 | 0.79 | 0.82 | 0.84 | 0.86 | 0.88 |
| 2.25 | 0.71 | 0.81 | 0.85 | 0.88 | 0.90 | 0.92 |
| 2.50 | 0.71 | 0.82 | 0.88 | 0.91 | 0.93 | 0.95 |
| 2.75 | 0.71 | 0.82 | 0.90 | 0.94 | 0.96 | 0.98 |
| 3.0 and above | 0.71 | 0.82 | 0.92 | 0.96 | 0.99 | 1.01 |

## Maximum shear stress

If the shear strength of the concrete beam is less than the nominal shear stress due to the loads coming on the beam ,then shear reinforcement is to be provided. The nominal shear stress in the beams with sear reinforcement shall not exceed maximum shear stress given in table below. If nominal shear stress is greater than the maximum shear stress( $\tau_v > \tau_{cmax}$ ) then the section is to be redesigned.

TABLE 20 (IS 456:2000)

| **Grade of concrete** | M15 | M20 | M25 | M30 | M35 | M40 and above |
|---|---|---|---|---|---|---|
| $\tau_{cmax}$ ($\mathrm{N/mm^2}$) | 2.5 | 2.8 | 3.1 | 3.5 | 3.7 | 4.0 |

**Minimum shear reinforcement**(Cl.26.5.1.6 of IS 456)

When the nominal shear stress($\tau_c$ ) is less than the design shear strength ($\tau_c$ )of concrete, then no shear reinforcement to be designed. But in such cases minimum shear reinforcement is to be provided in the form of stirrups such as

$$\frac{A_{sv}}{b \times s_v} \geq \frac{0.4}{0.87 \times f_y}$$

where

$A_{sv}$ = total cross-sectional area of stirrups legs effective in shear.

$s_v$ = stirrup spacing along with the Length of the member.

*b* = breadth of the beam or breadth of the web of the flanged beam. and

$f_y$ = characteristic strength of the stirrup reinforcement in N/mm2 which shall not be taken greater than 415N/mm2•

Where the maximum shear stress calculated is less than half the permissible value and in members of minor structural importance such as lintels. this provision need not be complied with.

## **Maximum spacing of stirrups**(Cl26.5.1.5 of IS 456)

The maximum spacing of vertical stirrups shall not exceed

i)0.75d or 300mm which ever is less .

ii)In the case of inclined stirrups at 45 degree the maximum spacing should not be greater than d or 300mm which ever is less.

## Design of shear reinforcement

When $\tau_v$exceeds $\tau_c$ , sear reinforcement is to be designed and can be provided in the following forms

I)vertical stirrups

ii)Bent up bars along with stirrups

shear reinforcement is provided to carry a sear force equals to$V_{us}$

$V_{us}$=$V_u$ - $\tau_c$bd

Vu=shear force due to design load

$\tau_c$bd=shear resistance of the concrete section

**i)** **Vertical stirrups :** The spacing of vertical stirrups is given by

$s_v$ =(.87$f_y A_{sv}$d)/$V_{us}$

$A_{sv}$ = total cross-sectional area of stirrups legs effective in shear.

$s_v$ = stirrup spacing along with the Length of the member.

*d =effective depth*

$f_y$ = characteristic strength of the stirrup reinforcement in N/mm2 which shall not be taken greater than 415N/mm2•

ii)**Bent up bars:** when bent up bars are provided their contribution towards shear resistance should not be more than half of the total sear reinforcement.($V_{us}$/2)

shear force taken by bent up bars is calculated as
$V_{us}'$=.87$f_y A_{sv}$sinα

Where $A_{sv}$ =area of bent up bars

α =angle between the bent up bars and the member axis $\alpha \geq 45^0$

## Minimumshearreinforcement(IS456:2000)

Theshearreinforcementintheformofstirrupsremainunstressedtillthediagonalcrackoccursatthecritical location.However,theinstantadiagonalcrackoccurs.Thewebreinforcementreceivessuddenincreasein stress.Ifwebreinforcementisnotprovided.Shearfailuremay occurwithoutgivinganywarning.Thecodetherefore,specifiesthatallthebeamsshouldbeprovidedwitha t leastsomeminimumreinforcementcallednominalshearreinforcementevenifnominalshearstressislesst han thedesignshearstress of concrete.

## Formsofshearreinforcement.

Shearreinforcementisnecessaryifthenominalshearstress($\tau_v$)exceedsthedesignshearstress.Ingenerals hearreinforcementisprovidedinanyoneofthefollowingthreeforms.

(a) Verticalstirrups
(b) Bendupbarsalongwiththestirrups.
(c) Inclinedstirrups.

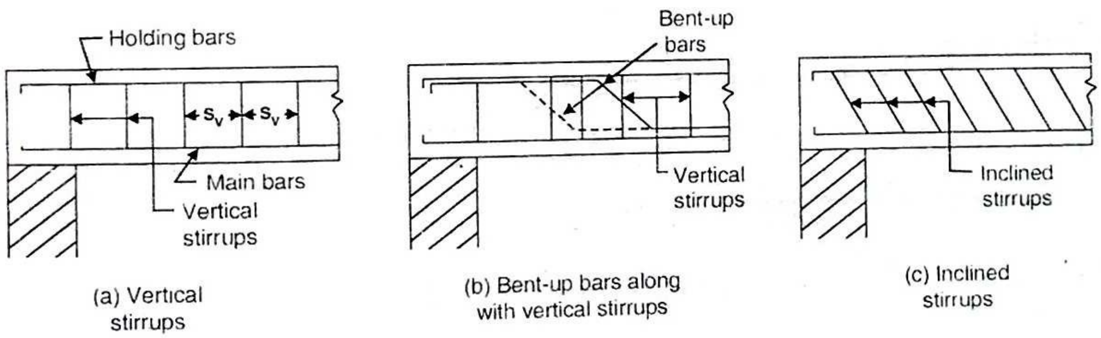



FIG. 5.8 TYPES OF SHEAR REINFORCEMENT

Wherebent-upbarsareprovided,theircontributiontowardsshearresistanceshallnotbemorethanhalfthatoftotalshearreinforcement.

## 4.2 BOND AND TYPES OF BOND, BOND STRESS, CHECK FOR BOND STRESS, DEVELOPMENT LENGTH IN TENSION AND COMPRESSION, ANCHORAGE VALUE FOR HOOKS 90° BEND AND 45° BEND STANDARDS LAPPING OF BARS, CHECK FOR DEVELOPMENT LENGTH.

**Bond:**One of the most important assumption in the behaviour of reinforced concrete structure isthat there isproper 'bond' between concrete and reinforcing bars. The force which prevents the slippage betweenthe two constituent materials is known as bond. In fact , bond is responsible for providing ' straincompatibility ' and composite action of concrete and steel. It is through the action of bond resistancethat the axial stress ( tensile or compressive) in a reinforcing bar can undergo variation from point topoint along its length. This is required to accommodate the variation in bending moment along thelength oftheflexuralmember.

## Typesofbond:-

Bond stress along the length of a reinforcing bar may be induced under two loading situations, andaccordinglybond stressesaretwo types:

1. FlexuralbondorLocalbond
2. Anchoragebondordevelopmentbond

**Flexural bond ($\tau_{bd}$)** is one which arises from the change in tensile force carried by the bar, along itslength, due to change in bending moment along the length of the member. Evidently, flexural bond iscritical at points where the shear (V= dM/dx) is significant. Since this occurs at a particular section,flexuralbondstressis knownaslocalbond stress[Fig-5.1(b)].

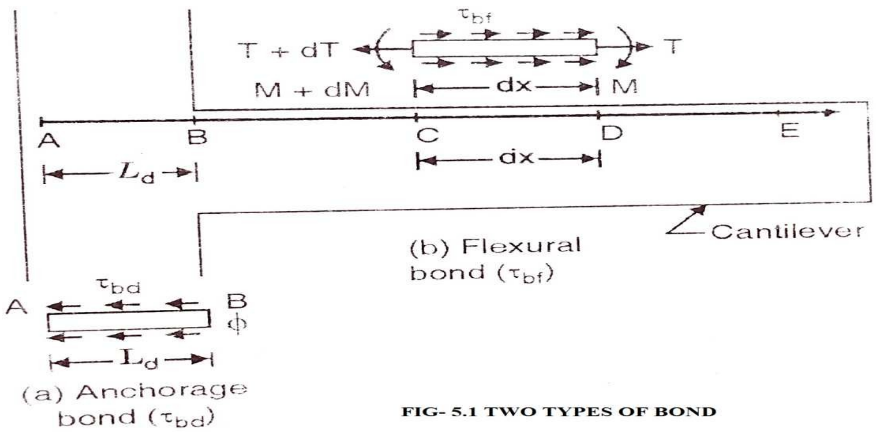



FIG- 5.1 TWO TYPES OF BOND

**Anchorage bond ($\tau_{bd}$)** is that which arises over the length of anchorage provided for a bar.It alsoarisesneartheendorcut offpointofreinforcingbar.Theanchoragebondresiststhe'pullingout'of the bar if it is in tension or 'pushing in' of the bar if it is in compression. Fig.[5.1 (a)] shows thesituation of anchorage bond over a length AB(=$L_d$ ). Since bond stresses are developed over specifiedlength$L_d$,anchoragebondstressisalsoknownasdevelopedoveraspecifiedlength$L_d$,anchoragebond stressis alsoknownasdevelopmentbondstress.

Anchoring ofreinforcing barsisnecessary whenthedevelopmentlengthofthereinforcementislargerthanthestructure.Anchorageisusedsothatthesteel'sintendedtensionloadcanbereachedandpop-outswillnotoccur.Anchorageshapescantaketheformof180or90degreehooks.

## Anchoragebondstress:

Fig- 5.2 shows a steel bar embedded in concreteAnd subjectedto a tensile force T. Due to this forceThere will be a tendency of bar to slip out and this tendency is resisted by the bond stress developedoverthe perimeterof thebar, alongitslengthofembedment.

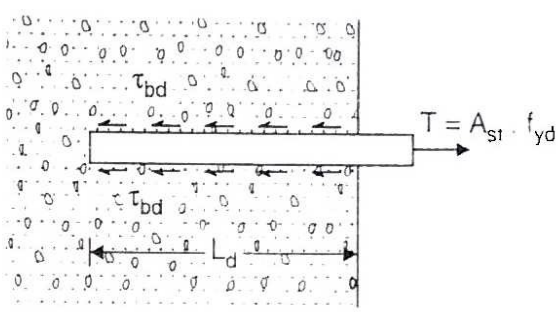



FIG- 5.2

Let us assume that average uniform bond stress is developed along the length. The required lengthnecessary todevelopfullresisting forceiscalled**Anchorage length** incaseofaxialtension orcompressionand**developmentlength**incaseofflexuraltensionandisdenotedby $L_d$.

**Designbondstress :-(IS 456:2000,CL26.2.1.1)**

Thedesignbondstressinlimitstatemethodforplainbarsintensionshallbeasgivenbelow(Table6.1)

Table-6.1

| Gradeofconcrete | M20 | M25 | M30 | M35 | M40andabove |
|---|---|---|---|---|---|
| Designbondstress $\tau_{bd}$(N/mm$^2$) | 1.2 | 1.4 | 1.5 | 1.7 | 1.9 |

**Designbondstressesfordeformedbarsintension:**FordeformedbarsconformingtoIS1786.Thesevaluesshallbeincreased by60%.

**Designbondstressforbarsincompression:**Forbarsincompression,thevaluesofbondstressforin tension shallbeincreased by25%.

Hencethevaluesof$k_d$forbarsIncompressionwillbe=0.87 $f_y$/5$\tau_{bd}$

Tablebelow gives thevaluesofdevelopmentlengthfactorforvariousgradesofconcrete and thevariousgradesofsteel,bothintensionaswellascompression.Thevalueshavebeenrounded-offtothehigherside

## Valuesofdevelopmentlengthfactor

| Gradeofconcrete | M20 | | | M25 | | |
|---|---|---|---|---|---|---|
| Gradeofsteel | Fe250 | Fe415 | Fe500 | Fe250 | Fe415 | Fe500 |
| Barsintension | 46 | 47 | 57 | 39 | 41 | 49 |
| Barsincomp. | 37 | 38 | 46 | 31 | 33 | 39 |

| Grade ofconcrete | M30 | | | M35 | | | M40 | | |
|---|---|---|---|---|---|---|---|---|---|
| Gradeof steel | Fe250 | Fe415 | Fe500 | Fe250 | Fe415 | Fe500 | Fe250 | Fe415 | Fe500 |
| Barsin tension | 37 | 38 | 46 | 32 | 34 | 40 | 29 | 30 | 36 |
| Barsin comp. | 29 | 31 | 37 | 26 | 27 | 32 | 23 | 24 | 29 |

**Note :** When the actual reinforcement provided is more than that theoretically required, so that theactualstress ($\sigma_s$) in steel is less than the full deign stress (0.87 $f_y$), the development length requiredmaybereducedbythefollowingrelation:

$$\text{Reduceddevelopmentlength} L_{dr} = L_d \times \frac{\text{Astrequired}}{\text{Astprovided}}$$

This principle is used in the design of footing and other short bending members where bond iscritical. Byprovidingmore steel,the bondrequirementsare satisfied.

**Bars bundled in contact :**The development length of each bar bundled bars shall be that forthe individual by 10% for two bars in contact,20% for three bars in contact and 33% for four bars incontact.

- Anchoring values :IS 456:2000 clause no.26.2.2 page no.34
- Lapping of bars :IS 456:2000 clause no.26.2.5.1 page no.45.
- Check for development length should be according to clause no.26.2.3.3(c)

**Problem1**: **A simply supported bam is 250mm wide with effective depth 500mm.it is reinforced with 4-20mm dia bars as tensile reinforcement. If the beam is subjected to a factored shear of 95kN at support. Design shear reinforcement consisting of vertical stirrups.**

Ans: Given b=250mm

d=500mm

$A_{st}$=4π/4×20×20=1256mm2

$V_u$=95KN

Nominal shear stress $\tau_v$= $V_u$/bd=95000/500×250=0.76N/mm$^2$

Design shear strength of concrete ($\tau_c$)

$P_t$=100 $A_{st}$/bd=100×1256/500×250=1%

For $P_t$=1% and M20 Concrete$\tau_c$ =0.62N/mm$^2$

Hence shear reinforcement is to be described.

$$\tau_v > \tau_c$$

Shear resistance of concrete= ($\tau_c$)×bd=.62×250×500=77500N

Shear to be taken by stirrups=$V_{u.}$ - Vc

=95000-7750=17500N

Using 8mm dia two legged stirrups

$A_{sv.} = 2 \times \frac{\pi}{4} \times 8^2 = 100.48$mm$^2$

Spacing of stirrups, $s_v$ = (0.87$f_y$ ×$A_{sv}$×d)/ $V_{us}$

=(0.87×415×100.48×500)/17500=1036.5mm

But maximum spacing

$s_v$ =(.87$f_y A_{sv}$)/0.4b=(.87×415×100.48)/(0.4×250)=362.7mm take it =363mm

Check for spacing: Minimum of following should taken

i).75×d=.75×500=375mm

ii)363mm

iii)300mm

Hence provide 8mm dia 2legged stirrups@300mm c/c.

**Problem-2A singly reinforced RC beam has an effective depth of 400mmand a breadth of 250mm.It contains 4-16mm bars. For M30 concrete and Fe 415 steel. Calculate the shear reinforcement for factored shear force of 250KN.**

Ans: Given b=250mm

d=400mm

$A_{st}$=4π/4×16^2=804.3mm2

$V_u$=250KN

Nominal shear stress $\tau_v$.= $V_u$/bd=250000/(400×250)=2.5N/mm$^2$

Design shear strength of concrete ($\tau_c$)

$P_t$=(100×$A_{st}$)/bd=(100×804.3)/(400×250)=0.803%

For $P_t$=0.75, $\tau_c$=0.56N/mm$^2$

For $P_t$=1, $\tau_c$ =0.62N/mm$^2$

Required, $\tau_c$={(0.803-0.75)×(0.62-0.56)/(1-0.75)}+0.56

=0.573N/mm$^2$

$$\tau_v > \tau_c$$

Hence shear reinforcement is to be described.

Shear resistance of concrete= ($\tau_c$)×bd=.573×250×400=57.3KN

Shear to be taken by stirrups=$V_{u.}$ - Vc

=250-57.3=192.7KN

Using 10mm dia two legged stirrups

$A_{sv.}$ =2×$\frac{\pi}{4}$×10$^2$=157mm2

Spacing of stirrups, $S_v$=(0.87$f_y$ ×$A_{sv}$×d)/$V_{us}$

=(0.87×415×157×400)/(192.7×10$^3$)=117.66mm=118mm

But maximum spacing

$S_v$<$\frac{0.87\text{fyAsv}}{0.4\times\text{b}}$ = $\frac{0.87\times415\times157}{0.4\times250}$ =566.84mm

Check for spacing: Minimum of following should taken

i)0.75×d=0.75×400=300mm

ii)566.86mm

iii)300mm

iv)118mm

Hence provide 10mm dia 2 -legged stirrups@110mm c/c.

# POSSIBLE SHORT TYPE QUESTIONS WITH ANSWER:

**1)The anchorage value of 90°hook is---?**

Ans:8×dia of bar.

**2.Write different forms of shear reinforcement.[S-2019]**

Ans: Different forms of shear reinforcement are

i)Verticalstirrups

ii) Bendupbarsalongwiththestirrups.

iii) Inclinedstirrups.

**3.The spacing of stirrups at the support is---?**

**Ans**: Minimum.

**4. Write the formula for development length of beam.[2010]**

**Ans:**$L_d = \phi\sigma_c/4$ $\tau_{bd}=k_d\phi$

Where $\phi$=nominaldiameterofthebar

$\sigma_c$=stressinbaratthesection

$\tau_{bd}$ =Design bond stress.

5.**Define development length.**

**Ans**: The minimum length of the bars which must be embedded in concrete to develop full stress is called development length.

**6.what is the anchorage value of U-type hook?[w-2019]**

**Ans:**The anchorage value of U-type hook is 16×dia of bar.

## POSSIBLE LONG TYPE QUESTIONS:

1.Write short note on Bond and Anchorage.[W-2014]

2.An RCC beam 250×400mm effective depth is carrying a UDL of 16KN/M. The beam is reinforced with 4-22mm dia bars.The clear span of the beam is 4m. Design the shear reinforcement. Use M20 concrete and mild steel.[2018,W-2019]

2.A singly reinforced RC beam has an effective depth of 400mmand a breadth of 250mm.It contains 4-16mm bars. For M30 concrete and Fe 415 steel. Calculate the shear reinforcement for factored shear force of 250KN.

3.:A simply supported bam is 250mm wide with effective depth 500mm.it is reinforced with 4-20mm dia bars as tensile reinforcement. If the beam is subjected to a factored shear of 95kN at support. Design shear reinforcement consisting of vertical stirrups.[2017]

4.Calculate development length required to be provided for M25 concrete and Fe415 steel of diameter Ø for bars in tension and compression. [S-2019]

# CHAPTER NO.-5

# ANALYSIS AND DESIGN OF T-BEAM (LSM)

## Learning objective:

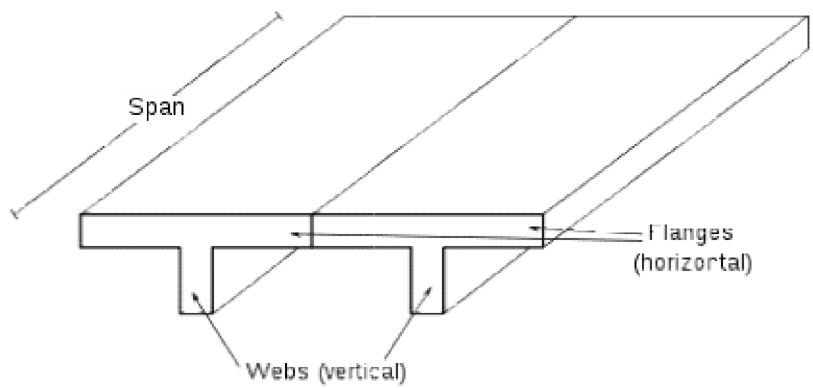
*5.1 General features, advantages, effective width of flange as per IS: 456-2000 code provisions.*

*5.2 Analysis of singly reinforced T-Beam, strain diagram & stress diagram, depth of neutral axis, moment of resistance of T-beam section with neutral axis lying within the flange.*

*5.3 Simple numerical problems on deciding effective flange width. (Problems only on finding moment of resistance of T-beam section when N.A. lies within or up to the bottom of flange shall be asked in written examination).*

## 5.1 GENERAL FEATURES, ADVANTAGES, EFFECTIVE WIDTH OF FLANGE AS PER IS: 456-2000 CODE PROVISIONS

**Genera Features: -** A T-Beam used in Construction is a load bearing structure of reinforced Concrete, Wood or Metal with a T Shaped Cross-section. The top of the T shaped cross section serves as a flange or Compression member in resisting compressive stresses. The web of the beam below the compression flange serves to resist shear stress and to provide greater separation for the couple forces of bending.




**Advantages**: -

1. Since the beam is cast monolithically with the slab, the flange also takes up the compressive stresses which means, it will be much more effective in resisting the sagging moment acting on the beam.
2. Better headroom is the direct outcome of the first point since the depth of the beam can be considerably decreased.

3. For larger spans, T beams are normally preferred rather than rectangular beams as the deflection is decreased to a good extent.

**Effective Width of Flange as per IS:456-2000 Code Provisions**:-

Refer to Page No.36, Clause No.23.1.2 of IS:456-2000

## 5.2 ANALYSIS OF SINGLY REINFORCED T-BEAM, STRAIN DIAGRAM & STRESS DIAGRAM, DEPTH OF NEUTRAL AXIS, MOMENT OF RESISTANCE OF T-BEAM SECTION WITH NEUTRAL AXIS LYING WITHIN THE FLANGE

### Analysis of T-beam:-

**Position of Neutral Axis: -** For a flanged beam, the N.A either lies in flange or lies in web. For a given section to decide whether the neutral axis lies in flange or web, the flange force and the total tension may be compared as

Let us assumer that the neutral axis lies at the bottom of flange

Total compression $F_{tc} = 0.36 f_{ck} b_f D_f$

Total Tension $F_{ts} = 0.87\ fy A_{st}$

Then

(i) If $F_{tc} > F_{ts}$ , N.A lies in the flange

(ii) If $F_{tc} = F_{ts}$ , N.A lies at the bottom of the flange

(iii) If $F_{tc} < F_{ts}$ , N.A lies in the web

By equating total compression with total tension the actual depth of neutral axis can be found out

(i) If $X_u > X_{umax}$ the section is over reinforced section

(ii) If $X_u = X_{umax}$, the section is balanced section

(iii) If $X_u < X_{umax}$, the section is under reinforced section

### Case-1 :- N.A lies in flange ( $X_u < D_f$ )

When the neutral axis lies in the flange, the size of the compression zone becomes ($b_f \times X_u$). As concrete doesn't resist any tension the width of tension zone has no effect on the moment of resistance of the section. Therefore this beam can be through out of as a rectangular beam of dimension ( $b_f \times d$ ). The formula derived for rectangular beam shall be applied.

For a single reinforced flanged beam

(i) $X_u = \frac{0.87\times fy \times Ast}{0.36\times fck\times bf}$

(ii) For under reinforced section

$M_u = 0.87\times fy\times A_{stt}$ (d-0.42$X_u$ )

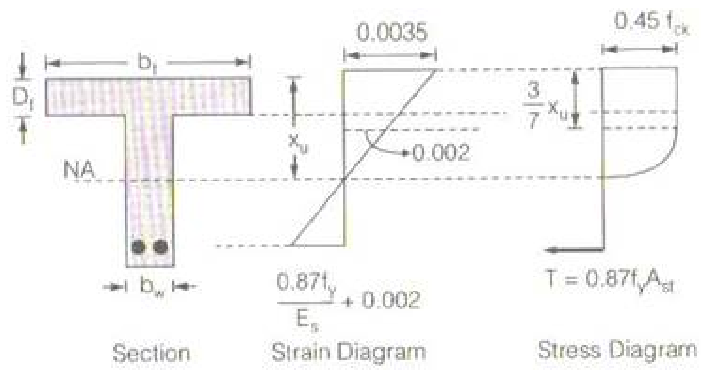
Or $M_u = 0.36\times f_{ck}\times b_f\times X_u\times$(d-0.42$X_u$ )

(iii) For balanced or over reinforced section

$M_{ulim} = 0.36\ f_{ck}\times b_f\times X_{umax}$ (d-0.42$X_{umax}$ )

Or $M_{ulim} = 0.87\ fy A_{stlim}$ (d-0.42$X_{umax}$ )

## <u>Case-2</u> :- N.A lies in Web ( $X_u > D_f$ )




(i) When mild steel is used stresses in concrete are uniform up to $\frac{3}{7}\times 0.53d = 0.227d$

(ii) When HYSD steel of Grade Fe415 is used the stresses in concrete are uniform up to $\frac{3}{7}\times 0.48d = 0.206d$

(iii) When HYSD reinforcement of Grade Fe500 is used the stress in concrete are uniform up to $\frac{3}{7}\times 0.46d$

When thickness of flange exceed 0.2d the stress in the flange are not uniform. The allowances for non-uniform stresses are made. If ($D_f$/d) < 0.2d $D_f$ is replaced by $y_f$,

$y_f = 0.15\ X_u + 0.65\ D_f$ but not greater than $D_f$

## (a) When ($D_f$/d) < 0.2

Total tension= $0.87 fy A_{st}$ Total Compression= Compression in rectangular beam of Size ($b_w$ x d)+ Compression in rectangular of size ( $b_f - b_w$)$D_f$

$= 0.36\ f_{ck}\times b_w X_{umax} + 0.446\times f_{ck}(b_f - b_w)D_f$

Limiting moment of resistance of the section can be found out by taking moment of compressive forces about centroid of tensile reinforcement

$M_{ulim} = 0.36\ f_{ck} b_w X_{umax}(d-0.42X_{umax}) + 0.446 f_{ck}(b_f - b_w)D_f(d - D_f/2)$

To find out the steel area total tension and compression are equated

$0.87\, fy A_{stlim} = 0.36\, f_{ck} b_w\, X_{umax} + 0.446 f_{ck}(b_f - b_w) D_f$

$A_{stlim} = (0.36\, f_{ck} \times b_w\, X_{umax} + 0.446 f_{ck}(b_f - b_w) D_f / 0.87\, fy$

**(b) When $D_f/d > 0.2$**

Total tension=$0.87\, f_y A_{stlim}$

Total compression= $0.36\, f_{ck} b_w X_{umax} + [0.446 f_{ck}(b_f - b_w) y_f]$

And $M_{ulim} = [0.36\, f_{ck} b_w X_{umax}(d - 0.42 X_{umax})] + [0.446 f_{ck}(b_f - b_w) y_f (d - y_f/2)]$

To find out the steel area equate total tension to total compression

$A_{stlim} = (0.36\, f_{ck}\, b_w\, X_{umax} + 0.446 f_{ck}(b_f - b_w) y_f) / 0.87\, fy$

**<u>Case-3</u>: - N.A lies in Web**

When the section is under reinforced the moment of resistance shall be found out using actual stress block

**(1) When $D_f \leq \frac{3}{7} X_u$,** in this case the stresses are uniform in flange

Total tension= $0.87\, fy A_{st}$

Total compression= $0.36 f_{ck} b_w X_{umax} + [0.446 f_{ck}(b_f - b_w) D_f]$

$M_u = 0.36\, f_{ck} b_w\, X_u(d - 0.42 X_u) + 0.446 f_{ck}(b_f - b_w) D_f (d - D_f/2)$

For given moment, the steel area required for beam

(a) Shall be treated as the sum of the steel area required for beam

(b) Resisting moment $M_{u1}$and steel $A_{st2}$ required for beam

(c) Resisting moment $M_{u2.}$

$M_u = M_{u1} + M_{u2.}$

$A_{st} = A_{st1} + A_{st2}$

Total compression = Total tension

$0.446 f_{ck}(b_f - b_w) D_f = 0.87\, fy A_{st2}$

$A_{st2} = 0.446 f_{ck}(b_f - b_w) D_f / 0.87\, fy$

$M_{u2} = 0.446 f_{ck}(b_f - b_w) D_f (d - D_f/2)$

$M_{u1} = M_u - M_{u2.}$

$100\, A_{st}/bd = 50[1 - \sqrt{(1 - 4.6\, M_u / f_{ck} bd^2)} / fy / f_{ck}]$

**(2) $D_f > \frac{3}{7} X_u$**

Total compression = $(0.36 f_{ck} b_w X_{umax})+\{0.446 f_{ck}( b_f - b_w) y_f\}$

Total tension= $0.87\ fy A_{st}$

$y_f = 0.15\ X_u + 0.65\ D_f$

$M_u = [0.36\ f_{ck} b_w\ X_u (d - 0.42 X_u)] + [0.446 f_{ck}( b_f - b_w) y_f (d - y_f/2)]$

**Problem-1**

Find the moment of resistance of a T beam of effective flange width 1200mm, thickness of slab 100mm and 300mm width of rib and 560mm effective depth reinforced with 5 nos of 25mm dia bar use M20 concrete and fe415 steel.

**Solution**: -

$A_{st} = 5 \times 491 = 2455 mm^2$

$F_{tc} = 0.36\ f_{ck} b_f D_f = 0.36 \times 20 \times 1200 \times 100 = 864 KN$

$F_{ts} = 0.87\ fy A_{st} = 0.87 \times 415 \times 2455 = 886.4 KN$

$F_{tc} < F_{ts}$

So N.A lies in Web

Assume $D_f > \frac{3}{7} X_u$

$y_f = 0.15\ X_u + 0.65\ D_f = 0.15\ X_u + 0.65 \text{ x } 100 = 0.15\ X_u + 65$

Total compression = $0.36\ f_{ck} b_w\ X_u + 0.446 f_{ck}( b_f - b_w) y_f$

$= (0.36 \times 20 \times 300 \times X_u) + [0.446 \times 20\ (1200-300) \times (0.15\ X_u + 65)]$

$= 3364.2\ X_u + 521820$

Total tension= $0.87\ fy A_{st} = 0.87 \times 415 \times 2455 = 886378$

Total tension = Total compression

$3364.2\ X_u + 521820 = 886378$

$X_u = 108.36 mm$

$\frac{3}{7} X_u = 46.44 < D_f$

$X_{umax} = 0.48 d = 0.48 \times 560 = 268.8 mm$

$X_u < X_{umax}$ so the section is under reinforced section.

$y_f = 0.15\ X_u + 65 = (0.15 \times 108.36) + 65 = 81.25 mm$

$M_u$= [0.36 $f_{ck} b_w$ $X_u$(d-0.42$X_u$ )]+[0.446$f_{ck}$( $b_f$-$b_w$)$y_f$(d- $y_f$/2)]

=[ 0.36 × 20 × 300 ×108.36 (560-0.42 )]+[0.446 × 20 ×900 × 81.25 (560- 40.63)]

= 459.2KNm

## 5.3 SIMPLE NUMERICAL PROBLEMS ON DECIDING EFFECTIVE FLANGE WIDTH. (PROBLEMS ONLY ON FINDING MOMENT OF RESISTANCE OF T-BEAM SECTION WHEN N.A. LIES WITHIN OR UP TO THE BOTTOM OF FLANGE SHALL BE ASKED IN WRITTEN EXAMINATION).

**Problem-2**

**A T beam floor system has 120mm thick slab supported on beams. The width of beam is 300mm and effective depth is 580mm. The beam is reinforced with 8 bars of 20mm diameter. Use M20 grade of concrete and Fe415 steel. The beams spaced 3m centre to centre, the effective span of beam is 3.6m.**

**Solution**: -

$A_{st}$= 8 × $\frac{\pi}{4}$ ×$(20)^2$=2513$mm^2$

$b_w$= 300mm

$D_f$=120mm

d=580mm

L=3.6mt

Effective width of flange

$b_f$= $\frac{l_0}{6}$+$b_w$+6×$D_f$

$l_0$= l=3.6mt

$b_f$=$\frac{l_0}{6}$+$b_w$+6×$D_f$

= $\frac{3600}{6}$+300+6×120

=1620mm

$b_f$ should not greater than 0.5($l_1$+$l_2$)+ $b_w$

=0.5(3000+3000)+300

=3300mm > 1620mm

Total tension = Total compression

Assuming neutral axis lies in flange

$X_u = X_u = \frac{0.87\times fy \times Ast}{0.36\times fck\times bf}$= (0.87×415×2513)/(0.36×20×1620)

= 77.8mm < $D_f$

Hence neutral axis lies in flange

$X_{umax}$=0.48d=0.48 x 580=278.4mm > $X_u$

Hence the section is under reinforced section.

$M_u$= 0.87 fy$A_{st}$(d-0.42$X_u$ )

= 0.87×415×2513(580-0.42×77.8)

= 496KNmt

## POSSIBLE SHORT TYPE QUESTIONS WITH ANSWER

**1. Write the formula of effective flange width, according to IS:456-2000 for simple supported beam?[W2015,W-2018]**

Answer:- $b_f$= $l_0$/6+$b_w$+6×$D_f$

$b_f$= Effective width of flange
$l_0$= Distance between points of zero moments in the beam
$b_w$= Width of the web
$D_f$= Thickness of the flange

**2. The overall depth of the simply supported T beam is assumed as what?**

**Answer**:- 1/12 to 1/15 of span

**3. The moment of resistance of a T beam is large due to large in which component?**

**Answer**:- The moment of resistance of a T beam is large due to large in compressive area of a flange.

## POSSIBLE LONG TYPE QUESTIONS

1. Find the moment of resistance of a T beam of effective flange width 1200mm, thickness of slab 120mm and 300mm width of rib and 500mm effective depth reinforced with 5 nos of 20mm dia bar use M20 concrete and fe415 steel.[W-2014]

2. A T beam floor system has 120mm thick slab supported on beams. The width of beam is 300mm and effective depth is 580mm. The beam is reinforced with 8 bars of 25mm diameter. Use M20 grade of concrete and Fe415 steel. The beams spaced 3mt centre to centre, the effective span of beam is 3.4mt.

3. A T beam floor system has 100mm thick slab supported on beams. The width of beam is 300mm and effective depth is 540mm. The beam is reinforced with 8 bars of 16mm diameter. The beams spaced 3mt centre to centre, the effective span of beam is 3.6mt. Use M20 grade of concrete and Fe415 steel.

# CHAPTER NO.-6

# ANALYSIS AND DESIGN OF SLAB & STAIR CASE (LSM)

*6.1 Design of simply supported one-way slabs for flexure check for deflection control and shear.*

*6.2 Design of one-way cantilever slabs and cantilevers chajjas for flexure check for deflection control and check for development length and shear.*

*6.3 Design of two-way simply supported slabs for flexure with corner free to lift.*

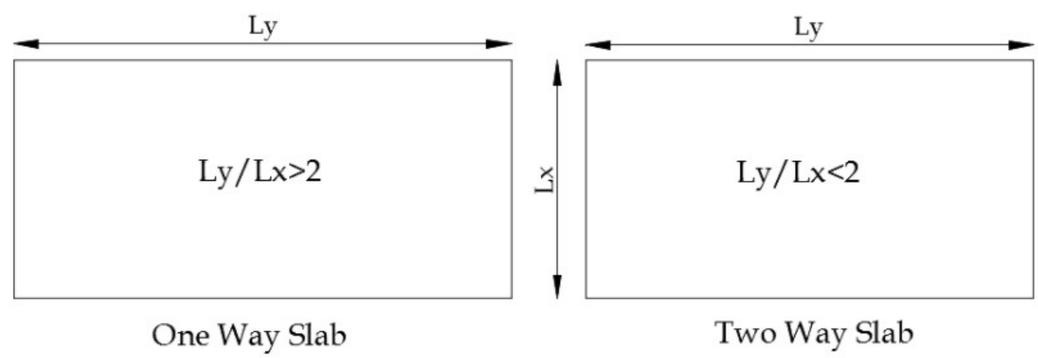
*6.4 Design of dog-legged staircase*

*6.5 Detailing of reinforcement in stairs spanning longitudinall*

## 6.1 DESIGN OF SIMPLY SUPPORTED ONE-WAY SLABS FOR FLEXURE CHECK FOR DEFLECTION CONTROL AND SHEAR

One way slabs are those slabs in which the $l_y/l_x$ ratio is greater than 2. This type of slab is also called as slab spanning in one direction as the bending takes place only along the short span. Therefore the main reinforcement is provided along the short span.

.




(i) The one way slab is analysed by assuming it to be a beam of 1mt width

(ii) The depth of slab can be assumed on the basis of control of deflection. Using balanced percentage of steel the l/d ratios are modified. To start with the span to depth ratios are approximated as following for initial depth trial calculation.

(a) For simply supported slab 25 to 30

(b) for cantilever slab 10

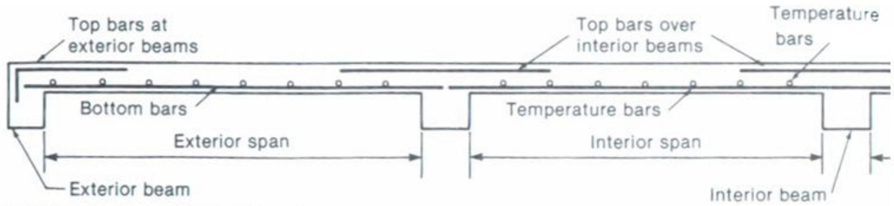
(iii) In addition to the main tensile reinforcement provided along short span, transverse reinforcement or distribution reinforcement is provided.

(iv) Some of the main bars in a slab are bent up near the supports. Shear is to be checked only. No shear reinforcement is provided

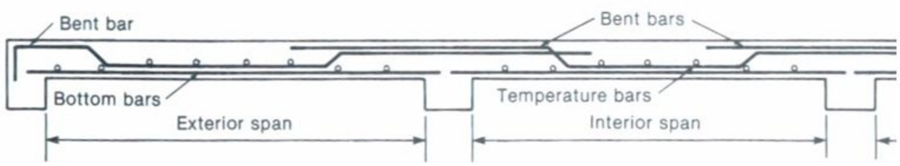
# One-Way Slab Design

## Reinforcement

Typical Reinforcement details in a one-way slab:



(a) Straight top and bottom bars.



(b) Alternate straight and bent bars.

**Deflection control**

For slabs the vertical deflection limits are specified by maximum l/d ratio

(a) For spans upto 10mt

| | |
|---|---|
| Cantilever | 7 |
| Simply supported | 20 |
| Continuous | 26 |

(b) For span greater than 10mt the above value may be multiplied by 10/span, except for cantilever, for which exact deflection calculation should be made.

**Shear design**

Slabs are safe in shear (Nominal shear stress is very low since b is larger) therefore no shear reinforcement is provided in slabs except that the alternate bars are bent up near the supports.

**Loads on slab**

(i) Self weight (dead load) of slab
(ii) Live loads as per use
(iii) Finishing and partition load

**Problem-1**

A Simply supported one way slab of effective span 4mt is supported on masonry walls of 230mm thickness. Design the slab take live load=2.5kN/mt$^2$ and floor finish = 1 kN/mt$^2$. The materials are M20 grade concrete and Fe415 steel.

**Solution**: -

Take 1 m along the long span direction
Breadth of the slab= 1000mm
Length=4 m

Assume overall depth of the slab (D)= 170mm
Effective depth= D-Clear cover=170-20=150

**Load calculation**

Self-weight=1×1×0.170×25=4.25kN/mt$^2$
Floor finish=1kN/mt$^2$
Live load= 2.5kN/mt$^2$
Total= 7.75kN/mt$^2$
Factored load= 1.5×7.75=11.6 KN/mt$^2$
Maximum moment= $Wl^2/8=11.6\times4^2/8=$ 23.2 kN.m

**Design calculation**

Check for depth:-
$M_u= 0.138\ f_{ck}bd^2$
$23.6\times10^6=0.138\times20\times1000\times d_{req}{}^2$
$d_{req}=\sqrt{23.6\times106/(0.138\times20\times1000)}$
= 91.68mm $<d_{assum}$ $M_u= 0.87\ fyA_{st}(d-A_{st}f_y/bf_{ck})$
$23.6\times10^6=0.87\times415\times A_{st}(150-A_{st}\times415/20\times1000)$
$A_{st}$=449mm$^2$
$A_{stmin}$=0.12/(100×1000×170)=204 mm$^2$
$A_{streq}>A_{stmin}$
So its ok
Provide 10mm dia bar
Spacing=$\frac{1000}{449}\times(\frac{\pi}{4}\times(10)^2)$
= 174.92mm

**Check for spacing**

Spacing should not be greater than (i) 300mm
(ii) 3×d =3×150=450mm
So provide 10mm dia main bar @170mm centre to centre along the short span direction
Distribution bars (8mm dia bar)
Spacing=(1000/204) × ($\frac{\pi}{4}\times8^2$)
=246mm

Spacing should not be greater than (i) 450mm
(ii) 5×d =5×150=750mm
So provide 8mm dia main bar @230mm centre to centre

**6.2 Design of one way cantilever slabs and chajjas**

One way cantilever slab or chajja is designed as cantilever beam of 1mt width. The points to be considered in designed of one way cantilever slab are following

(i) The effective span of cantilever slab shall be taken as its length upto the face of the support plus half the effective depth except where it forms the part of the continuous slab where the length from centre to centre of supports is taken

(ii) The effective depth at fixed end is maximum and is assumed to be about span/10 to span/12 (From deflection consideration)

(iii) The depth required at the free end is minimum and is kept ½ to 1/3 of the depth at fixed end

(iv) The main reinforcement is provided at the top and is to be curtailed at appropriate point

## Cantilever slab

**Problem-1**

**Design a Cantilever slab for an overhang of 1.2mt . The imposed load on slab consist of 1 KN/mt$^2$ of live load and floor finish 0.8 KN/mt$^2$. Use m20 Concrete and Fe415 steel.**

**Solution**: -

L=1.2mt

Assume D= 150mm

d =150-25=125mm

Effective span= 1.2+0.125/2=1.26mt

$f_{ck}$ = 20kN/m$^2$

$F_y$ = 415kN/m$^2$

**Load calculation**

Self-weight of slab= 1×1×0.15×25=3.75 KN/m

Imposed load=1×1=1 kN/m

Floor finish= 0.8×1=0.8 kN/m

Total weight=5.55 KN/m=5550N/m

Factor load=5550×1.5=8325N/m

Factored moment=$M_u$=8325×(1.26)$^2$/2=6608Nmt

Factored shear force $V_u$=W×l=8325×1.20=10406.25N

$d_{req} = \sqrt{\frac{6.608\times10^6}{0.138\times20\times1000}}$

==49mm<125mm

Hence ok.

$M_u$= 0.87 fy$A_{st}$(d-$A_{st}$fy/b$f_{ck}$)

6.608×10$^6$= 0.87×415×$A_{st}$(125-415×$A_{st}$/1000×20)

$A_{st}$=148mm$^2$

This area of steel should be more than minimum area of reinforcement

$A_{stmin}$= (0.12×b×D)/100=(0.12×1000×150)/100=180mm$^2$>148mm$^2$

Provide$A_{st}$=180mm$^2$

Using 8 mm dia bar

Spacing=(1000×50.3)/180=279mm say 270mm

Maximum spacing is limited to (i) 300mm

(ii) 3×d=3×125=375mm

Provide 8mm dia bar @270mm centre to centre as main reinforcement

Also provide 8mm dia bar @270mm centre to centre as distribution reinforcement

## Check for shear

$V_u$=10406.25N

$A_{stprovide}$=1000×50.3/270=186.3mm$^2$

Nominal shear stress $\tau_v$= $V_u$/bd=10406.25/(1000×125)=0.08N/mm$^2$

Design shear strength of concrete ($\tau_c$)

$P_t$=100 $A_{st}$/bd=(100×186.3)/(1000×125)=0.15%

For $P_t$=0.15% and M20 Concrete $\tau_c$=0.28N/mm$^2$

k= 1.3

k $\tau_c$=1.3×0.08=0.36 N/mm$^2$>>0.08 N/mm$^2$

Hence ok.

## Check for development length

The code requires that the reinforcement must extend at least $L_d$ into support

$L_d$= $\theta$×0.87$f_y$/4 $\tau_{bd}$=(8×0.87×415)/(4×1.6×1.2)=376mm

Therefore extending the bars into the support after giving a 90° bend

## Check for deflection

$P_t$=0.15%

$F_s$=0.58$f_y$[$A_{strequd}$/ $A_{stprovide}$]=0.58×415×[180/186.3]=232.56N/mm$^2$

l/$d_{max}$=7×2=14 (f$_s$=223, k$_t$=2 approximately)

$(l/d)_{provided}$=1250/125=10

l/$d_{max}$>$(l/d)_{provided}$

Hence ok.

## 6.3 DESIGN OF TWO WAY SIMPLE SUPPORTED SLAB FOR FLEXURE WITH CORNER FREE TO LIFT

Two way slabs are those slabs which are supported on all the four edges and having $l_y/l_x<2$. In this type of slabs bending occurs in both the direction long direction as well as short direction. There are two kinds of two way slabs

(i) Restrained slabs (Corners get lifted up)

(ii) Unrestrained slabs (Corners are not prevented from lifting)

### Analysis

Simply supported slabs (prevents the corners from lifting). In this case the maximum bending moment per meter width are given by the following equation

$M_x= \alpha_x W l_x^2$

$M_y= \alpha_y W l_x^2$

$M_x$=Bending moment for short span strip

$M_y$= Bending moment for long span strip

$\alpha_x$& $\alpha_y$ are moment coefficient depending on the ratio $r=l_y/l_x$ (Given in Table 27 IS:456-2000).

At least 50% of the tension reinforcement provided at mid span should extend to the support. The remaining 50% should extend to within $0.1l_x$ or $0.1l_y$ of the support.

For a two way slab the depth is assumed on the basis of span/depth ratio based on deflection control

**Problem-1**

**Design a slab over a room 4mtx6mt. The edges of the slab are simply supported and the corners are not held down. The live load on the slab is 3KN/mt$^2$. The slab has a bearing of 150mm on the supporting walls. Use M20 concrete and fe415 steel.**

**Solution: -**

Short span between centres of bearing=4+0.15=4.15mt

Long span between centres of bearing=6+0.15=6.15mt

Assuming 0.3% steel, modification factor=1.43

Effective depth required=Short span/20×M.F=4150/(20×1.43)=145mm

Providing 8mm dia bars at a clear cover of 15mm

Effective cover to reinforcement= 15+19mm

Overall depth= 145+19=164mm

Providing overall depth of 170mm

Actual effective depth of short span= 170-19=151mm

Actual effective depth of long span=151-8=143mm

Effective short span $l_x$=4.15mt

Effective long span $l_y$=6.143mt

$r= l_y/ l_x$=6.143/4.15=1.48

$\alpha_x$=[(1.48-1.4)×(0.104-0.099)/(1.5-1.4)]+0.099=0.103

$\alpha_y$=[(1.48-1.4)×(0.051-0.046)/(1.5-1.4)]+0.046=0.047

dead load on slab=25x0.170=4250N/mt$^2$

20mm Floor finish= 24x20=480 N/mt$^2$

Live load=3000 N/mt$^2$

Total load=7730 N/mt$^2$

Factored load=1.5×7730=11595 N/mt$^2$

$M_{ux}$= $\alpha_x W l_x^2$=0.103×11595×4.15$^2$=20568.57N.mt

$M_{uy}$= $\alpha_y W l_x^2$=0.047×11595×4.15$^2$=9385.66N.mt

Equating $M_{ulim}$ to $M_{ux}$

0.138 $f_{ck}bd^2$=20568.57x10$^3$

d=86mm

Provide $d_{effective}$=151mm

Steel for short span

$M_{ux}/bd^2$=20568.57×1000/(1000×151$^2$)=0.902

$P_t$=50[1-√ (1-4.6×0.902/20)/415/20]=0.264%

$A_{st}$=0.264/[100×1000×151]=398.64mm$^2$

Spacing of 8mm dia bar=( 1000×50.3)/398.64=125mm

Provide 8mm dia bars @120mm centre to centre in short span direction

**Long span**

$M_{uy}/bd^2$=9385.66×10$^3$/1000×143$^2$=0.459

$P_t$=50[1-√ (1-4.6×0.459/20)/415/20]=0.131%

$A_{st}$=0.131/100×1000×143=187.33mm$^2$

Spacing of 8mm dia bar= 1000x50.3/187.33=267mm

Provide 8mm dia bars @260mm centre to centre in long span direction

In each principal direction alternate bars are bent up at 1/10$^{th}$ of the respective span between centres of bearings.

**Check for shear**

**Short span**

$V_u$=$Wl_x$/2

Here clear span is considered

= 11595×4/2=23190N

$\tau_v$= $V_u$/bd=23190/1000×150=0.15N/mm$^2$

Actual area of bottom steel near support =1000/120×50.3/2=208.35mm$^2$

Percentage of bottom steel near support= 208.35×100/100×151=0.14%

$\tau_c$=0.28N/mm$^2$

For a slab having 170mm thickness k=1.26

k $\tau_c$=1.26×0.28=0.352 N/mm$^2$

$\tau_v$< k $\tau_c$Thus slab is safe in shear

**Check for development length**

$A_{st}$= 208.35mm$^2$

Moment of resistance to the above quantity of steel=$M_1$=0.87$f_y A_{st}$d[1-$A_{st} f_y$/bd$f_{ck}$]

0.87×415×208.35×151[1-208.35×415/(1000×151×20)]=11.034×10$^6$Nmm

$V_u$=23190N

$l_0$=Anchorage length beyond the centre of support=150/2-15=60mm

1.3$M_1/v_u$+$l_0$=(1.3×11.0.34x10$^6$/23190)+60=678mm

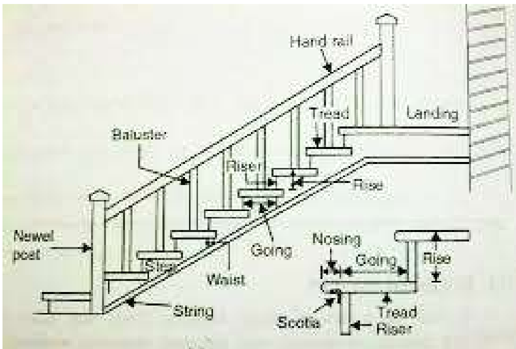
Development length $l_d$= ϕ×0.87$f_y$/4 $\tau_{bd}$=0.87×415×8/(4×1.92)=376mm

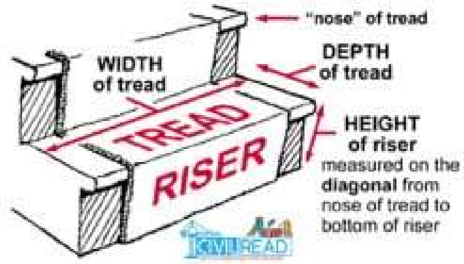
We find $l_d$<1.3$M_1/v_u$+$l_0$

The design is safe in anchorage.

## 6.4 design of Dog legged Stair case

## Terminology




**Flight:-** Flight is the length of the stair case between two landings. The number of steps in a flight varies from 3 to 12.

**Landing:-** Landing is the intermediate horizontal portion provided in a stair case.

**Rise:-** the vertical height of the step is called rise or riser. It varies from 150mm to 180mm for a residential landing and 120 to 150mm for public building.

**Trade:-** The horizontal distance between two risers on a step is called as trade. The width of the trade kept as 200 to 250mm for residential building and 200 to 300mm for public building.

**Going and Nosing:-** The horizontal distance between the two risers is known as going and the portion projecting out from the riser surface is called as nosing.

**Proportioning of stair case:-**

A stair case is proportional on the basis of space available and some thumb rules mentioned below

(i) 2×riser+tread=600 to 640mm

(ii) Riser ×tread= 40000 to 42000mm$^2$

(iii) Width of private stair is about 900mm and that of public stairs is kept about 1800mm to 2400mm.

(iv) Clear height between a flight and the other vertically above it should not be less than 2.1mt.

(v) The angle of flight with the horizontal should be between 25°to 40°

(vi) The free flow of users the width of landing should be equal to width of the stairs

## 6.5 Stair slab spanning longitudinally

The stair slab supported at the bottom and top of the flight is called as stair slab spanning longitudinally. Dog legged, open well and Quarter turn stairs come under this category.

**Clause No.33 of IS:456-2000 gives guidelines about the design of such stairs which are to be referred**

### Design of Stairs Spanning Longitudinally

The stairs spanning longitudinally are supported at the bottom and the top of the flights.

These types of stairs are not supported at the sides by means of beams. Here, for the design purpose, the bending moment is calculated per unit width as $wl^2/10$, where 'w' is the load per unit horizontal area and 'l' is the effective horizontal span.

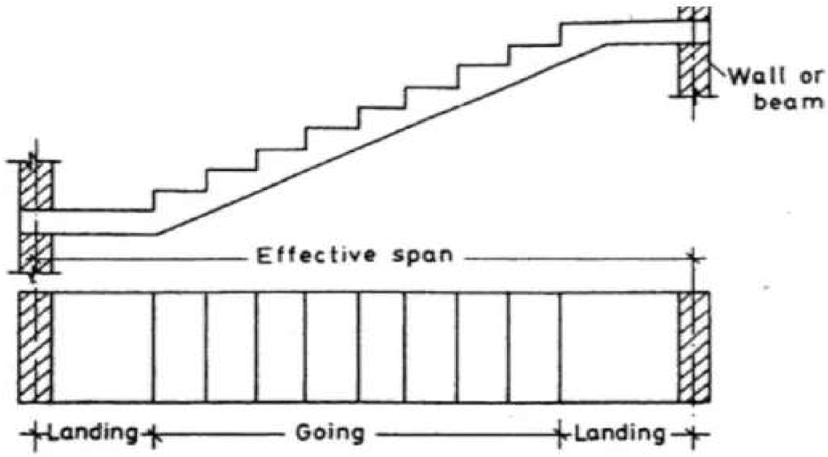



Figure-1: Stair Spanning Longitudinally

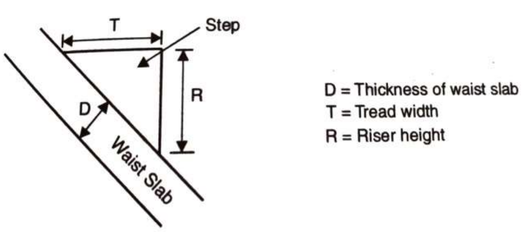
Sometimes, stairs are cast along with the landings that are supported on the walls (Figure-1). In such cases, the effective span is taken as the horizontal distance between the centres of the bearings. Here, the maximum bending moment is calculated as $wl^2/8$.

**Loads on stairs**

(a) Live loads:- As per IS:875 (i) Stairs liable for overcrowding =5KN/mt$^2$

(ii) Not liable for overcrowding =3 KN/mt$^2$

(b) Dead loads:- dead loads are to be calculated per unit horizontal area. The dead load of stairs consists of (i) Dead weight of steps (ii) dead weight of waste slab




(i) Dead weight of 1 step (for 1 m width)

$W_1$=Area of steps x 1 x unit weight of RCC =R $\times\frac{T}{2}\times 1 \times 25 = 25RT\frac{1}{2}$

(ii) Weight of Steps per metre length in plan

$w_1 = 25 \times R \times T/2T = 25/2 \times R$

(ii) Weight of waist slab

$w_2 = \sqrt{R^2+T^2} \times D \times 1 \times 25 = 25D\sqrt{R^2+T^2}$

Weight of waste slab per metre length in plan=

$w_2 = (25D\sqrt{R^2+T^2})/T$

Total load W= $w_1 + w_2$

**Problem-1**

**The main stair of an office building has to be located in a stair measuring 3.5mt×5.5mt. The vertical distance between the floors is 3.75mt. Design the stair allows a live load of 2000N/mt$^2$. Use M20concrete and Fe415 steel.**

**Solution**: -

Height of each flight=3.75/2=1.875mt

Assuming 150mm Risers

No. of Risers required=1875/150=12Nos

Hence the actual rise of each riser=1875/12=156.2mm

No. of treads in each flight=No. of risers-1

=12-1=11nos

Let the width of the stairs be 1600mm

Treads of the steps be 270mm

**Design of first flight**

Let the bearing for the flight be 150mm

Effective span=2.97+1.6+0.15/2=4.645mt

Let the thickness of the waist be 220mm

**Load Calculation**

Dead load of 220mm waist=25×220=5500N/mt$^2$

Ceiling finish (12.5mm) = 24×12.5=300N/mt$^2$

Total=5800 N/mt$^2$

Load per square metre on plan= Slope×$\frac{\sqrt{(R2+T2)}}{T}$

=5800×√(156.2$^2$+270$^2$)/270=6700 N/mt$^2$

Dead load of steps(156.2/2mm average)=78.1×25=1950 N/mt$^2$

Top finish(12.5mm)=12.5×24=300 N/mt$^2$

Live load=2000 N/mt$^2$

Total=10952 N/mt$^2$

Maximum bending moment per metre width of stairs=10952×4.645$^2$/8=29538Nmt

Ultimate moment $M_u$=1.529538=44307Nmt

0.138$f_{ck}$bd$^2$=44307×10$^3$

0.138×20×1000×d$^2$=44307×10$^3$

d=127mm

**Steel calculation**

Provide 10mm dia bars

Effective cover=15+5=20mm

Overall depth required=127+20=147mm

Provide an overall depth of 220mm

Effective depth=d=220-20=200mm

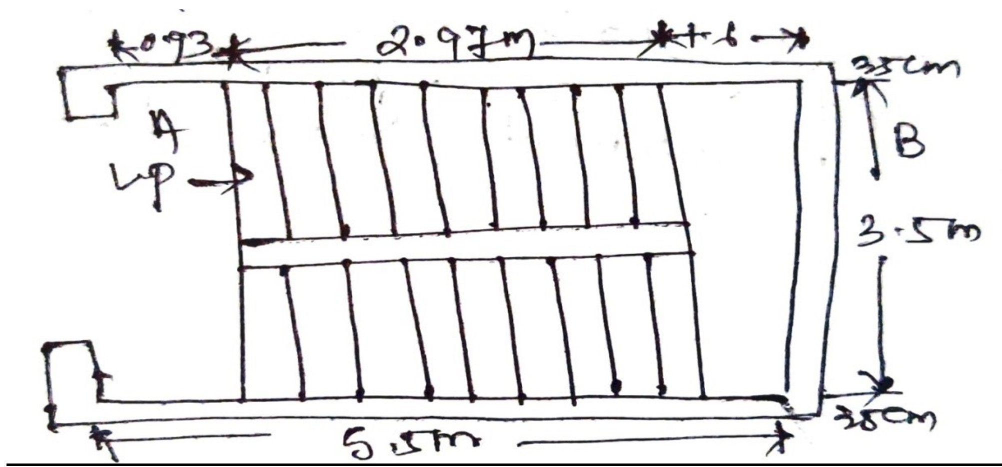
$M_u$/bd$^2$=44307×10$^3$/1000×200$^2$=1.11

$$p_t = 50\left[\frac{\sqrt{1-4.6\times\frac{1.11}{20}}}{\frac{415}{20}}\right]$$

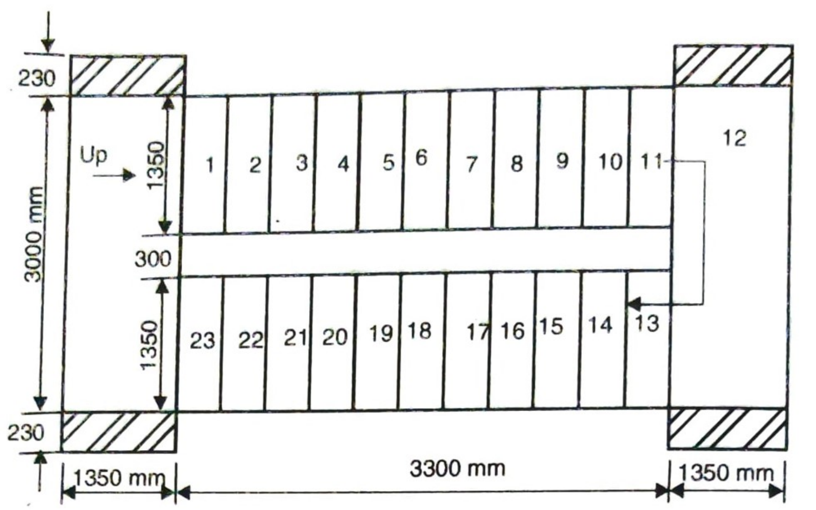
=0.33%

$A_{st}$=0.33×1000×200/100=660mm$^2$

Spacing of 10mm dia bars=1000×79/660=120mm
Provide 10mm dia bars @120mm C/C
Distribution steel=0.12/100×1000×220=264mm$^2$
Spacing= $\frac{1000\times50.3}{264} = 189mm$
Provide 8mm dia bars @180mm C/C




**6.5 Problem-2: Design the staircase shown in fig., if the landing slab is supported on sides instead of ends, by brick walls 230mm thick.**




**Solution.**

- Effective span of flight

$$X = Y = \frac{1350}{2} = 675\text{mm} < 1.0\text{m}$$

Hence Effective span of flight $= 3300 + 675 + 675 = 4650$mm
Thickness of waist slab $= \frac{4650}{20} = 232.5$ say $225$mm
Overall thickness $= 250$mm and $d = 225$mm

- Loading
  (a) Waist slab

Weight of waist slab in plan $= 0.25\sqrt{1 + \dfrac{150^2}{300^2}} \times 25 = 6.98\text{kN/m}$

Weight of steps (per m width) $= \dfrac{1}{2} \times 0.15 \times \dfrac{0.30}{0.30} \times 25 = 1.88\text{kN/m}$

Total dead load $= 6.98 + 1.88 = 8.86\text{kN/m}$

Live lead $= 5\text{kN/m}$

$DL + LL = 13.86\text{kN/m}$

Factored load $= 1.5 \times 13.86$

$= 20.8\text{kN/m}$

$DL$ of slab $= 0.25 \times 1.0 \times 25$

$= 5.75\text{kN/m}$

$LL$ of slab $= 5.0 \times 1.0$

$= 5\text{kN/m}$

$DL + LL = 10.75\text{kN/m}$

Landing $= 1.5 \times 10.75$

Factored load $= 16.13\text{kN/m}$


Weight of steps (per m width) $= \frac{1}{2} \times 0.15 \times \frac{0.30}{0.30} \times 25 = 1.88\text{kN/m}$

Total dead load $= 6.98 + 1.88 = 8.86\text{kN/m}$

Live lead $= 5\text{kN/m}$

$DL + LL = 13.86\text{kN/m}$

Factored load $= 1.5 \times 13.86$

$= 20.8\text{kN/m}$

$DL$ of slab $= 0.25 \times 1.0 \times 25$

$= 5.75\text{kN/m}$

$LL$ of slab $= 5.0 \times 1.0$

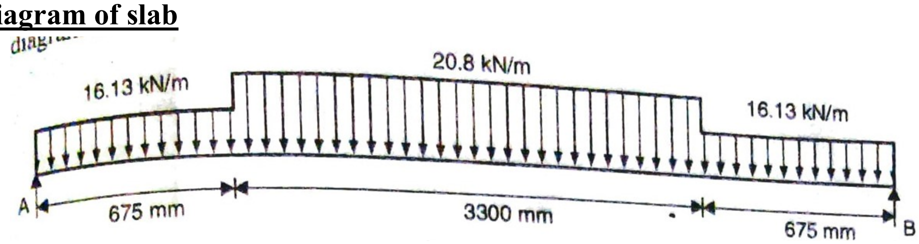
$= 5\text{kN/m}$

$DL + LL = 10.75\text{kN/m}$

Factored load $= 1.5 \times 10.75$

$= 16.13\text{kN/m}$


**Load diagram of slab**




$$R_A = R_B = \frac{16.13 \times (0.675 \times 2) + (20.8 \times 3.3)}{2} = 45.2\text{kN}$$

Maximum BM, Mu

$$= [45.2 \times (0.675 + 1.65)] - \left[16.13 \times 0.675\left(\frac{3.3 + 0.675}{2}\right) + \left(20.8 \times \frac{3.3}{2} \times \frac{3.3}{4}\right)\right]$$

$= 55.14\text{kNm}$

$M_{\text{ulim}} = 0.138 f_{dd} b d^2$

$= 0.138 \times 20 \times 1000 \times 225^2$

$= 139.73 \times 10^6\text{Nm} = 139.73\text{kNm} > 55.14 kNm$

Hence section can design as singly reinforced .

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{b d f_{ck}}\right)$$

$$\begin{aligned}
55.14 \times 10^6 &= 0.87 \times 415 \times A_{st} \times 225 \left(1 - \frac{A_{st} \times 415}{1000 \times 225 \times 15}\right) \\
A_{st}^2 - 8131.76 A_{st} + (5.52 \times 10^6) &= 0 \\
A_{st} &= \frac{8131.76 \pm \sqrt{8131.76^2 - (4 \times 5.52 \times 10^6)}}{2} \\
&= 748 \text{mm}^2
\end{aligned}$$

using $12\phi$ bars spacing= $\frac{113}{748} \times 1000 = 151mm$

**provide12$\phi$ bars @150mm c/c.**

Distribution steel@ 0.12% $= \frac{0.12}{100} \times 1000 \times 250 = 300\text{mm}^2$

Spacing of 8 Ø bars $= \frac{50}{300} \times 1000 = 166.67\text{mm}$

Provide 8 Ø @ 160 mm c/c.

- **Design of landing slab**

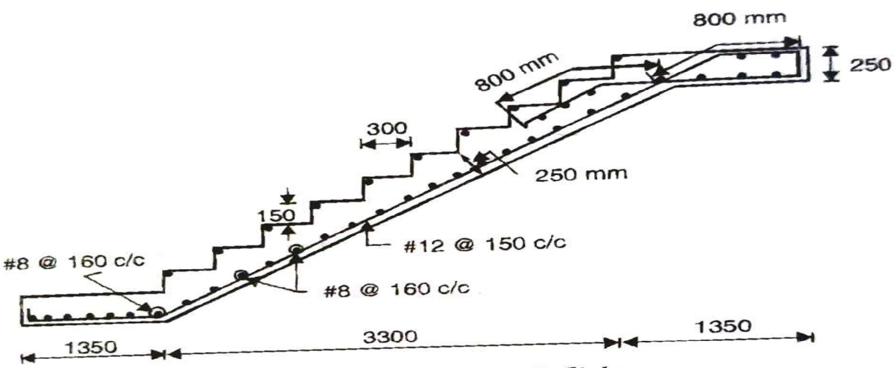
$$\text{Effective span } = 3.00 + \frac{0.23}{2} + \frac{0.21}{2} = 3.23\text{m} = 3230\text{mm}$$

Factored load $= 16.13\text{kN/m}$

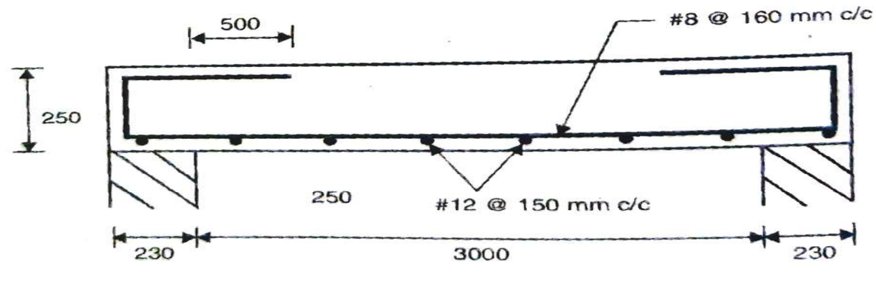
BM= $= \frac{wl^2}{8} = 16.13 \times \frac{(3.23)^2}{8} = 21.04\text{kNm}$

$$\begin{aligned}
21.04 \times 10^6 &= 0.87 \times 415 A_{st} \times 225 \left[1 - \frac{A_{st} \times 415}{1000 \times 225 \times 20}\right] \\
&= 81235.25 A_{st} - 7.49 A_{st}^2 \\
A_{st}^2 - 10845.83 A_{st} + 2.81 \times 10^6 = 0 & \\
A_{st} &= \frac{10845.83 \pm \sqrt{(10845.83)^2 - 4 \times 2.81 \times 10^5}}{2} \\
&= \frac{10845.83 - 10314.65}{2} = 265.59\text{mm}^2 \\
\text{Minimum steel} &= 0.12\% = 300\text{mm}^2 \text{ as above}
\end{aligned}$$

hence provide 8 Ø @ 160mm c/c.



(a) Reinforcement in stair flight



Note: All dimensions are in mm.

(b) Reinforcement in landing flight

## POSSIBLE SHORT QUESTIONS WITH ANSWER

**1. Define one way slab.[W-2015,S-2019]**

Answer:- One way slabs are those slabs in which the $l_y/l_x$ ratio is greater than 2. This type of slab is also called as slab spanning in one direction as the bending takes place only along the short span.

**2. Define two way slab.**

Answer:- Two way slabs are those slabs which are supported on all the four edges and having $l_y/l_x<2$. In this type of slabs bending occurs in both the direction long direction as well as short direction.

**3. Write the recommendation of code for effective span of a simply supported slab.[W-2018]**

Answer:- The effective span is taken as the smaller of the following (i) Centre to centre of support (ii) Clear distance between the support plus the effective depth.

**4. What will be the minimum reinforcement for slab?**

Answer:- The area of reinforcement in either direction in a slab should be less than 0.15% of the total cross sectional area in case of mild steel reinforcement. In the case of high strength deformed bars the value can be reduced to 0.12%.

**5. What will be the maximum diameter of reinforcement in slab?[W-2017]**

Answer:- The maximum diameter of the reinforcing bar in a slab should not exceed $1/8^{th}$ of the total thickness of the slab.

**6. What is stair case?[W-2017,W-2019]**

Answer:- A stair case is a passage through which consecutive floors exist. It is a material of RCC or wood.

**7. Write the limiting values of risers?**

Answer:- The vertical height of a step is called riser. It varies from 150mm to 180mm for residential building and 120mm to 150mm for public building.

**8. Write the accepted relationships between tread and risers.**

Answer:- Riser plus tread= 600mm

**9. Write the formula for effective span of stair spanning in same direction as landing.[S-2019]**

Answer:- Going+landing width on each side+1/2thickness of wall on each side.

# POSSIBLE LONG TYPE QUESTIONS

1. Design a Dog legged stair case with the following parameters (i) Width=1.2mt (ii) Floor to floor height=3.3mt (iii) Size of stair hall=3mt×6mt (iv) Thickness of walls=23cm on each side. Use M20 Concrete and Fe415 steel.[W-2015,W-2016]

2. The main stair of an office building has to be located in a stair measuring 3mt×6mt. The vertical distance between the floors is 3.75mt. Design the stair allows a live load of 2000N/$mt^2$. Use M20concrete and Fe415 steel.

3. Design a simply supported RCC slab to carry a uniformly distributed load of 2 KN/$mt^2$ (including its self-weight) over an effective span of 3.5mt. Use M20 Concrete and Fe415 steel.[W-2019]

4. Design a RCC slab with a clear span of 3mt supported on the sides over a 230mm thick wall and is carrying a live load of 4 KN/$mt^2$. Use M20 Concrete and Fe415 steel.[W-2019]

5. Design a RCC slab of size 5mtx6mt simply supported on all four edges with corners held down. The slab is carrying a load of 4 KN/$mt^2$ including floor finish. Use M20Concrete and Fe415 steel. [W-2016,W-2018]

6. Write the difference between one way slab and Two way slab.

# CHAPTER NO. -07

# DESIGN OF AXIALLY LOADED COLUMNS AND FOOTINGS (LSM)

## Learning objective

*7.1 Assumptions in limit state of collapse- compression.*

*7.2 Definition and classification of columns, effective length of column. Specification for minimum reinforcement; cover, maximum reinforcement, number of bars in rectangular, square and circular sections, diameter and spacing of lateral ties.*

*7.3 Analysis and design of axially loaded short square, rectangular and circular columns (with lateral ties only).*

*7.4 Types of footing, Design of isolated square column footing of uniform thickness for flexure and shear.*

## 7.1 ASSUMPTIONS IN LIMIT STATE OF COLLAPSE-COMPRESSION:-

**(IS:456-2000 Clause no.39)**

In addition to assumptions given in clause no.38.1 (a) to 38.1(e) for flexure.

The following shall be assumed

(a) The maximum compressive strain in concrete in axial compression is taken as 0.002

(b) The maximum compressive strain at the highly compressed extreme fibre in concrete subjected to axial compression and bending and when there is no tension on the section shall be 0.0035 minus 0.75 times the strain at the least compressed extreme fibre.

## 7.2 DEFINITION AND CLASSIFICATION OF COLUMNS, EFFECTIVE LENGTH OF COLUMN. SPECIFICATION FOR MINIMUM REINFORCEMENT; COVER, MAXIMUM REINFORCEMENT, NUMBER OF BARS IN RECTANGULAR, SQUARE AND CIRCULAR SECTIONS, DIAMETER AND SPACING OF LATERAL TIES.

### Definition of column:-

A column is defined as a vertical compression member which is mainly subjected to axial loads and the effective length of which exceeds 3 times its least lateral dimension.

### Classification of columns:-

Columns are classified based on different criteria such as

1. Shapes of cross-section.
2. Material of construction.
3. Type of loading.
4. Slenderness ratio.
5. Type of lateral reinforcement.

## 1. Shapes of cross-section:-

On the basis of shapes of the cross-section of the column, the column may be classified as (i) Square (ii) Rectangular (iii) Circular (iv) Pentagonal (v) Hexagonal (vi) Octagonal (vii) T shape or L shape

## 2. Material of construction:-

Columns may be classified as following as per the material used for construction (i) Timber columns (ii) Masonry columns (iii) RCC columns (iv) Steel columns (v) Composite columns

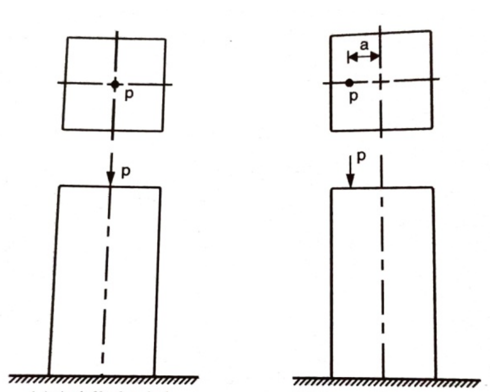
**3. Type of loading:** A column may be classified as follows based on type of loading

(i)Axially loaded column:-

The columns which are subjected to loads acting along the longitudinal axis or centroid of the column section are called axially loaded columns.

(ii) Eccentrically loaded columns:-

Eccentrically loaded columns are those columns in which the loads don't act on the longitudinal axis of the column. They are subjected to direct compressive stress and bending stress both.




(a) Axially loaded column

(b) Eccentrically loaded column (uniaxial bending)

**4. Slenderness ratio:-**

The slenderness ratio of a compressive member is defined asthe ratio of effective length to the least lateral dimension. The columns are classified as following depending upon the slenderness ratio

(i) **Short column:-** The column is considered as short when the slenderness ratio i.e ratio of effective length to its least lateral dimension is less than or equal to 12.

(ii) **Long column:-** If the slenderness ratio of the column is greater than 12, it is called as long or slender column.

## 5. Type of lateral reinforcement:-

(i) Column with longitudinal steel and lateral ties

(ii) Column with longitudinal steel and spiral ties

## Effective length of column:-

The effective length of column should be according to IS:456-2000 Table-28

## Specification for minimum reinforcement:-

## Longitudinal reinforcement

The longitudinal reinforcement consists of steel bars placed longitudinally in a column. It is also called as main reinforcement. The specification for longitudinal reinforcement is given in **IS:456-2000 Clause No. 26.5.3.1**

## Transverse reinforcement

The transverse reinforcement is provided along the lateral direction of the column in the form of ties or spirals enclosing the main steel. The specifications for transverse reinforcement is given in **IS:456-2000 Clause No. 26.5.3.2**

## Cover

The nominal cover for a longitudinal reinforcing bar in a column shall not be less than any of the following (i) 40mm (ii) the diameter of the bar.

In the case of small size of columns of minimum dimensions of 200mm or under whose reinforcing bars don't exceed 12mm, a nominal cover of 25mm may be used.

**Maximum reinforcement, No. of bars in rectangular, square and circular sections, diameter and spacing of lateral ties**

All specifications are given in **IS:456-2000 Clause No. 26.5.3**

# 7.3 ANALYSIS AND DESIGN OF AXIALLY LOADED SHORT SQUARE, RECTANGULAR AND CIRCULAR COLUMNS (WITH LATERAL TIES ONLY).

Given: Factored load ($P_u$)

Material- Grade of concrete and steel

(i) Assume suitable percentage of area of steel $A_{sc}$(Say 0.8% to 4%), $A_{sc}$=p×$A_g$

(ii) Determine area of concrete ($A_c$) in terms of gross area ($A_g$)

$A_c = A_g - A_{sc}$ (iii) Calculate Ag as follows

$P_u = 0.4 f_{ck}(A_g - A_{sc}) + 0.67 f_y A_{sc}$

(iv) Calculate dimensions of column as following

For Square column, $B^2 = A_g$

For Rectangular column, $B \times D = A_g$ (Assume B and Calculate D)

(v) Provide area of reinforcement ($A_{sc}$)

(vi) design lateral ties as per IS Specification

**Problems:1 A reinforced concrete short column is 400×400mm and has 4 bars of 20mm diameter. Determine the ultimate load carrying capacity of column if M20 concrete and Fe 415 steel is used. Assume $e_{min} < 0.05D$.**

**Solution :** given b=400mm ,d= 400mm

$A_{sc} = 4 \times \frac{\pi}{4} 20^2 = 1256.6\ mm^2$

$f_{ck} = 20$ N/mm$^2$

$f_y = 415$ N/mm$^2$

Ag=400×400=160000mm$^2$

$A_c = A_g - A_{sc}$

=16000-1256.6=158743.4 mm$^2$

Load on column $P_u = 0.4 f_{ck}(A_g - A_{sc}) + 0.67 f_y A_{sc}$

=(0.4×20×158743.4)+ (0.67×415×1256.6)=1619344.8N=1619.3KN

∴The ultimate load carrying capacity of the column is 1619.3kN

**Problem:2.Determine the steel required to carry a load of 980kN on a rectangular column of size 300 × 400 mm. The grade of concrete and steel are M20 and Fe 415 respectively. Assume that the column is short.**

$f_{ck}$ = 20 MPa, $f_y$= 415 MPa, P = 980 kN Area of steel $A_{sc}$ = ?

Area of concrete $A_c = A_g - A_{sc}$ = (300 × 400 - $A_{sc}$)

Ultimate load carried by the column

$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$

$980 \times 1.5 \times 1000 = 0.4 \times 20 \times (300 \times 400 - A_{sc}) + 0.67 \times 415 A_{sc}$

$= 960000 - 8 A_{SC} + 278.06 A_{SC}$

$\therefore A_{SC} = 1888.5$ mm$^2$,

Percentage of steel = 100Asc/bD = $\frac{100 \times 1888.5}{300 \times 400}$ = 1.57 % which is more than 0.8% and less than 6% and therefore ok.

Use 20 mm dia. bas, No. of bars = 1888.5/314 = 6.01 say 6

**Problem:3**. **Design a column of size $450\text{mm} \times 600\text{mm}$ and having $3\text{m}$ unsupported length. The column is subjected to a load of $2000\text{kN}$ and is effectively held in position but not restrained against rotation. Use M20 concrete and Fe 415 steel.**

**Solution**. Given: $b = 450\text{mm}$, $d = 600\text{mm}$

$= 3\text{m} = 3000\text{mm}$

$$
\begin{aligned}
L &= 3\text{m} \\
P &= 2000\text{kN} \\
f_{ck} &= 20\text{N/mm}^2 \\
f_y &= 415\text{N/mm}^2 \\
l &= 10L
\end{aligned}
$$

Hence, it is a short column.

- Minimum eccentricity ( $e_{\min}$ )
  For $d = 600\text{mm}$

$$
\begin{aligned}
e_{\min} &= \frac{L}{500} + \frac{D}{30} \\
&= \frac{3000}{500} + \frac{600}{30} = 26.0\text{mm} > 20\text{mm} \\
e_{\min} &= 20\text{mm} \\
0.05D &= 0.05 \times 600 \\
&= 30.0\text{mm} > e_{\min}
\end{aligned}
$$

For $b = 450\text{mm}$

$$
\begin{aligned}
e_{\min} &= \frac{L}{500} + \frac{D}{30} \\
&= \frac{3000}{500} + \frac{450}{30} \\
&= 21.0\text{mm} > 20\text{mm}
\end{aligned}
$$

$$
\begin{aligned}
\therefore \quad e_{\min} &= 20\text{mm} \\
0.05b &= 0.05 \times 450 = 22.5\text{mm} > \epsilon_{\min}
\end{aligned}
$$

Hence it is designed as a short axially loaded column.

- Factored load, $P_\omega = 0.5P = 1.5 \times 2000 = 3000\text{kN or } 3000 \times 10^3\text{N}$
  Area of longitudinal steel $(A_\kappa)$

$$
A_g = 450 \times 600 = 270000\text{mm}^2
$$
$$
A_c = A_g - A_{sc} = 270000 - A_{sc}
$$

For an axially loaded short column

$$
\begin{aligned}
P_w &= 0.4 f_{ck} \cdot A_c + 0.67 f_y A_{sc} \\
3000 \times 10^3 &= 0.4 \times 20(270000 - A_{sc}) + 0.67 \times 415 \times A_{sc} \\
270.05 A_{sc} &= 840000
\end{aligned}
$$

$$
A_{sc} = 3110.5\text{mm}^2
$$

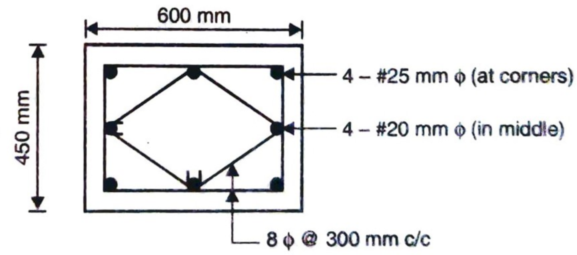
Percentage of reinforcement $= \frac{A_{sc}}{A_g} = \frac{3110.5}{270000} = 1.15\%$

It is between 0.8 to 4%. Hence OK

Using $425\text{mm}\phi$ bars $= 4 \times 490.6 = 1962.5\text{mm}^2$

and $4-20\text{mm}\phi$ bars $= 4\times 314 = 1256$
$A_{sc\text{ Provided}} = 3218.5\text{mm}^2 > 3110.5\text{mm}^2$ Hence OK

- Lateral ties
  The diameter of lateral ties should be more than.
- $\frac{1}{4}\times 25 = 6.25$
  (ii) 6mm
  Using 8mm dia ties
- The pitch of ties should not exceed the following.
  (i) Least lateral dimension $= 450$mm
  (ii) $16\times 20 = 320$mm
  (iii) 300mm
  ∴ Provide $8\text{mm}\phi$ ties @ 300 mm c/c
  The arrangement of reinforcement is shown in fig.




## Design of circular column

Problem:3 **Design a circular column of diameter 400mm subjected to a load of 1200kN. The column is having spiral ties. The column is 3m long and is effectively held in position at both ends but not restrained against rotation. Use M25 concrete and Fe415 steel.**

**Solution :** Given L=3m

P=1200kN

D=400mm

$f_{ck} = 25$ MPa, $f_y = 415$ MPa

effective length, $l_{eff} = 1.0l$ [since the column effectively held in position at both ends but not restrained against rotation]

hence it is a short column. Minimum eccentricity $(e_{min}) = \frac{l}{500}+\frac{D}{30} = \frac{3000}{500}+\frac{400}{30}$

$$= 19.33 > 20mm$$

$$e_{min}\text{=20mm}$$

$$\frac{e_{min}}{D} = \frac{20}{400} = 0.05$$

∴ It is designed as axially loaded column.

**Area of steel ($A_{sc}$)**

$$P_u = 1.5\times P$$

=1.5×1200=1800kN=1800× $10^3 N$

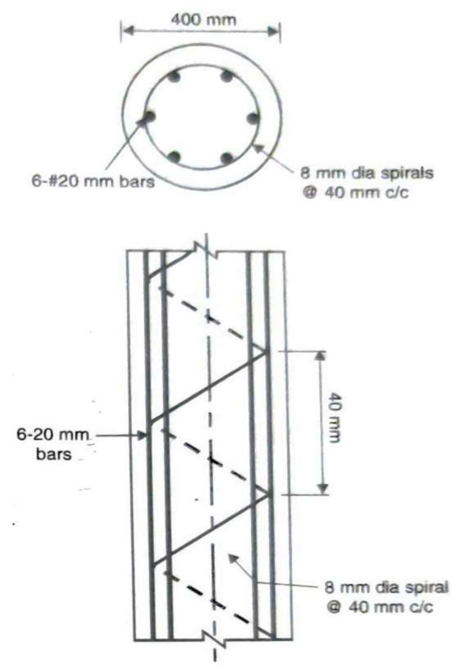
For a column with helical ties, the load carrying capacity is given by

$$\text{Percentage of steel } = \frac{1793 \times 100}{125663.7} = 1.43\%$$

$$P_u = 1.05(0.4 f_{ck} A_c + 0.67 f_y A_{sc})$$
$$A_g = \frac{\pi}{4} \times 400^2$$
$$= 125663.7\text{mm}^2$$
$$A_c = A_g - A_{sc} = 125663.7 - A_{sc}$$
$$1800 \times 10^3 = 10.5[0.4 \times 25 \times (125663.7 - A_s) + 0.67 \times 415 \times A_{sc}]$$
$$268.05 A_{sc} = 480531.15 = 1793\text{mm}^2$$

It is between 0.8 and 4%. Hence OK
Using 20mm dia bars $A_\phi = 314\text{mm}^2$
No. of bars reqd. $= \frac{1793}{314} = 4.7$ say 6
∴ Provide $6 - 20$mm dia bars $(A_{sc\text{ provided}} = 6 \times 314 = 1884\text{mm}^2)$

- **Helical Reinforcement**




Core diameter, $D_c = 400 - 2 \times 50$
$= 300\text{mm}$

[Assuming clear cover = 50 ]

$$\text{Area of core} = \frac{\pi}{4} \times 300^2 - 1884$$
$$= 6880.8\text{mm}^2$$
$$\text{Assuming pitch} = p$$

Volume of core per pitch=6880.8×p

Using 8mm$\phi$ spiral
Volume of one spiral per pitch $= \frac{\pi}{4} \times 8^2 \times \pi(300 - 8)$

$$= 46110.8\text{mm}^2$$

$$\frac{\text{Volume of Helical reinforcement}}{\text{Volume of core}} = \frac{46110.8}{68801.8p}$$
As per IS code

$$\frac{46110.8}{68801.8p} \leq 0.36\left(\frac{A_g}{A_c} - 1\right)\frac{f_{ck}}{f_y}$$

$$\leq 0.36\left(\frac{125663.7}{68801.8} - 1\right) \times \frac{25}{415}$$

**Maximum pitch:**

It should not be more than 75mm or $\frac{\text{Core diameter}}{6}$

$\frac{300}{6} = 50\text{mm}$

**Minimum pitch:**

(i) 25mm

(ii) 3 × dia of helical reinforcement = 3 × 8 = 24mm

∴ Provide 8mm$\phi$ spirals @ 40mm c/c as shown in Fig.

# 7.4 TYPES OF FOOTING, DESIGN OF ISOLATED SQUARE COLUMN FOOTING OF UNIFORM THICKNESS FOR FLEXURE AND SHEAR.

## Types of footing:-

Following are some of the common types of footings

**(i) Isolated footings**

(a) Square footings

(b) Rectangular footings

(c) Circular footings

**(ii) Combined footings**

(a) Oval footing

(b) Rectangular footing

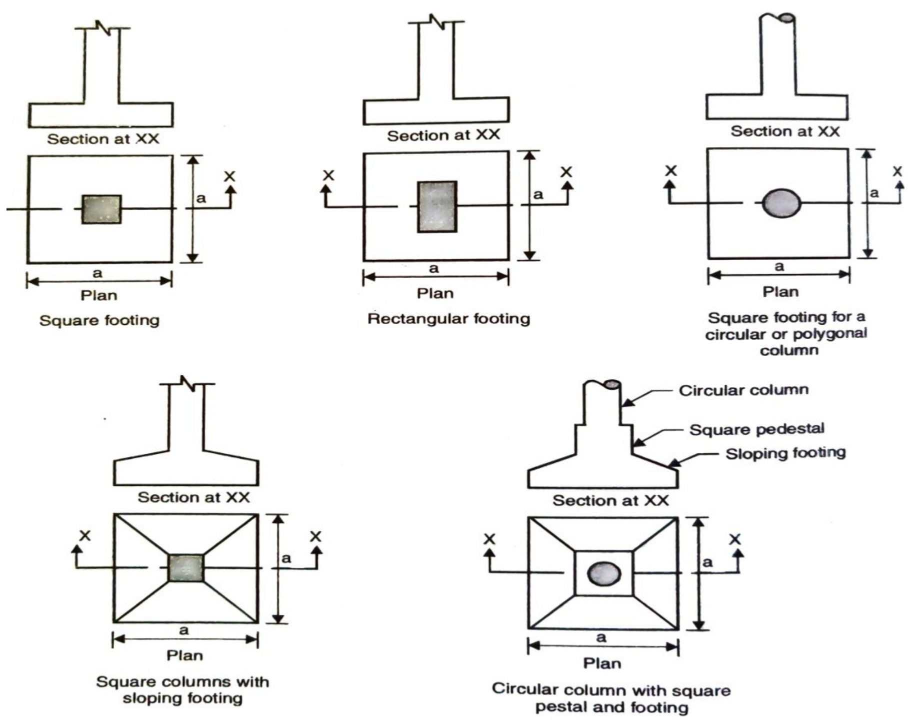
(c) Trapezoidal footing

**(iii) Continuous/wall footings/Strip footings**

**(iv) Strap footing**

**(v) Raft/Mat footing**

**(vi) Pile footing**



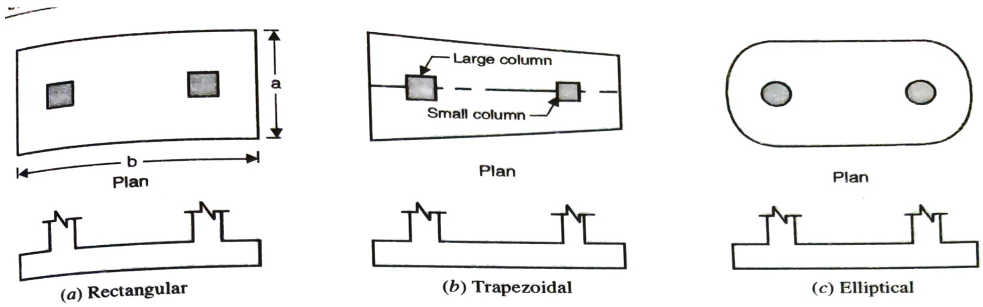



**Fig. 14.2. Combined footings.**

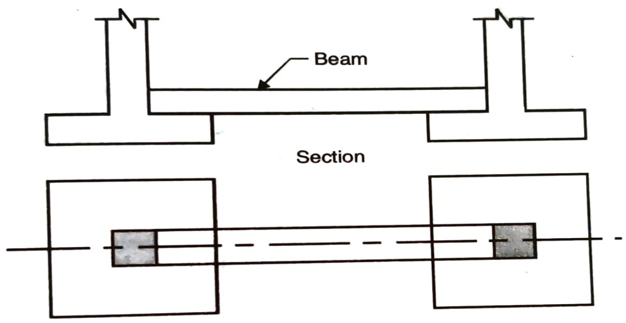



**Fig. 14.3. Strap foundation.**

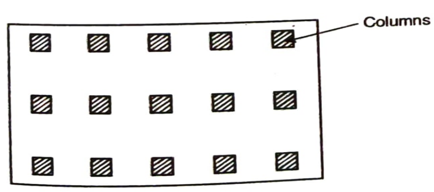



**Fig. 14.4. Raft foundation.**

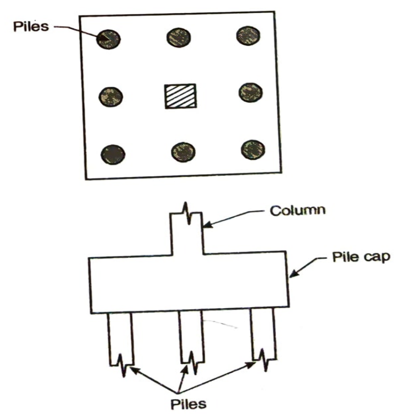



**Fig. 14.5. Pile foundation.**

## Design of Isolated square column footing of uniform thickness for flexure and shear:-

The footings which are provided under single columns are called as isolated footings. These are usually square or rectangular and rarely circular. Isolated footings are ideally provided when loads are small and the soil is not very poor. Isolated footings are of 2 types

(a) Uniform thickness footing

(b) Tapered thickness footing

Given Load on column, Safe bearing capacity of soil, Grade of concrete and steel

1. Find design constants $X_{umax}/d$ and $R_u$ for given steel and concrete grades.

2. Calculate area of footing as follows:

where $w_c$ = load on column

$w_c$ = load on column

$A = \frac{w_c + w_f}{q_0}$ on column wt of footing + pedestal if provided (usually sidered as 10% of $w_c$ )

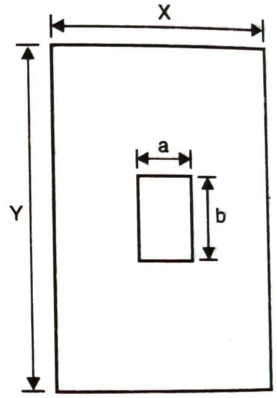
bearing capacity of soil ring:

3. Calculate the size of footing: (a) For square footing, side of footing $S = \sqrt{A}$, round off (b) For rectangular footing, assume one dimension (say $X$ ) and calculate the other dimensions (say $\boldsymbol{Y}$ ) as follows:

$Y = \frac{A}{X}$ round off to nearest 5 or 10cms.

Alternatively, if ratio of width to length of column and footing are assumed to be similar (say $/b$ ), then bending moment is same in both directions.

4. Calculate the soil pressure due to factored column load only, as follows:




$$P_u = \frac{1.5w_c}{X.Y}$$

where

$w_c$ = column load
$X$ = shorter dimension of footing
$Y$ = longer dimension of footing

5 Depth of footing is calculated by the following three criteria and highest value so calculated is adopted in the design:
**(a) By one way shear criterion:** The critical section for one way shear is taken at a distance $d$ (effective depth) from the column's face (fig.14.9).

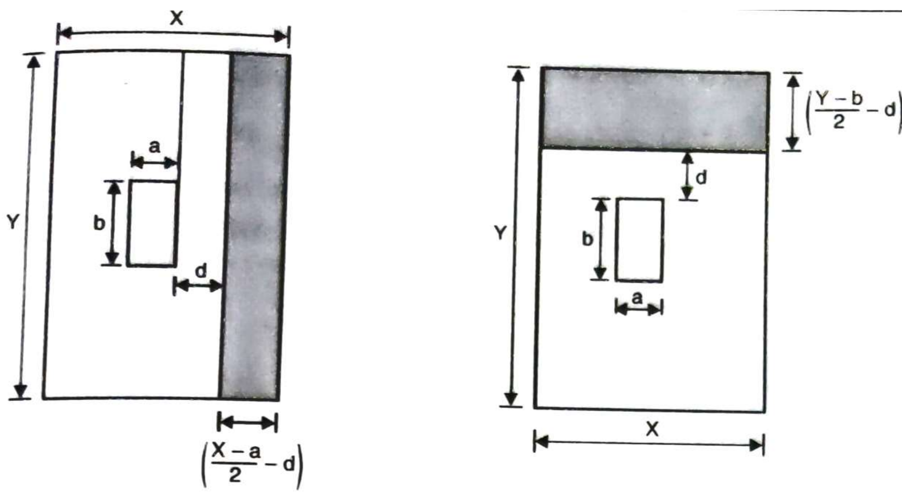
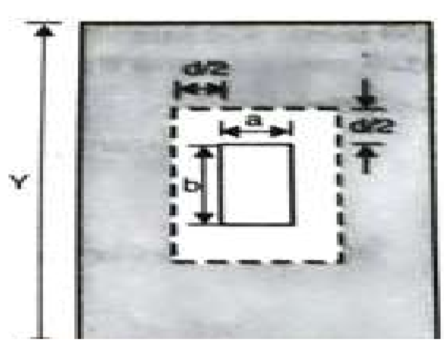



**Fig. 14.9**




Shear force at the critical section

$V_u = p_u \times X \times \left(\frac{Y-b}{2} - d\right)$……………………(i)
Shear force resisted by concrete $= \tau_c X d$ ... (ii)
Equating (i) and (ii)

$$\tau_c X d = p_u \times X \times \left(\frac{Y-b}{2} - d\right)$$

As exact percentage of reinforcement to be provided is not yet known, $\tau_c$ may be assumed as that corresponding to minimum reinforcement, i.e., 0.2%. For M20, this value may be taken as $0.32\text{N/mm}^2$.

**(b) By two way shear criterion:** The critical section for two way shear or punching shear as it is commonly called, is at a distance $d/2$ from the face of the column.
Referring to Fig. 14.10, perimeter of critical section

$$= 2\left(a + \frac{d}{2} + \frac{d}{2} + b + \frac{d}{2} + \frac{d}{2}\right)$$
$$= 2(a + b + 2d)$$

Area of concrete resisting punching shear

A= 2(a+b+2d)× d

Punching shear on the critical section=$p_u\big(XY - (a+d) \times (b+d)\big)$ … … … … $(iii)$

Punching shear resisting by the section =$\tau_c \times A$

$$= \tau_c \times 2(a + b + 2d) \times (b + d)$$

Where $\tau_c = 0.25\sqrt{f_{ck}}$

By equating the two expressions (iii) and (iv) we can calculate the depth of footing.

**(c) By bending moment criterion:** The critical section for bending moment is shown in Fig. 14.11.

$$p_u\left(\frac{X-a}{2}\right)\left(\frac{X-a}{4}\right) = \frac{p_w}{8}(X-a)^2$$

B.M. in $Y$ direction $= \frac{p_u}{8}(Y-b)^2$

$$= 0.36 f_{ck} \cdot \frac{x_{ulim}}{d}\left(1 - \frac{0.42 x_{u\,\text{lim}}}{d}\right) Y d^2$$

equating the (v) and (vi) with the moment of resistance we get value of $d$.

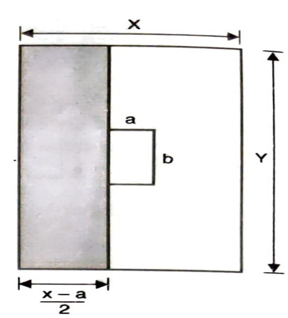


Fig. 14.11.

above shall be adopted as effective depth of the footing.
and substitute $X = Y$ and $a = b$

$$M_u = 0.87 f_y A_{st} d\left(1 - \frac{A_{st} f_y}{X d} f_{ck}\right)$$

The reinforcement area so calculated should not be less than the minimum reinforcement and distributed as per IS code provisions.

NOTE: For design of square footing, follow the above mentioned procedure and substitute $X = Y$ and $a = b$

**Example 1. Design a square footing of uniform thickness for an axially loaded column of $450\text{mm} \times 450\text{mm}$ size. The safe bearing capacity of soil is $190\text{kN/m}^2$. Load on column is $850\text{kN}$. Use M20 concrete and Fe 415 steel.**

Solution. Given,

$$w_c = 850\text{kN}$$

$$\text{safe bearing capacity of soil} = 190\text{kN/m}^2$$

$f_{ck} = 20\text{kN/mm}^2$

$F_y = 415\text{kN/mm}^2$

Load calculation

$$\begin{aligned} w_c &= 850\text{kN} \\ \text{Self wt. of footing, } w_f &= 10\% \text{ of } w_c = 85\text{kN} \\ w_c + w_f &= 850 + 85 = 935\text{kN} \end{aligned}$$

Area of footing

$$A = \frac{w_c + w_f}{q_u} = \frac{935}{190} = 4.92\text{m}^2$$

Side of square footing $= \sqrt{4.92} = 2.22\text{m}$ say $2.25\text{m}$

Factored soil pressure due to column load only

$$p_u = \frac{1.5 \times 850}{2.25 \times 2.25} = 251.85\text{kN/m}^2$$

## Depth of footing by one way shear criterion (Fig. 14.12)

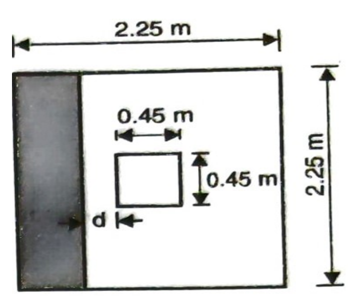


Fig. 14.12.

Critical section shall be at a distance $d$ from the face of the column. Shear force due to factored soil pressure at critical section

$$= 2.25 \times \left(\frac{2.25 - 0.45}{2} - d\right) \times 251.85$$

$= 566.66(0.9 - d)$………………..(i)

Assuming 0.2% steel, $\tau_c = 0.32\text{N/mm}^2$
Shear force resisted by the section
$= \tau_c X d$

$$\tau_c \quad = \frac{0.32 \times 10^6}{10^3} \times 2.25 \times \text{d} \left.\frac{\text{kN}}{\text{m}^2}\right]$$
$$= 720d \ldots \ldots \ldots \ldots (ii)$$

Equating $(i)$ and $(ii)$ we get

$$566.66(0.9 - d) \quad = 720d$$
$$1286.66d \quad = 509.99$$
$$d \quad = \frac{509.99}{1286.66} = 0.396\text{m}$$

$$566.66(0.9 - d) \quad = 720d$$
$$1286.66d \quad = 509.99$$
$$d \quad = \frac{509.99}{1286.66} = 0.396\text{m} \ldots \ldots \ldots \ldots \ldots \ldots \ldots . . (\text{A})$$

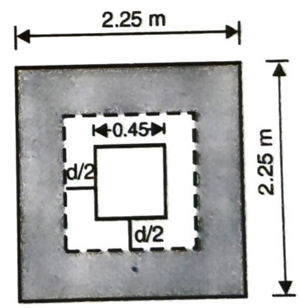


Fig. 14.13.

- **Depth of footing by two way shear (fig.14.13)**
  Considering critical section is at $\frac{d}{2}$ from the face of column.
  Perimeter of critical section $= 4(0.45 + d) = 1.80 + 4d$
- Shear force at critical section $= 251.85 \times (2.25 \times 2.25 - (0.45 + d)^2)$
  $=1274.99 - 251.85(0.2025 + d^2 + 0.9d)$………………(iii)
  Shear force resisted by the critical section=$0.25\sqrt{f_{ck}}$

$$= 0.25\sqrt{20} = 1.118\text{N/mm}^2 = 1118\text{kN/m}^2$$

Shear force resisted=1118 (1.18 + 4d) × d =$2012.4d + 4472d^2$…………(iv)

Equating (iii) and (iv)

$$1274.99 - 251.85(0.2025 + d^2 + 0.9d) \quad = 2012.4d + 4472d^2$$
$$d^2 - 0.423d - 29 \quad = 0$$
$$d \quad = \frac{-0.423 \pm \sqrt{(-0.423)^2 + 4 \times (0.29)}}{2}$$
$$d \quad = 0.367\text{m} \ldots \ldots \ldots \ldots \ldots \ldots \ldots . . (\text{B})$$

- **Depth of footing by bending moment criterion**
  Critical section is an the face of column.
  Bending moment at the critical section

$$M_u = 251.85 \times 2.25 \times \left(\frac{2.25-0.45}{2}\right) \times \left(\frac{2.25-0.45}{4}\right)$$
$$= 566.66 \times \frac{1.80^2}{8}$$
$$= 229.498\text{kNm}$$
$$= 229.498 \times 10^6\text{Nmm} \ldots\ldots\ldots..(\text{v})$$

Moment of resistance at criterion section:

$$\frac{x_{\text{umax}}}{d} = 0.48 \text{ and } R_{\text{u}} = 2.76 \text{ for M20 concrete and Fe 415 steel}$$
$$M_{\text{ulim}} = R_{\text{u}} \times bd^2$$
$$= 2.76 \times 2250 \times d^2 = 6210d^2 \ldots\ldots\ldots\ldots\ldots\ldots..(vi)$$

Equating $(v)$ and $(vi)$, we get

$$229.498 \times 10^6 = 6210d^2$$
$$d = 192.24\text{mm}$$
$$= 0.192\text{m} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(\text{C})$$

From Eqs. [A], [B] and [C], the highest value of $d$ obtained is 0.396m.
Let us adopt

$d = 400$mm.
Overall depth $= 400 + 8 + 50$
$= 458$ say 460mm [Taking clear cover $= 50$mm and 16mm dia. bars]

- Area of steel reinforcement

$$M_u = 0.87 f_y A_{st} d \left(1 - \frac{A_{st} f_y}{b d f_{ck}}\right)$$

$$229.498 \times 10^6 = 0.87 \times 415 \times A_{st} \times 400 \left(1 - \frac{A_{st} \times 415}{2250 \times 400 \times 20}\right)$$
$$A_{st}^2 - 43369.37 A_{st} + 68918318.32 = 0$$
$$A_{st} = \frac{43369.37 \pm \sqrt{43369.37^2 - (4 \times 68918318.32)}}{2}$$
$$= 1652.03\text{mm}^2$$

Minimum reinforcement reqd. $= \dfrac{0.12 \times 2250 \times 460}{100}$
$= 1242\text{mm}^2 < 1652.03\text{mm}^2$

Using 16Ø bars $A_\varnothing = \frac{\pi}{4} \times 16^2 = 201\text{mm}^2$

$$\text{Spacing} = \frac{201 \times 2250}{1652.03} = 273\text{mm}$$

Provide 16$\phi$bars @270mmc/c in each direction.

Check for development length

$$L_d = \frac{0.87 f_y \phi}{4\tau_{bd}} \quad [\tau_{bd} = 1.92\text{N/mm}^2 \text{ for M20 Fe 415. Table 5.5}]$$

$$= \frac{0.87 \times 415 \times 16}{4 \times 1.92} = 752.2\text{mm}$$

$$\text{Available length of bars} = \frac{2250 - 0.45}{2} \times 1000 = 900\text{mm. Hence OK}$$

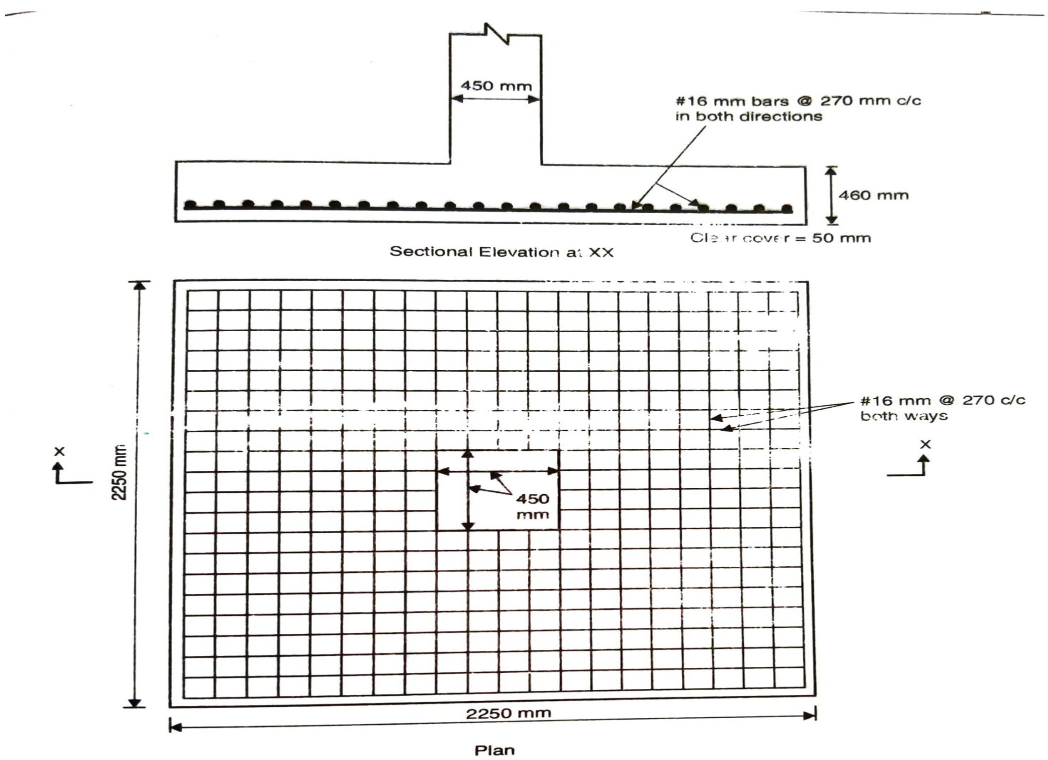



**Fig. 14.14.**

## Numerical problems on wall footing

A brick masonry wall 230mm thick carries a load of 370kN/m inclusive of its own weight. Design the footing of the wall, take bearing capacity of soil as 150KN/mt$^2$ at 1mt depth. Use M20 Concrete and Fe415 steel.

Solution. Given: Thickness of wall $= b = 230\text{mm}$
$w = 370\text{kNm}$ including self weight

$$w \equiv 370\text{kNm}$$
$$q_0 = 150\text{N/mm}^2$$
$$f_y = 415\text{N/mm}^2$$
$$f_{ck} = 20\text{N/mm}^2$$

- Size of footing

$$w_u = 1.5 \times 370$$
$$= 555\text{kN/m}$$
$$\text{Width of footing} = \frac{555}{150} = 3.7\text{m}$$

Hence providing a width of 3.8m i.e., $B = 3.8\text{m}$

Taking 10% of total load as self weight of footing and subtracting it from total ultimate load

Net downward load on soil $= 555 \times 0.9$

$$= 500\text{kN/m}$$

$$\text{Net upward pressure} \quad = \frac{500}{3.8}$$

$$p_0 = 132\text{kN/m}^2/\text{m length of footing}$$

- **Bending moment calculation**
  in the case of brick masonry wall, the critical section for maximum bending is taken at a section midway between the edge of the wall and centre of wall: between the edge of the wall and centre of wall:

$$\begin{aligned} M_u &= \frac{P_0}{2}\left[\frac{B-b}{2}+\frac{b}{4}\right]^2 \\ &= \frac{132}{2}\left[\frac{3.8-0.23}{2}+\frac{0.23}{4}\right]^2 \\ M_u &= 224.1\text{kNm per m} \\ &= 224.1 \times 10^6\text{Nmm} \end{aligned}$$

[$R_u = 2.76$ for M20 and Fe 415]

$$\begin{aligned} d_{\text{reqd}} &= \sqrt{\frac{224.1 \times 10^6}{2.76 \times 1000}} \\ &= 285\text{mm} \end{aligned}$$

Taking 50mm clear cover and 20mm diameter bars.

$$\begin{aligned} D &= 350\text{mm} \\ d &= 350 - 50 - 10 = 290\text{mm} \end{aligned}$$

- **Area of steel**

$$\begin{aligned} M_u &= 0.87 f_y A_{ut} d \left(1 - \frac{f_y A_{st}}{f_{ck} bd}\right) \\ 224.1 \times 10^6 &= 0.87 \times 415 \times A_{st} \times 290 \left(1 - \frac{415 A_{st}}{20 \times 1000 \times 290}\right) \\ A_{st} &\cong 2500\text{mm}^2 \end{aligned}$$

Using 20mm dia bars $A_\emptyset = 314mm^2$

Spacing required $= \frac{314 \times 1000}{2500} = 125\text{mm}$

Hence providing 20mm dia bars @ 120mm c/c.

$$A_{st\ \text{provided}} = \frac{314 \times 1000}{120} = 2616\text{mm}^2$$

$$p_t = \frac{100A_z}{bd} = \frac{100 \times 2616}{1000 \times 290}$$

$$p_t \equiv 0.9\%$$

Minimum steel required $= 0.12\%$

$$= \frac{0.12 \times 1000 \times 350}{100}$$

$= 420\text{mm}^2 < 2616\text{mm}^2$. Hence OK

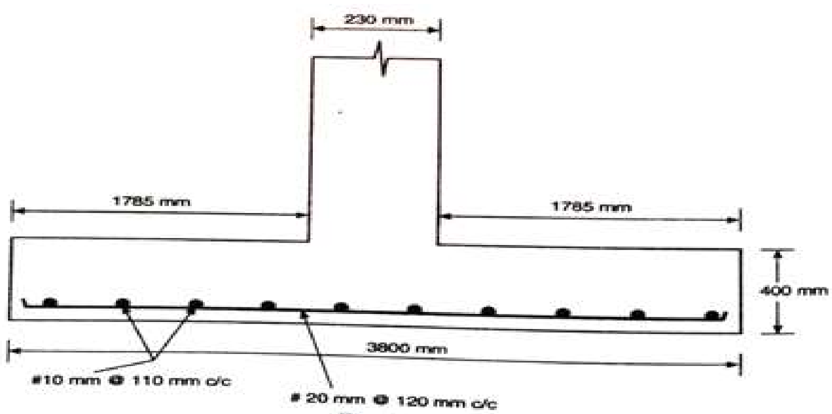
Distribution steel is provided @0.12% $= 420\text{mm}^2$
Using 10mm diameter bars
Spacing required $= 119\text{mm}$
Hence providing 10mm dia bars @ 110mm c/c in the longitudinal direction.

- **Check for shear (one way shear)**
  The critical section for shear is at a distance ' $d$ ' from the face of the wall as shown below:

$$V_u = p_0\left[\frac{(B-b)}{2} - d\right]$$

$$= 132\left[\left(\frac{3.8-0.23}{2}\right) - 0.29\right]$$

$$V_u = 197.34\text{kN per m}$$




$$\tau_v = \frac{V_u}{bd} = \frac{197.34 \times 1000}{1000 \times 290}$$

$$\tau_v = 0.68\text{N/mm}^2$$

$$\tau_c = k.\tau_c$$

For $p_t = 0.9\%$ and M20 concrete
$\therefore$ $\tau_c = 0.60\text{N/mm}^2$ and $k = 1$ for 300mm or more thickness.
$\tau_v > \tau_c$

Hence the footing is not safe in shear therefore revising its depth.

$$\frac{197.34 \times 1000}{1000 \times d} = 0.60\text{N/mm}^2$$

$$d \cong 330\text{mm}$$

$$D = 400\text{mm}$$

$$d = 400 - 50 - 10$$

$$d = 340\text{mm}$$

Hence providing

- **Check for development length**

$$L_d = \frac{\phi(0.87 f_y)}{4\tau_{bd}}$$

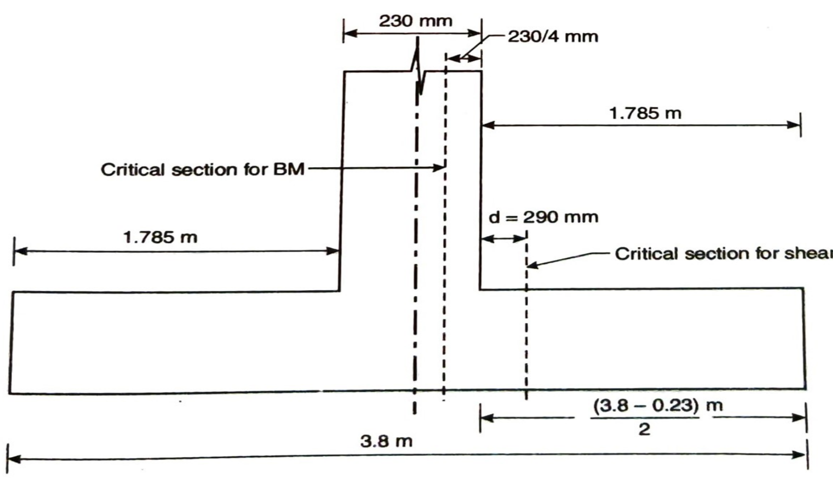
$$= \frac{20 \times 0.87 \times 415}{4 \times 1.92}$$

$$L_d = 941\text{mm} = 0.941\text{m}$$

Providing 50mm clear cover, length of bar available

$$= \left[\frac{1}{2}(B - b) - 0.50\right] = \left[\frac{1}{2}(3.8 - 0.23) - 0.5\right]$$

$= 1.285\text{m} > 0.941\text{m}$. Hence OK




## POSSIBLESHORT QUESTIONS WITH ANSWER

**1. What is Short column?**

**Answer:-** The column is considered as short when the slenderness ratio i.e ratio of effective length to its least lateral dimension is less than or equal to 12.

**2. What is Long column?**

**Answer**:- If the slenderness ratio of the column is greater than 12, it is called as long or slender column.

**3. Write the codal provision of cover for columns.[2015,S-2019]**

**Answer:-** The nominal cover for a longitudinal reinforcing bar in a column shall not be less than any of the following (i) 40mm (ii) the diameter of the bar.

In the case of small size of columns of minimum dimensions of 200mm or under whose reinforcing bars don't exceed 12mm, a nominal cover of 25mm may be used.

**4. Write the formula for minimum eccentricity for all columns.[W-2016]**

**Answer:-** All columns shall be designed for minimum eccentricity given by

$e_{min}$=Unsupported length/500+lateral dimension/30

$e_{min} \geq 20mm$

**5. Write the minimum number of steel bars for rectangular, square, circular column.[W-2015]**

**Answer:-** Minimum number of steel bars for rectangular is 4, for square is 4 and for circular is 6.

**6. What is the main purpose of footing?**

**Answer**:- The main purpose of footing is to transfer the loads from column to soil.

**7. What would be the minimum reinforcement to be provided for Fe415 steel?**

**Answer**:- Minimum reinforcement to be provided in the Isolated footing is 0.12% of the cross-sectional area for Fe415 steel.

8.**where is the critical section for maximum bending moment of a RCC footing**

**Ans :** The critical section for maximum bending moment is at the face of the support of the footing.

# POSSIBLE LONG TYPE QUESTIONS

1. A short column 450mmx450mm is reinforced with 8-16mm diameter bars. Determine the ultimate loads and allowable load carried by the column. Use M20 Concrete and Fe415 steel. Solve by LSM method.[2011,W-2018]
2. Design a circular column 4mt height is effectively held in position but restricted against rotation at one and only. It is carrying an axial load of 1200KN. Design the column if its diameter is restricted to 500mm. Use M20 Concrete and Fe415 steel.
3. Give IS Specification regarding reinforcement in a column.[W-2019]
4. Design a square footing with uniform depth for an axially loaded column with 750KN load. Size of column is 300mmx300mm and safe bearing capacity of soil is 150KN/mt$^2$. Use M20 Concrete and Fe415 steel.
5. Design a rectangular footing of an axial loaded column carrying 1200KN load. Size of column is 600mmx400mm. The safe bearing capacity of the soil is 180Kn/mt$^2$. Use M20 Concrete and Fe415 steel. [W-2018,S-2019]
6. Design a footing for a brick wall 30cm thick carrying a load of 150KN/m per unit length. The bearing capacity of the soil is 100KN/mt$^2$. Use M20 Concrete and Fe415 Steel.[W-2017]
7. what are the codal provision to provide pitch and diameter of lateral ties in column.
[W-2019]